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Finding the Minimize Summation for Location of Facility in a Convex Set with Fuzzy Data

F. Hosseinzadeh Lotfi[1]

Department of Mathematics, Science and Research Branch
Islamic Azad University, P.O. Box 14155/775 and 14155/4933
Post code: 1477893855, Tehran, Iran

G. R. Jahanshahloo

Department of Mathematics
Teacher Training University, Tehran, Iran

F. Rezai Balf

Department of Mathematics
Islamic Azad University, Qaemshahr, Iran

H. Zhiani Rezai

Department of Mathematics
Islamic Azad University, Mashhad, Iran

Abstract

In this paper we first show a model using L_1-Norm for locating a facility in a convex set and then we use it for fuzzy data in linear programming problems and especial case of integer linear programming. Locating a unit is connected to its replacing in a set, which is achieved with regard to other facilities. Different ideas can exist in facilities locating decision. In this paper we intend to minimize weight summation of distance between new facilities and other facilities. Note that it is important to select the distance kind in locating problems.

Keywords: Facilities location, Norm, Linear Programming

[1]Corresponding author. e-mail: hosseinzadeh_lotfi@yahoo.com

1. Introduction

The facilities programming determine how to use the best case possible for achievement of activities with respect to the existing a tools in our purpose direction [1, 3, 4, 5]. This aim is mostly contained of locating facilities and designing them. In this paper we present a model for locating a facility in a convex set.

Holmes (1930) [2], introduced an acceptable purpose for locating industry facilities, which is related to the total effective factors over cost considering both the value of capital and eligible quantity.

In section two we have glance to using of a quantity model in locating facilities. In section three we introduce a model for locating a facility in a convex set. In section four we give a practical example. A method for fuzzy linear programming problems and location for fuzzy data are introduced in sections 5 and 6, respectively. In finally conclusion is achieved in the last section.

2. Use of Quantity Models in Locating Facilities

Consider $P_1, P_2, ..., P_m$ to be a set of m machine in a workshop. Suppose that $P_1 = (a_1, b_1), P_2 = (a_2, b_2), ..., P_m = (a_m, b_m)$ show their situations. Also, consider a new machine Z with coordinates Z=(x,y). Now the question is what the best place for the created machine Z as, considering the minimization of the cost of relationship between them.

Regarding the performance, we assume the movement cost between the new and the $i-th$ machine to be proportional to the distance between Z and P_i with weight of w_i . Therefore, the objective function will be as follow:

$$f(x,y) = \sum_{i=1}^{m} w_i d(Z, P_i)$$

Where $d(Z, P_i)$ show distant between the new machine and $i-th$ machine. Then $f(x,y)$ will be the total cost of the new machine interaction with other machines. Therefore, the purpose of this problem will be to minimize $f(x,y)$, and with use of L_1 -Norm we have:

$$Min\ \ f(x,y) = Min\ \ \sum_{i=1}^{m} w_i\{|x-a_i| + |y-b_i|\}$$

Since variables x and y are independent, then we obtain the below formula:

$$Min\ \ f(x,y) = Min\ \ \sum_{i=1}^{m} w_i|x-a_i| + Min\ \ \sum_{i=1}^{m} w_i|y-b_i|$$

Note that the above objective function is nonlinear which will be linear with an eligible transformation.

3. Locating Facilities in a Convex Set

Suppose that we have a set of L machines in a workshop for which the captured spaced are as intervals. Assume that $(x_j, y_j), j = 1, ..., L$ correspond to their coordinates. Also, suppose that we divide the remaining region of the workshop to K convex regions (Here we consider convex regions as rectangle ones) which are given as follows:

$$S_j = \{(x, y)|a_j \leq x \leq b_j, c_j \leq y \leq d_j\} \text{ for } j = 1, ..., K.$$

