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Cost and Profit Analysis of M / E_k / 1 Queueing System with Removable Service Station

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Abstract

In this paper, an attempt is made to investigate cost and profit analysis of an M/E_k /1 queuing model with a removable service station under N-policy and steady state conditions. We construct the total expected cost function per unit time for two different cases. In first case, we analyze the total expected cost of the system under the condition when the system is turned off and turned on and in second case, we discuss the total expected cost of the system with utilization of idle time and finally total optimal costs in both of cases have been obtained. The notion of the total revenue is introduced to analyze the total profit of the system with respect to the total cost of the system. Finally, observations on TOC and TEP have been drawn with the help of numerical tables and graphic representations.

Keywords: Cost and Profit / Expected Revenue / Computing / Removable Service
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Introduction

The cost analysis of M/E_k/1 queueing system with removable service station is very useful to provide basic framework for efficient design and analysis of several practical situations including various technical systems. For any queueing system cost and profit analysis constitutes a very important aspect of its investigation. Many types of queueing models with a single removable server have been investigated so far. Bell [4] discussed about optimal operation of an M/G/1 queue with removable server. Kimura [10] analyzed an M / G / 1 queueing system with removable server via diffusion approximation. Yadin and Naor [8] discussed queueing system with a

removable service station and introduced the concept of an N policy which turns the server on when the number of customers in the system reaches a certain number N ($N \geq 1$) and turns the server off when there are no customers in the system. Pearn and Chang [9] considered a sensitivity investigation of the N-policy $M/E_k/1$ queueing system with removable service station. Wang [6] discussed about optimal control of $M/E_k/1$queyeing system. Baburaj and Manoharan [3] analyzed a bulk service queueing system with removable servers and obtained steady state probabilities of the system. Bell [1] contributed his work to the analysis of queueing system M/M/2 with removable server. Also, Bell [2] presented characterization and computation of optimal policies for operating an M/G/1 queueing system with removable server. Levy and Yechiali [11] discussed utilization of idle time in M/G/1 queuing system where idle time of the server is utilized for additional work in a secondary system.
Gross and Harris [5] discussed about an ordinary $M/E_k/1$ queueing system. In this series of investigations, Wang and Huang [7] have made an attempt to study the optimal operation of an M/E_k /1 queueing system with a removable service station. They have defined total expected cost function with respect to N and finally they have computed the number of customers N in the system when system is in the states of turned on and turned off. Authors made no attempt to optimize total expected cost and as well as profit with respect to optimal service rate of the system which is immutably considered to be a main function of total cost. Moreover, service level is a function of μ and hence by optimizing μ, we can have a better service level which, in turn, leads to optimization of total expected cost by reducing customers' waiting time. In this paper, an attempt has been made to analyze the total optimal cost of an $M/E_k/1$ queueing system with a removable service station under steady state condition. We construct here the total expected cost function per unit time for two different cases. In the first case, we analyze the total expected cost (TEC) of the system under the condition when the system is turned off and turned on. In the second case, we discuss the total expected cost of the system under the utilization of idle time, when server is idle. Since the problem is too complex to solve manually, a fast converging N-R method (with C++ language) has been applied to obtain the total optimum cost of the system. The notion of the total revenue has also been introduced and finally total expected profit (TEP) of the system has also been evaluated. The paper has been organized in various important sections such as introduction, description of the model, cost analysis of the system, profit analysis of the system, observations and conclusion.

Description of the Model

We consider an $M / E_k / 1$ queuing system with a removable service station in which idle fraction of server's time is optimized by shifting the server to another service station in the turned off state of the system. It is assumed that customer arrival follows Poisson process with parameter λ and with service time according to an

Erlang distribution with mean $1/\mu$ and stage parameter k.The Erlang type k distribution is made up of k independent and identical exponential stages; each with mean$1/k\mu$. The service operates when N customers have accumulated and is shutdown (turn off) when no customers are present. Further, it is assumed that there is no any case of reneging and balking in the system.

