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General Composite Implicit Iteration Processes for Common Fixed Points of Asymptotically Nonexpansive Mappings

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Abstract. The purpose of this paper is to introduce the following general composite modified implicit iteration schemes:

$$\begin{cases} x_n = \alpha_n x_{n-1} + (1-\alpha_n)T_n^n y_n, \\ y_n = r_n x_{n-1} + s_n x_n + t_n T_n^n x_{n-1} + w_n T_n^n x_n, \\ r_n + s_n + t_n + w_n = 1,\ \{\alpha_n\}, \{r_n\}, \{s_n\}, \{t_n\}, \{w_n\} \in [0,1], \end{cases}$$

where $T_n = T_{nmodN}$, for common fixed points of a finite family of asymptotically nonexpansive mappings $\{T_i\}_{i=1}^N$ in Banach spaces, and to prove weak and strong convergence theorems. The general composite modified implicit iteration scheme presented in this paper included various previous concrete iteration schemes. Hence, the results presented in this paper extend, generalize and improve the results of Xu and Ori, Oilike, Zhou and Chang, and other authors.

Keywords: General composite implicit iterative; Asymptotically nonexpansive; Weak convergence; Strong convergence; Opial's condition; Demiclosed principle

1. Introduction and Preliminaries

In recent years, the implicit iteration scheme for approximating fixed points of nonlinear mappings has been introduced and studied by various authors.

In 2001, Xu and Ori [1] introduced the following implicit iteration scheme for common fixed points of a finite family of nonexpansive mappings $\{T_i\}_{i=1}^N$ in Hilbert spaces:

$$x_n = \alpha_n x_{n-1} + (1-\alpha_n)T_n x_n, n \geq 1 \tag{1.1}$$

where $T_n = T_{nmodN}$, and proved weak convergence theorem.

In 2004, Osilike [2] extended results of Xu and Ori from nonexpansive mappings to strictly pseudocontractive mappings. By this implicit iteration scheme (1.1), he proved some weak and strong convergence theorems in Hilbert spaces and Banach spaces.

In 2002, Zhou and Chang [3] introduced the following modified implicit iteration scheme for common fixed points of a finite family of asymptotically nonexpansive mappings $\{T_i\}_{i=1}^N$ in Bnanch spaces:

$$x_n = \alpha_n x_{n-1} + \beta_n T_n^n x_n, n \geq 1, \tag{1.2}$$

where $T_n = T_{nmodN}$, by this modified implicit iteration scheme (1.2), Zhou and Chang proved some weak and strong convergence theorems in Banach spaces.

In this paper, we introduce the general composite modified implicit iteration schemes

$$\begin{cases} x_n = \alpha_n x_{n-1} + (1-\alpha_n)T_n^n y_n, \\ y_n = r_n x_{n-1} + s_n x_n + t_n T_n^n x_{n-1} + w_n T_n^n x_n, \\ r_n + s_n + t_n + w_n = 1, \ \{\alpha_n\}, \{r_n\}, \{s_n\}, \{t_n\}, \{w_n\} \in [0,1], \end{cases} \tag{1.3}$$

where $T_n = T_{nmodN}$, for common fixed points of a finite family of asymptotically nonexpansive mappings $\{T_i\}_{i=1}^N$ in Banach spaces and prove weak and strong convergence theorems. The general composite modified implicit iteration scheme presented in this paper included various previous concrete iteration schemes

Observe that if K is a nonempty closed convex subset of a real Banach space E and $T : K \to K$ is an asymptoticaaly nonexpansive mapping, then for every $u \in K$, $\alpha, r, s, t, w \in [0,1]$ and positive integer n , the operator $S = S_{(\alpha,r,s,t,w,n)} : K \to K$ defined by

$$Sx = \alpha u + (1-\alpha)T^n(ru + sx + tT^n u + wT^n x), n \geq 1$$

satisfies

$$\|Sx - Sy\| = (1-\alpha)\|T^n(ru+sx+tT^nu+wT^nx) - T^n(ru+sy+tT^nu+wT^ny)\|$$

$$\leq (1-\alpha)k_n\|(ru+sx+tT^nu+wT^nx) - (ru+sy+tT^nu+wT^ny)\|$$

$$\leq (1-\alpha)(wk_n^2 + sk_n)\|x-y)\|$$

for all $x, y \in K$. Thus, if $(1-\alpha)(wk_n^2+sk_n) < 1$ the S is a contractive mapping, then S has a unique fixed point $x^* \in K$. Thus there exists a unique $x^* \in K$ such that