Now, we want to find a point in one of the K regions for a new machine such the objective function is minimized. To gain this, we introduce the below nonlinear model:

$$\begin{array}{ll}
Min\ \ d = \sum_{j=1}^{L}\{|x - x_j| + |y - y_j|\} & \\
\text{S.t.} & (1)\\
a_1 \leq x \leq b_1, c_1 \leq y \leq d_1 & \\
or & \\
a_2 \leq x \leq b_2, c_2 \leq y \leq d_2 & \\
or & \\
\vdots & \\
a_k \leq x \leq b_k, c_k \leq y \leq d_k &
\end{array}$$

Model (1) is convertible to below model (model (2)):

$$\begin{array}{ll}
Min\ \ d = \sum_{j=1}^{L}\{|x - x_j| + |y - y_j|\} & \\
\text{S.t.} & (2)\\
x \leq b_1 + M\alpha_1 & \\
x \geq a_1 - M\alpha_1 & \\
y \leq d_1 + M\alpha_1 & \\
y \geq c_1 - M\alpha_1 & \\
x \leq b_2 + M\alpha_2 & \\
x \geq a_2 - M\alpha_2 & \\
y \leq d_2 + M\alpha_2 & \\
y \geq c_2 - M\alpha_2 & \\
\vdots & \\
x \leq b_k + M\alpha_k & \\
x \geq a_k - M\alpha_k & \\
y \leq d_k + M\alpha_k & \\
y \geq c_k - M\alpha_k & \\
\alpha_1 + \alpha_2 + \cdots + \alpha_k = k - 1 & \\
\alpha_j \in \{0, 1\}, \quad j = 1, ..., k &
\end{array}$$

Note, the objective function in model (2) is nonlinear. We use the below Shapiro transformation for its linearization.

$x - x_j = u_j - v_j, \;\; y - y_j = w_j - z_j, \quad j = 1, ..., L$

With respect to the above transformation, above we have:

$$u_j v_j = 0, u_j + v_j > 0 \text{ and } z_j w_j = 0, z_j + w_j > 0, \quad j = 1, ..., L \qquad (3)$$

and also we have:

$$|x - x_j| = u_j + v_j, |y - y_j| = w_j + z_j, \quad j = 1, ..., L \qquad (4)$$

With using relations (3) and (4) in model (2), we obtain below model :

$$
\begin{aligned}
& Min\;\; d = \sum_{j=1}^{L} \{(u_j + v_j) + (w_j + z_j)\} \\
& \text{S.t.} \qquad\qquad\qquad\qquad\qquad\qquad (5) \\
& x \leq b_t + M\alpha_t, \quad t = 1, ..., k \\
& x \geq a_t - M\alpha_t, \quad t = 1, ..., k \\
& y \leq d_t + M\alpha_t, \quad t = 1, ..., k \\
& y \geq c_t - M\alpha_t, \quad t = 1, ..., k \\
& \alpha_1 + \alpha_2 + \cdots + \alpha_k = k - 1 \\
& x - x_j = u_j - v_j, y - y_j = w_j - z_j, \quad j = 1, ..., L \\
& \alpha_t \in \{0, 1\}, \quad t = 1, ..., k \\
& u_j, v_j, z_j, w_j \geq 0, \quad j = 1, ..., L
\end{aligned}
$$

4. Examples

4.1. A Numerical Example

Consider the problem of locating a new machine in an existing layout consisting of five machines P_1, P_2, P_3, P_4 and P_5. The coordinate of machines above are presented in Table 1. Also consider the four possible regions S_1, S_2, S_3 and S_4, in order to create a new machine in Table 2. The problem is to obtain an optimal location for the region $S_j, j = 1, ..., 4$, so that the sum of the distances (street distance) of the new machine from the five other machines is minimized.

