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A Time Dependent Deteriorating Order Level Inventory Model for Exponentially Declining Demand

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Abstract

An order-level lot-size inventory model is formulated for a time - dependent deterioration and exponentially declining demand. The analytical development is provided to obtain the optimal solution to minimize the total cost per time unit of an inventory system. It is illustrated with the help of numerical example. The effect of changes in the values of different parameters on the decision variable and objectives function is studied.

Keywords: Order – level, inventory, time dependent deterioration, exponentially declining demand

1. Introduction

Now a days, researches are analyzing effect of deterioration and the variations in the demand rate with time in logistics. Silver and Meal (1969) gave a heuristic solution procedure for the inventory model with time varying demand Donaldson (1977) exhibited a very complicated solution procedure taking demand to be linear. Ritchie (1980, 1984, 1985) obtained on exact solution for linearly trended demand. Mitra et al. (1984) developed a simple procedure for adjusting the economic order quantity model for linearly increasing and decreasing demand.

Dave and Patel (1981) derived a lot size model for constant deterioration of items with time proportional demand. Sachan (1984) allowed shortages in Dave and Patel (1981)'s model. Related articles by Bahari Kashani (1989) Deb and Chaudhuri (1987), Mudreshwar (1988), Goyal (1986, 1988), Dave (1989),Hariga (1994), Xu and Wang (1991) Mehta and Shah (2006), Chung and Ting (1993), Hariga (1995, 1996) Jalan et al. (1996, 1999) and their references. Covert and Philip (1973) developed inventory model using a two parameter Weibull distribution for deterioration of units.

Philip (1974) formulated inventory model when deterioration start after some time and used a three-parameter Weibull distribution for deterioration of units. Misra (1975) extended Covert and Philip (1973)'s model for finite rate of replenishment. For further references, refer to review articles by Raafat (1991), Shah and Shah (2000) and Goyal and Giri (2001).

In the proposed study, demand is considered to be decreasing exponentially. Deterioration of units follows a two-parameter Weibull distribution and shortages are allowed. The objective is to minimize the total cost per time unit of an inventory system. The effect of changes in the model parameters on decision variables and total cost of an inventory system is studied through numerical example.

2. Assumptions and Notations

The mathematical model is developed using following notations and assumptions:

2.1 Notations :

C : purchase cost of a unit.

h : inventory holding cost per unit per time unit.

A : ordering cost per order

π : shortage cost per unit short

Q_0 : initial inventory

R(t): = $a\,e^{-bt}$ $a > 0$, $b > 0$, $a >> b$, demand rate at any instant of time t. a is constant demand. b denotes the rate of change of demand.

t_1 : time point of positive inventory, $0 < t_1 < T$

T: cycle time (decision variable)

$\theta(t)$: rate of deterioration at any instant of time t.

2.2 Assumption:

- The inventory system deals with single item.
- The demand R(t) is exponentially decreasing. i.e. $R(t) = ae^{-bt}$, where $a > 0$, $b > 0$, $a >> b$ are constants. a denotes constant demand and b denotes the rate of change of demand.
- The replenishment rate is infinite
- Shortages are allowed and completely backlogged.
- Lead-time is zero or negligible.
- The deterioration of units follows the two parameter Weibull distribution (say) $\theta(t) = \alpha\beta t^{\beta-1}$ where $0 < \alpha < 1$ is the scale parameter and $\beta > 0$ is the shape parameter.

- The deteriorated units can not be repaired or replaced during a period under review.

3. Mathematical Model

We analyze one cycle.