$$x^* = \alpha u + (1-\alpha)T^n(ru + sx^* + tT^n u + wT^n x^*).$$

This implies that, if $(1-\alpha_n)(w_n k_n^2 + s_n k_n) < 1$, the general composite modified implicit iteration scheme (1.3) can be employed for the approximation of common fixed points of a finite family of asymptotically nonexpansive mappings.

New, we give some definitions and lemmas for our main results.

• Recall that E is said to satisfy *Opial′s condition* if, whenever $\{x_n\}$ is a sequence in E which converges weakly to $x \in E$, then

$$\limsup_{n\to+\infty} \|x_n - x\| < \limsup_{n\to+\infty} \|x_n - y\|, \quad \forall\ y \in E, x \neq y.$$

Let D be a closed subset of a real Banach space E and $T : D \to D$ be a mapping:

• T is said to be $demi-closed$ at the origin if, for any sequence $\{x_n\} \subset D$ which converges weakly to x_0 and $\{Tx_n\}$ converges strongly to 0, then $Tx_0 = 0$.

• T is said to be $semi-compact$ if, for any bounded sequence $\{x_n\} \subset D$ with $\lim_{n\to\infty} \|x_n - Tx_n\| = 0$, then there exists a subsequence $\{x_{n_k}\} \subset \{x_n\}$ such that $\{x_{n_k}\}$ converges strongly to $x^* \in D$.

• T is said to be *asymptoticallynonexpansive* if, there exists a real sequence $\{k_n\} \subset [1, +\infty)$ with $\lim_{n\to\infty} k_n = 1$ such that

$$\|T^n x - T^n y\| \leq k_n \|x - y\|, \quad \forall\ x, y \in D, n \geq 1.$$

• Let $\{T_i\}_{i=1}^N$ are N asymptotically nonexpansive mappings, it is easy to see that, there exists real sequence $\{k_n\} \subset [1, +\infty)$ with $\lim_{n\to\infty} k_n = 1$ such that

$$\|T_i^n x - T_i^n y\| \leq k_n \|x - y\|, \quad \forall\ x, y \in D, n \geq 1,\ i = 1, 2, 3 \cdots, N.$$

Where, real sequence $\{k_n\}$ is called the $uniformly\ asymptotically\ nonexpansive\ coefficient$ of $\{T_i\}_{i=1}^N$.

Lemma 1.1. [4,5] Let E be a uniformly convex Banach space, K be a nonempty closed convex subset of E and $T : K \to K$ be an asymptotically nonexpansive mapping. Then $I - T$ is demi-closed at zero.

Lemma 1.2. [4] Let $\{a_n\}$ and $\{b_n\}$ be two nonnegative sequences which satisfying the following conditions:

$$a_{n+1} \leq (1 + b_n)a_n, \quad n \geq n_0, \quad \sum_{n=1}^{+\infty} b_n < +\infty.$$

Then the limit $\lim_{n\to+\infty} a_n$ exists.

Lemma 1.3. [6] Let E be a uniformly convex Banach space and a, b be two constants with $0 < a < b < 1$. Suppose that $\{t_n\} \subset [a, b]$ is a real sequence and $\{x_n\}, \{y_n\}$ are two sequence in E. Then the conditions

$$\lim_{n\to+\infty} \|t_n x_n + (1-t_n)y_n\| = d,$$

and

$$\limsup_{n\to+\infty} \|x_n\| \le d, \quad \limsup_{n\to+\infty} \|y_n\| \le d,$$

imply that $\lim_{n\to+\infty} \|x_n - y_n\| = 0$, where $d \ge 0$ is a constant.