Table 1. The coordinates of machines

	x	y
P_1	4	4
P_2	2	2
P_3	2	5
P_4	4	1
P_5	6	5

Table 2. The interval coordinates of regions

	x^L	x^U	y^L	y^U
S_1	1	2	6	7
S_2	2	3	3	4
S_3	4	5	5	7
S_4	5	6	2	3

Using model (5) we obtain:
$u_1^* = v_1^* = v_2^* = v_3^* = v_4^* = u_4^* = u_5^* = z_1^* = z_2^* = z_3^* = z_4^* = z_5^* = w_3^* = w_5^* = \alpha_3^* = 0, u_2^* = 2, u_3^* = 2, v_5^* = 2, w_1^* = 1, w_2^* = 3, w_4^* = 4, \alpha_1^* = 1, \alpha_2^* = 1, \alpha_4^* = 1, x^* = 4, y^* = 5, d^* = 14.$

Hence, with regard to obtained solution we find $(x^*, y^*) = (4, 5) \in S_3$ and the optimal value $d^* = 14$.

4.2. A Practical Example

Consider 6 important industrial regions in a city which receive compulsory services from a fire station. These industrial regions are located in Table 3. Also three sites are considered for building a fire station according to Table 4. The problem is to determine a site considering of three sites for building a fire station, so that the sum of the distances (street distance) of the fire station from the six industrial regions is minimized.

Table 3. The coordinates of industrial regions

	x	y
A	20	15
B	25	25
C	13	32
D	25	14
E	4	21
E	18	8

Table 4. The interval coordinates of sites

	x^L	x^U	y^L	y^U
S_1	4	6	8	10
S_2	10	12	18	23
S_3	32	33	18	20

Using model (5) we obtain:
$u_1^* = u_2^* = u_3^* = u_4^* = v_5^* = v_6^* = z_1^* = z_4^* = z_5^* = z_6^* = w_2^* = w_3^* = w_5^* = \alpha_2^* = 0, v_1^* = 8, v_2^* = 13, v_3^* = 1, v_4^* = 13, v_5^* = 8, v_6^* = 6, z_2^* = 4, z_3^* = 11, w_1^* = 6, w_4^* = 7, w_6^* = 13, \alpha_1^* = 1, \alpha_3^* = 1, x^* = 12, y^* = 21, d^* = 90.$

Therefore the optimal solution is $(x^*, y^*) = (12, 21)$ which lies in site S_3, and the nearest distance of instituted site from the industrial regions is $d^* = 90$.

Note that in the above example, we assume that the weights of the industrial regions and sites are equal.

5. A Method for Fuzzy Linear Programming

5.1 Preliminaries

A Fuzzy Linear Programming (FLP) is concerned with the optimization (minimization or maximization) of a fuzzy linear function while satisfying a set of linear equality and/or inequality fuzzy constraints. In this paper FLP with Right Hand Solution (R.H.S) is considered (T. Allahviranloo) [6].

5.1.1 Definition

An ordered pair of functions $T = (\underline{u}(r), \bar{u}(r)), 0 \leq r \leq 1$, is called a fuzzy number if and only if it satisfied in the following requirements.
(1) $\underline{u}(r)$ is a bounded left continues non-decreasing function over [0,1].
(2) $\bar{u}(r)$ is a bounded left continues non-increasing function over [0,1].
(3) $\underline{u}(r)$ and $\bar{u}(r)$ are right continues in 0.
(4) $\underline{u}(r) \leq \bar{u}(r), 0 \leq r \leq 1$.
where $\underline{u}(r) = wr + (c - w)$ and $\bar{u}(r) = -wr + (c + w), 0 \leq r \leq 1$. which $c, w \in R, c = Core(T)$ and $w = W(T) \geq 0$.
$T = (c, w)$ is called Symmetric Triangular Fuzzy Number (STFN). Let ST be the set of all SFTN.
A crisp number is simply represented by $\underline{u}(r) = \bar{u}(r) = \alpha, 0 \leq r \leq 1$.