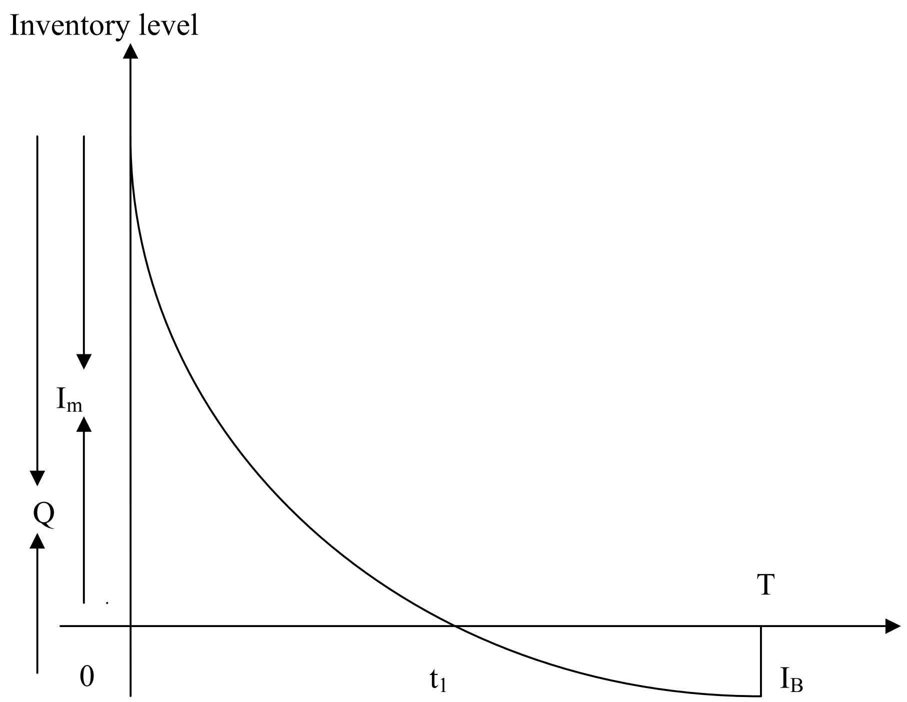

Fig. 3.1 Inventory – time representation

The on hand inventory depletes due to demand and deterioration. At any instant of time, the inventory level I(t) is governed by the differential equation

$$\frac{dI(t)}{dt} + \theta(t)\, I(t) = -R(t), \quad 0 \le t \le t_1 \tag{3.1}$$

with $I(0) = I_m$ and $I(t_1) = 0$, and

$$\frac{dI(t)}{dt} = -R(t), \qquad t_1 \le t \le T \tag{3.2}$$

with $I(t_1) = 0$

The solution of differential equation (3.1) is

$$I(t)\, e^{\alpha t} = \int_0^t -R(u)\; e^{-\alpha u}\, du \tag{3.3}$$

Since α is very small using series expansion (ignoring α^2 and higher powers), the solution (3.3) is I(t) =

$$a\left[t_1 - t + \frac{\alpha}{\beta+1}\left(t_1^{\beta+1} - t^{\beta+1}\right) + \frac{b}{2}\left(t^2 - t_1^{\,2}\right) + \frac{b\alpha}{\beta+2}\left(t^{\beta+2} - t_1^{\,\beta+2}\right) + \frac{b\alpha}{2}\left(t_1^{\,2}t^{\beta} - t^{\beta+2}\right) - \alpha t_1 t^{\beta} + \alpha t^{\beta+1}\right] \tag{3.4}$$

Using $I(0) = I_m$, we have

$$I_m = a\left[t_1 + \frac{\alpha t_1^{\beta+1}}{\beta+1} - \frac{b t_1^{\,2}}{2} - \frac{b\alpha t_1^{\,\beta+2}}{\beta+2}\right] \tag{3.5}$$

and inventory holding cost in the system during $[0, t_1]$ is

$$IHC = h\int_0^{t_1} I(t)\, dt$$

$$= ha\left[\frac{t_1^{\,2}}{2} - \frac{b t_1^{\,3}}{3} - \frac{\alpha t_1^{\,\beta+2}}{(\beta+1)(\beta+2)} - \frac{b\alpha t_1^{\,\beta+3}}{\beta+2} + \frac{b\alpha t_1^{\,\beta+3}}{(\beta+2)(\beta+3)} + \frac{\alpha t_1^{\,\beta+2}}{\beta+2} + \frac{b\alpha t_1^{\,\beta+3}}{2(\beta+1)} - \frac{b\alpha t_1^{\,\beta+3}}{2(\beta+3)}\right] \tag{3.6}$$