The queueing system can be formulated as a continuous time parameter Markov chain with states {(n, i)/n=0,1,2,3…….and i=0, 1,2…k} where n denotes the number of customers in the system, i denotes that the customer in service is in stages i.

In this paper, the following notations: ρ=traffic intensity, P^0_{00}=probability that there are no customers in the system and zero stage of service when the service station is turned-off, P^0_{nk} = probability that there are n customers in the system and the customer in service is in stage k when the service station is turned-off, where n=1, 2, ……, N-1.

P^1_{ni} = probability that there are n customers in the system and the customer in service is in stage i when the service station is turned-on and in operation, where n=1, 2, ….. and i =1,2,….,k, L_N= the expected number of customers in the system.

L_{off} = expected number of customers in the system when the service station is turned off. L_{on} = expected number of customers in the system when the service station is turned on. E (I) = expected length of the idle period, E(B) = expected length of the busy period. E(C) = expected length of the busy cycle, E(I) / E(C)= the long-run fraction of time when the service station is turned off, k = no. of phases, N=no. of customers for starting system.

C_s= service cost per unit time, C_o = holding cost for each customer per unit time when system is turned off, C_n = holding cost per unit time when system is turned on, T_1 =total cost of the system when the system is turned off and turned on, T_2 = total cost of the system under utilization of idle time, C_h= holding cost per unit time for each customer present in the system, C_u = cost per unit time associated with utilization of idle period of the system for an auxiliary task, R= earned revenue for providing service per customer.

Cost Analysis of the System

Here, in view of Wang and Huang [7], for an M/E_k/1 queueing system following results in steady states are given.

$$L_{of} = \sum_{n=1}^{N-1} nP^0{}_{nk} = \frac{(N-1)(1-\rho)}{2}, L_{on} = \sum_{n=1}^{\infty} \sum_{i=1}^{k} nP_{ni}^{1} = \frac{\rho(N+1-\rho N+\frac{\rho}{k})}{2(1-\rho)} \tag{1}$$

$$L_N = L_{off} + L_{on} \Rightarrow L_N = \frac{N-1}{2} + \frac{\rho^2(1-k)+2k\rho}{2k(1-\rho)}, \frac{E(I)}{E(C)} = (1-\rho) \tag{2}$$

Now, we construct total expected cost function and analyze its solution in following two cases,

Case—I

$T_1 = C_s K\mu + C_o L_{off} + C_n L_{on}$ .This implies that

$$T_1 = C_s K\mu + C_o \frac{(N-1)(1-\rho)}{2} + C_n \frac{x}{z} \quad \text{where } x = \rho(N+1) - \rho^2 (N - \frac{1}{k}) \tag{3}$$

$$z = 2(1-\rho),\; x' = -\frac{1}{\mu}\left[\rho(N+1) + 2\rho^2 (N - \frac{1}{k})\right], x'' = \frac{1}{\mu^2}\left[2\rho(N+1) - 6\rho^2 (N - \frac{1}{k})\right]$$

Also $z' = \frac{1}{\mu}(2\rho)$ and $z'' = -\frac{1}{\mu^2}(4\rho)$

Eqn (3), after differentiating w. r. t. μ , gives that

$$T_1' = C_s k + C_o \frac{(N-1)}{2}\frac{\rho}{\mu} + C_n \frac{zx' - xz'}{z^2} \tag{4}$$

$$T_1'' = -C_o (N-1)\frac{\rho}{\mu^3} + C_n \left[\frac{z^2 x'' - xzz'' - 2x'zz' + 2x(z')^2}{z^3}\right]$$

Now, for optimum μ , we have

$$T_1' = C_s k + C_o \frac{(N-1)}{2}\frac{\rho}{\mu} + C_n \frac{zx' - xz'}{z^2} = 0 \tag{5}$$