5.1.2 Theorem

If $T = (c_1, w_1), U = (c_2, w_2)$ be SFTNs, $k \in R, \tilde{X} \in ST$ and A be a matrix then:
(1) $T = U$ if and only if $c_1 = c_2$ and $w_1 = w_2$.
(2) $T + U = (c_1 + c_2, w_1 + w_2)$.
(3) $kT = (kc_1, |k|w_1)$.
(4) $A\tilde{X} = (A\,Core(\tilde{X}); |A|W(\tilde{X}))$, which $|A|_{ij} = |a_{ij}|$.

5.1.3 Definition

Let $T = (c_1, w_1), U = (c_2, w_2)$ be SFTNs. We say $T \tilde{<} U$ if and only if
(1) $c_1 < c_2$ or
(2) $c_1 = c_2$ and $w_1 < w_2$.
And $T \tilde{\leq} U$ if and only if $T \tilde{<} U$ or $T = U$.

5.2 Fuzzy Linear Problem

Consider fuzzy linear programming as follows:

$$\begin{aligned} &Min\ C\tilde{X} \\ &S.t.\ A\tilde{X} = \tilde{b} \\ &\quad \tilde{X} \in ST, \tilde{X} \tilde{\geq} 0 \end{aligned} \tag{6}$$

which $A \in R^{m\times n}, C \in R^n$ and $\tilde{b}$ is an triangular fuzzy vector. Now, we reduce problem (6) to two following problems.

$$\begin{aligned} &Min\ \ CX \\ &S.t.\ \ AX = Core(b) \\ &\qquad X \geq 0 \end{aligned} \qquad (7)$$

and

$$\begin{aligned} &Min\ \ |C|Y \\ &S.t.\ \ |A|Y = W(b) \\ &\qquad Y \geq 0 \end{aligned} \qquad (8)$$

where $|A|_{ij} = |a_{ij}|, |C|_i = |c_i|$.

5.2.1 Theorem

$\tilde{X}$ is a feasible solution of problem (6) if and only if $X = Core(\tilde{X})$ is a feasible solution of problem (7) and $Y = W(\tilde{X})$ is a feasible solution of problem (8).

5.2.2 Theorem

$\tilde{X}^*$is an optimal solution of problem (6) if and only if $X^* = Core(\tilde{X}^*)$ is an optimal solution of problem (7) and $Y^* = W(\tilde{X}^*)$ is an optimal solution of problem (8).

5.2.3 Theorem

Problem (6) is infeasible if and only if problem (7) is infeasible or problem (8) is infeasible.

5.2.4 Theorem

If problem (6) be feasible then problem (6) has unbounded optimal solution if and only if problem (7) has unbounded optimal solution or problem (8) has unbounded optimal solution.

The proofs of all the above theorems are given T. Allahviranloo (2005) [6].

5.2.5 Example

Consider the fuzzy linear programming problem as below:

$$\begin{aligned} &Min\quad -\tilde{x_1} - 2\tilde{x_2} + \tilde{x_3} \\ &S.t\quad \tilde{x_1} + \tilde{x_2} + \tilde{x_3} + \tilde{x_4} = (6r-2, -6r+10) \\ &\quad -\tilde{x_1} + 2\tilde{x_2} - 2\tilde{x_3} + \tilde{x_5} = (4r+2, -4r+10) \\ &\quad 2\tilde{x_1} + \tilde{x_2} + \tilde{x_3} + \tilde{x_6} = (5r, -5r+10) \\ &\quad \tilde{x_1}, \tilde{x_2}, \tilde{x_3}, \tilde{x_4}, \tilde{x_5}, \tilde{x_6} \tilde{\geq} 0 \end{aligned}$$

Now we should be two problems in below:

$$Min\quad -x_1 - 2x_2 + x_3$$

$$\begin{aligned} S.t \quad & x_1+x_2+x_3+x_4=4 \\ & -x_1+2x_2-2x_3+x_5=6 \\ & 2x_1+x_2+x_3+x_6=5 \\ & x_1,x_2,x_3,x_4,x_5,x_6 \geq 0 \end{aligned}$$