The solution of differential equation (3.2) is

$$I(t) = \frac{a}{b}\left(e^{-bt} - e^{-bt_1}\right) \tag{3.7}$$

Using $I(T) = I_b$ (maximum back ordered units), we have

$$I_B = \frac{a}{b}\left(e^{-bT} - e^{-bt_1}\right) \tag{3.8}$$

and shortage cost incurred during period $[t_1, T]$ is

$$SC = \pi\int_{t_1}^{T} I(t)\,dt = \frac{\pi a}{b^2}\left[e^{-bt_1} - e^{-bT} + b(t_1 - T)e^{-bt_1}\right] \quad (3.9)$$

In the beginning of the cycle, Q (= $I_m + I_B$) units are purchased. Hence, number of units deteriorated is Q - $at_1e^{-bt_1}$

Hence cost due to deterioration is

$$CD = Ca\left[t_1 - \frac{bt_1^2}{2} + \frac{\alpha t_1^{\beta+1}}{\beta+1} - \frac{b\alpha t_1^{\beta+2}}{\beta+2} + \frac{1}{b}\left(e^{-bT} - e^{-bt_1}\right) - t_1e^{-bt_1}\right] \quad (3.10)$$

Ordering cost; OC per cycle is A.

The total cost K(t_1,T) per time unit of an inventory system is

$$K(t_1, T) = \frac{1}{T}[IHC + SC + CD + OC] \quad (3.11)$$

The necessary condition for total cost K(t_1,T) to be minimum is

$$\frac{\partial k}{\partial t_1} = 0 \text{ and } \frac{\partial k}{\partial T} = 0 \quad (3.12)$$

Provided $\left(\frac{\partial^2 k}{\partial t_1^2}\right)\left(\frac{\partial^2 k}{\partial T^2}\right) - \left(\frac{\partial^2 k}{\partial t_1 \partial T}\right)^2 > 0$. The non-linear equations in (3.12) can be solved simultaneously by the mathematical software. Once t_1 and T are obtained, the minimum total cost K(t_1, T) per time unit of an inventory system can be calculated.

4 Numerical Illustrations:

Consider the following parametric values in the proper units:
[a, b, α, β, C, h, A, π] = [1000, 0.9, 0.005, 0.5, 30, 2, 250, 55]

The following table 1 shows the effect of changes in the parameters on decision variables and total cost per time unit of an inventory system.

Increase in deterioration rate; α decreases time of positive inventory and increases number of units to be procured and total cost of an inventory system. The increase in shape parameter. β decreases the on hand stock time t_1 whereas optimal purchase unit and total cost of an inventory system. The objective function and decision variables are very sensitive to changes in the fined demand; a, and rate of change of demand. Increase in shortage cost increases total cost of an inventory system and purchase quantities significantly.

Table 1

Changes in		t_1	T	Q	$K(t_1, T)$
α	0.005	2.4868	8.7001	425.82	6505.69
	0.010	2.4841	8.7169	434.27	6521.98
	0.015	2.4813	8.7336	442.63	6538.14
	0.020	2.4787	8.7502	450.90	6554.17
β	0.500	2.4868	8.7001	416.82	6505.69
	0.750	24858	8.7020	419.02	6511.33
	1.250	2.4836	8.7133	419.94	6524.67
	1.500	2.4822	8.7232	420.52	6532.98
a	1000	2.4868	8.7001	425.82	6505.69
	1500	2.4805	8.7013	627.90	9813.82
	2000	2.4797	8.6904	835.28	13094.67
	2500	2.4792	8.6838	1042.66	16375.54
b	0.5	4.8494	15.7057	3204.52	24261.08
	0.7	3.3094	11.1830	1725.50	19321.87
	0.9	2.4868	8.7001	418.62	
	1.2	1.7758	6.4910	548.25	13586.67
π					
	60	1.8049	6.4839	624.09	14310.91
	65	1.8319	6.4744	697.24	15005.55
	70	1.8571	6.4635	767.95	15673.45

5. Conclusion

Most of the models are derived under assumption of linearly increasing or exponentially increasing demand. However, in market of commodities like food grains, fashion apparels, electronic equipments decreases with time during the end of season. The goal of this research is to study the optimal policy of the retailer when there is a decline in the demand.

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