This finally implies that

$$\mu^4 \left(\frac{2C_s k}{\lambda^3}\right) + \mu^3 \left(\frac{-4C_s k}{\lambda^2}\right) + \mu^2 \left(\frac{2C_s k}{\lambda} + \frac{C_o (N-1)}{\lambda^2}\right) + \mu \left(\frac{-2C_o (N-1) - 2C_n (N - \frac{1}{k})}{\lambda}\right) +$$

$$C_o (N-1) + 3C_n (N - \frac{1}{k}) = 0. \quad \textit{This may be written that } f(\mu) = 0 \qquad (\textit{say})$$

Here, f (μ) is a non-linear function in μ which is quite difficult to solve manually. This non-linear equation is solved by N-R method after using a C++ programming language. The value of μ obtained in such a way is optimal and denoted as μ* that optimizes the total expected cost function and gives the total optimal cost (TOC) of the system which is given in table 1.

Case-II

$$T_2 = C_s k\mu + C_h L_N + C_u \frac{E(I)}{E(C)}$$

Which, after substituting the values of set-up, holding cost, queue length and ratio E(I) / E(C), turns out to be

$$T_2 = C_s k\mu + C_h \frac{(N-1)}{2} + C_h \frac{x}{z} + C_u(1-\rho) \tag{6}$$

where $x=\rho^2(1-k)+2k\rho \; and \quad z=2k(1-\rho$

And

$$x' = -\frac{1}{\mu}\left[2(1-k)\rho^2 + 2k\rho\right], x'' = \frac{1}{\mu^2}\left[6\rho^2(1-k)+4k\rho\right], z' = \frac{1}{\mu}\left[2k\rho\right],\; z'' = -\frac{1}{\mu^2}\left[4k\rho\right]$$

Now, differentiating equation (6) w.r.t.μ, we get

$$T_2' = C_s k + C_h\left(\frac{zx' - xz'}{z^2}\right) + C_u \frac{\rho}{\mu} \;, T_2'' = C_h\left[\frac{z^2x'' - xzz'' - 2x'zz' + 2x(z')^2}{z^3}\right] - C_u(2\frac{\rho}{\mu^3})$$

For optimum μ, we have

$$C_s k + C_h\left(\frac{zx' - xz'}{z^2}\right) + C_u \frac{\rho}{\mu} = 0 \tag{7}$$

This implies that

$$\mu^4\left(\frac{4C_s k^3}{\lambda^3}\right) + \mu^3\left(\frac{8C_s k^3}{\lambda^2}\right) + \mu^2\left(\frac{4C_s k^3}{\lambda} + \frac{4C_u k^2}{\lambda^2}\right) + \mu\left(\frac{2C_h k(k-2) - 8C_u k^2}{\lambda}\right) + 2C_h k(1-k) + 4C_u k^2 = 0$$

$f(\mu) = 0 \qquad (say)$

This is a non-linear equation in μ and solved for obtaining the total optimal cost of the system by making use of the similar method as described in case I. Final results are given in table 2.

Profit Analysis of the System

In this section, we discuss total expected profit (TEP) of the system on the basis of the total revenue earned by the system in rendering its service to the customers.
Let R be the earned revenue for providing service to each customer then total expected revenue (TER) is given as
TER= $R L_N$ and also total expected cost (TEC) is given as TEC= $C_s \; k\,\mu + C_h \, L_N$.
Then TEP is expressed as

$$\text{TEP=TER – TEC=} \left[\frac{N-1}{2} + \frac{\lambda^2(1-k)+2k\lambda\mu}{2k\mu(\mu-\lambda)}\right](R-C_h) - C_s k\mu$$

$$= \left(\frac{N-1}{2} + \frac{\theta}{\phi}\right)(R-C_h) - C_s \text{, where, } \theta = \lambda^2(1-k) + 2k\lambda\mu, \phi = 2k\mu(\mu-\lambda) \tag{8}$$

Thus, the total expected profit of the system is given by the equation (8). Now, we evaluate TEP of the system by using the programming in C++ and closely analyze the variation effect of various parameters upon it in the table 3.