By using simplex method we obtain: $X^* = (\frac{2}{3}, \frac{10}{3}, 0, 0, 0, \frac{1}{3})^t$
and

$$\begin{aligned} Min \quad & y_1+2y_2+y_3 \\ S.t \quad & y_1+y_2+y_3+y_4=6 \\ & y_1+2y_2+2y_3+y_5=4 \\ & 2y_1+y_2+y_3+y_6=5 \\ & y_1,y_2,y_3,y_4,y_5,y_6 \geq 0 \end{aligned}$$

Then we have: $Y^* = (0,0,0,6,4,5)^t$
Therefore the optimal solution of the above example will be:

$$\tilde{X}^* = ((\frac{2}{3},\frac{2}{3}),(\frac{10}{3},\frac{10}{3}),(0,0),(6r-6,-6r+6),(4r-4,-4r+4),(5r-\frac{14}{3},-5r+\frac{16}{3}))^t$$

6. Locating Facilities in a Convex Set with Fuzzy Data

Suppose that we have a set of L machines in a workshop for which the captured spaced are as intervals. Assume that $(\tilde{x}_j, \tilde{y}_j), j = 1, ..., L$ correspond to their coordinates, (the sign "$\sim$" is used for fuzzy numbers). Also, suppose that we divide the remaining region of the workshop to K convex regions (Here we consider convex regions as rectangle ones) which are given as follows:
$S_j = \{(x,y) |\ \tilde{a_j} \tilde{\leq} x \tilde{\leq} \tilde{b_j}\ ,\ \tilde{c_j} \tilde{\leq} y \tilde{\leq} \tilde{d_j}\}$ for $j = 1, ..., K$.
where $\tilde{a_j}, \tilde{b_j}, \tilde{c_j}$ and $\tilde{d_j}$ for $j = 1, ..., K$ are triangular systematic fuzzy numbers. It is obvious that (x, y) also is fuzzy number. Now, we want to find a point in one of the K regions for a new machine such the objective function is minimized. To gain this, we introduce the below nonlinear model:

$$\begin{aligned} & Min \ \ d = \textstyle\sum_{j=1}^{L} \{|\tilde{x} - \tilde{x_j}| + |\tilde{y} - \tilde{y_j}|\} \\ & \text{S.t.} \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (9) \\ & \tilde{a_1} \tilde{\leq} \tilde{x} \tilde{\leq} \tilde{b_1}, \tilde{c_1} \tilde{\leq} \tilde{y} \tilde{\leq} \tilde{d_1} \\ & \text{or} \\ & \tilde{a_2} \tilde{\leq} \tilde{x} \tilde{\leq} \tilde{b_2}, \tilde{c_2} \tilde{\leq} \tilde{y} \tilde{\leq} \tilde{d_2} \\ & \text{or} \\ & \vdots \\ & \tilde{a_k} \tilde{\leq} \tilde{x} \tilde{\leq} \tilde{b_k}, \tilde{c_k} \tilde{\leq} \tilde{y} \tilde{\leq} \tilde{d_k} \end{aligned}$$

Equivalently, according model (5) in section 3, we have the below model for fuzzy numbers:

$$Min\ \ d=\sum_{j=1}^{L}\{(\tilde{u_j}+\tilde{v_j})+(\tilde{w_j}+\tilde{z_j})\}$$
S.t. (10)
$$\tilde{x}\tilde{\leq}\tilde{b_t}+\tilde{M}\alpha_t,\quad t=1,...,k$$
$$\tilde{x}\tilde{\geq}\tilde{a_t}-\tilde{M}\alpha_t,\quad t=1,...,k$$
$$\tilde{y}\tilde{\leq}\tilde{d_t}+\tilde{M}\alpha_t,\quad t=1,...,k$$
$$\tilde{y}\tilde{\geq}\tilde{c_t}-\tilde{M}\alpha_t,\quad t=1,...,k$$
$$\alpha_1+\alpha_2+\cdots+\alpha_k=k-1$$
$$\tilde{x}-\tilde{x_j}=\tilde{u_j}-\tilde{v_j},\tilde{y}-\tilde{y_j}=\tilde{w_j}-\tilde{z_j},\quad j=1,...,L$$
$$\alpha_t\in\{0,1\},\quad t=1,...,k$$
$$\tilde{u_j},\tilde{v_j},\tilde{z_j},\tilde{w_j}\tilde{\geq}0,\quad j=1,...,L$$

The problem (10) can be converted to standard form as follows:

$$Min\ \ d=\sum_{j=1}^{L}\{(\tilde{u_j}+\tilde{v_j})+(\tilde{w_j}+\tilde{z_j})\}$$
S.t. (11)
$$\tilde{x}+\tilde{p_t}-\tilde{M}\alpha_t=\tilde{b_t},\quad t=1,...,k$$
$$\tilde{x}-\tilde{q_t}+\tilde{M}\alpha_t=\tilde{a_t},\quad t=1,...,k$$
$$\tilde{y}+\tilde{r_t}-\tilde{M}\alpha_t=\tilde{d_t},\quad t=1,...,k$$
$$\tilde{y}-\tilde{s_t}+\tilde{M}\alpha_t=\tilde{c_t},\quad t=1,...,k$$
$$\alpha_1+\alpha_2+\cdots+\alpha_k=k-1$$
$$\tilde{x}-\tilde{u_j}+\tilde{v_j}=\tilde{x_j},\tilde{y}-\tilde{w_j}+\tilde{z_j}=\tilde{y_j},\quad j=1,...,L$$
$$\alpha_t\in\{0,1\},\qquad t=1,...,k$$
$$\tilde{p_t},\tilde{q_t},\tilde{r_t},\tilde{s_t}\tilde{\geq}0,\quad t=1,...,k$$
$$\tilde{u_j},\tilde{v_j},\tilde{z_j},\tilde{w_j}\tilde{\geq}0\quad j=1,...,L$$

In order to obtain the optimal solution of problem (11), we solve two problems in below:

$$Min\ \ d=\sum_{j=1}^{L}\{(u_j+v_j)+(w_j+z_j)\}$$
S.t. (12)
$$x+p_t-M\alpha_t=Core(b_t),\quad t=1,...,k$$
$$x-q_t+M\alpha_t=Core(a_t),\quad t=1,...,k$$
$$y+r_t-M\alpha_t=Core(d_t),\quad t=1,...,k$$
$$y-s_t+M\alpha_t=Core(c_t),\quad t=1,...,k$$
$$\alpha_1+\alpha_2+\cdots+\alpha_k=k-1$$
$$x-u_j+v_j=Core(x_j),y-w_j+z_j=Core(y_j),\quad j=1,...,L$$
$$\alpha_t\in\{0,1\},\qquad t=1,...,k$$
$$p_t,q_t,r_t,s_t\geq 0,\quad t=1,...,k$$
$$u_j,v_j,z_j,w_j\geq 0\quad j=1,...,L$$
and

$$
\begin{aligned}
&Min\ \ d=\textstyle\sum_{j=1}^{L}\{(u_j+v_j)+(w_j+z_j)\}\\
&\text{S.t.} \qquad\qquad (13)\\
&x+p_t+M\alpha_t=W(b_t), \quad t=1,...,k\\
&x+q_t+M\alpha_t=W(a_t), \quad t=1,...,k\\
&y+r_t+M\alpha_t=W(d_t), \quad t=1,...,k\\
&y+s_t+M\alpha_t=W(c_t), \quad t=1,...,k\\
&\alpha_1+\alpha_2+\cdots+\alpha_k=k-1\\
&x+u_j+v_j=W(x_j), y+w_j+z_j=W(y_j), \quad j=1,...,L\\
&\alpha_t\in\{0,1\}, \qquad t=1,...,k\\
&p_t,q_t,r_t,s_t\geq 0, \quad t=1,...,k\\
&u_j,v_j,z_j,w_j\geq 0 \quad j=1,...,L
\end{aligned}
$$