Observations and Conclusion

In this section, following remarks follow. From graph-1, we observe that in the beginning TOC decreases as N increases upto a certain number of N thereafter TOC begins to increase as N increases. Graph -2 indicates that TOC decreases as holding cost for each customer increases when system is turned off. Graph-3 shows that TOC decreases as holding cost for each customer increases when system is turned on.

It is also noticed from graph-4 that when we increase the number of customers required for starting service, TEP of the system also gets increased. When revenue per customer is increased then TEP of the system also increases, it is evident from graph-5. A fast converging numerical computing has been used in this analysis that consumes least computing time in performing the output. The analysis of utilization of idle time and evaluation of total expected profit of the system bring the model closure to the real life situations leading to a good deal of potential to the applications in various fields including inventory management, production management, computer and telecommunications etc.

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Table-1

λ	Cs	C_0	C_n	k	N	$\mu*$	TOC
10	12	15	20	3	10	23.3	930.2
10	11	15	20	3	10	21.6	807.9
10	13	15	20	3	10	25.2	1073.4
10	12	16	20	3	10	22.6	909.1
10	12	14	20	3	10	24.0	953.3
10	12	15	21	3	10	22.8	915.3
10	12	15	19	3	10	23.9	947.7
10	12	15	20	3	10	24.2	962.6

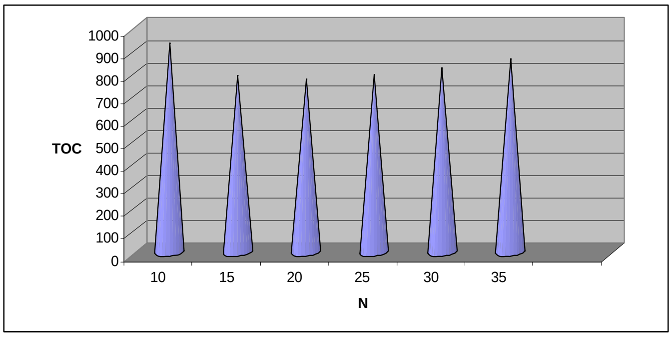
Table-2

λ	Cs	Ch	Cn	k	N	$\mu*$	TOC
10	15	20	25	3	10	3.63	207.4
10	16	20	25	3	10	3.63	218.3
10	15	21	25	3	10	3.64	211.9
10	15	20	26	3	10	3.63	205.4
10	14	20	25	3	10	3.63	196.4
10	15	20	23	3	10	3.64	211.1
10	13	19	24	3	10	3.63	182.8
10	15	20	25	3	10	3.65	208.2

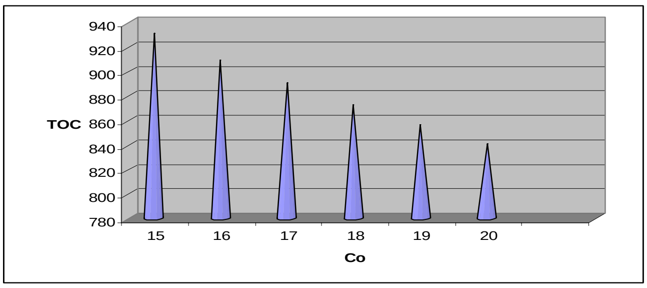
Table-3

μ	λ	R	ch	cs	k	N	TEP
4	10	300	15	20	3	10	963.3
4	11	300	15	20	3	10	1005.1
4	13	300	15	20	3	10	1076.8
4	16	300	15	20	3	10	1169.1
4	10	400	15	20	3	10	1385.5
4	10	500	15	20	3	10	1807.7
4	10	300	16	20	3	10	959.1
4	10	300	18	20	3	10	950.6

Graph 1



Graph-2



Graphs 3, 4 and 5