Note that, constrain $\sum_{i=1}^{k}\alpha_i=k-1,\ \alpha_i\in\{0,1\}$, has repeated in the models (12) and (13). We call it as "*equivalents conserving constrain*" between two model (12) and (13), because any constrain in model (12) is correspond to one constrain in model (13), for each variable (binary variable) of equivalents conserving constrain. Also note the theorem 5.2.2 is satisfied when the optimal values of the variables of equivalents conserving constrain in models (12) and (13) are same.

6.1 Example

Consider an example is given in section 4.1 with symmetric triangular fuzzy number as follow (Tables 5 and 6):

Table 5. The coordinates of machines

	$(Core(\tilde{x}),W(\tilde{x}))$	$(Core(\tilde{y}),W(\tilde{y}))$
P_1	(2,4)	(2,4)
P_2	(1,2)	(1,4)
P_3	(2,1)	(3,5)
P_4	(3,4)	(2,1)
P_5	(2,6)	(4,5)

Table 6. The interval coordinates of regions

	$(Core(\tilde{x}^L),W(\tilde{x}^L))(Core(\tilde{x}^U),W(\tilde{x}^U))$		$(Core(\tilde{y}^L),W(\tilde{y}^L))(Core(\tilde{y}^U),W(\tilde{y}^U))$	
S_1	(1,2)	(12,1)	(6,2)	(7,1)
S_2	(2,2)	(13,3)	(2,4)	(4,3)
S_3	(4,1)	(5,2)	(4,2)	(2,6)
S_4	(5,2)	(6,4)	(1,2)	(2,3)

Using models (12) and (13), the optimal solution is obtained as follows:

$x^*=(5,1), y^*=(1,1), p_1^*=(7,0), p_2^*=(8,2), p_3^*=(0,1), p_4^*=(1,3), q_1^*=(4,1), q_2^*=(3,1), q_3^*=(1,0), q_4^*=(0,1), r_1^*=(6,0), r_2^*=(3,2), r_3^*=(3,5), r_4^*=(1,2), s_1^*=(5,1), s_2^*=(1,3), s_3^*=(3,1), s_4^*=(0,1), u_1^*=(4,0), u_2^*=(4,1), u_3^*=$

$(3,0), u_4^* = (2,3), u_5^* = (3,2), v_1^* = (0,0), v_2^* = (0,0), v_3^* = (0,0), v_4^* = (0,0), v_5^* = (0,0), w_1^* = (0,3), w_2^* = (0,3), w_3^* = (0,4), w_4^* = (0,0), w_5^* = (0,4), z_1^* = (1,0), z_2^* = (0,0), z_3^* = (2,0), z_4^* = (1,0), z_5^* = (4,0), \alpha_1^* = 1, \alpha_2^* = 1, \alpha_3^* = 1, \alpha_4^* = 1$, and $d^* = 24$.

Since optimal values of the variables related to equivalents supervisor constrain is same in models (12) and (13), therefore the obtained optimal solution from example 6.1 is acceptable for a fuzzy problem.

7. Conclusion

In this paper we presented a model for finding an optimal location as a linear program for a facility with respect to the other facilities and then use it for fuzzy data. Here we use street distance (L_1-Norm) for finding the optimal location. Also in order to avoid congestion, we suppose that an eligible site must be as interval. Finally, in order to locate n $(n > 1)$ new facilities in a convex set, we suggest a method by which the distance between a new facility and all the other existing facilities and the new ones is minimized.

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