2. Main Results

Theorem 2.1 *Let E be a real uniformly convex Banach space which satisfying Opial's condition, K be a nonempty closed convex subset of E, $\{T_i\}_{i=1}^N : K \to K$ be N asymptotically nonexpansive mappings with nonempty common fixed points set F. Let $\{\alpha_n\}, \{r_n\}, \{s_n\}, \{t_n\}, \{w_n\}$ be five real sequences in $[0,1]$ and $\{k_n\}$ be the uniformly asymptotically nonexpansive coefficient of $\{T_i\}_{i=1}^N$ satisfying the following conditions*

(i) $\sum_{n=1}^{+\infty}(k_n - 1) < +\infty$,

(ii) there exists constants a, b such that $0 < a \le \alpha_n \le b < 1$ and $0 < a \le r_n$,

(iii) For any bounded sequence $\{x_n\} \subset K$,

$$\lim_{n\to\infty} \|T_i^n x_n - T_{i+1}^n x_n\| = 0, \ i = 1, 2, 3, \cdots, N.$$

Then there exists sufficient large positive n_1, such that for any initial $x_{n_1} \in K$, the general composite modified implicit iteration processes $\{x_n\}_{n=n_1+1}^{\infty}$ defined by (1.3) converges weakly to a common fixed point of $\{T_i\}_{i=1}^N$.

Proof Firstly, from the condition (ii) we know that, there exists sufficient large positive n_1 such that, if $n \ge n_1$ then $(1-\alpha_n)(w_n k_n^2 + s_n k_n) < 1$, so that (1.3) can be employed for the approximation of common fixed points of a finite family of asymptotically nonexpansive mappings.

For any given $p \in F$ and $n > n_1$, we have

$$\begin{aligned}
\|y_n - p\| &\le \|r_n(x_{n-1} - p) + s_n(x_n - p) + t_n(T_n^n x_{n-1} - p) + w_n(T_n^n x_n - p)\| \\
&\le r_n\|x_{n-1} - p\| + s_n\|x_n - p\| + t_n\|T_n^n x_{n-1} - p\| + w_n\|T_n^n x_n - p\| \\
&\le r_n\|x_{n-1} - p\| + s_n\|x_n - p\| + t_n k_n\|x_{n-1} - p\| + w_n k_n\|x_n - p\| \\
&\le (r_n + t_n k_n)\|x_{n-1} - p\| + (s_n + w_n k_n)\|x_n - p\|.
\end{aligned} \tag{2.1}$$

In addition

$$\begin{aligned}
\|x_n - p\| &= \|\alpha_n(x_{n-1} - p) + (1-\alpha_n)(T_n^n y_n - p)\| \\
&\le \alpha_n\|x_{n-1} - p\| + (1-\alpha_n)\|T_n^n y_n - p\| \\
&\le \alpha_n\|x_{n-1} - p\| + (1-\alpha_n)k_n\|y_n - p\|.
\end{aligned} \tag{2.2}$$

Substituting (2.1) into (2.2), we get

$$\begin{aligned}
\|x_n - p\| &\le \alpha_n\|x_{n-1} - p\| + (1-\alpha_n)k_n[(r_n + t_n k_n)\|x_{n-1} - p\| + (s_n + w_n k_n)\|x_n - p\|] \\
&\le [\alpha_n + (1-\alpha_n)k_n(r_n + t_n k_n)]\|x_{n-1} - p\| + (1-\alpha_n)k_n(s_n + w_n k_n)\|x_n - p\|.
\end{aligned}$$

Which leads to

$[1-(1-\alpha_n)k_n(s_n+w_nk_n)]\|x_n-p\| \le [\alpha_n+(1-\alpha_n)k_n(r_n+t_nk_n)]\|x_{n-1}-p\|$

Using the condition (ii) , if n sufficient large then $1-(1-\alpha_n)k_n(s_n+w_nk_n) \ge \frac{a}{2} > 0$, so that

$$\begin{aligned}
\|x_n-p\| &\le \frac{\alpha_n+(1-\alpha_n)(r_n+t_nk_n)k_n}{1-(1-\alpha_n)k_n(s_n+w_nk_n)}\|x_{n-1}-p\| \\
&\le [1+\frac{(1-\alpha_n)k_n(s_n+w_nk_n)-1+\alpha_n+(1-\alpha_n)(r_n+t_nk_n)k_n}{1-(1-\alpha_n)k_n(s_n+w_nk_n)}]\|x_{n-1}-p\| \\
&\le [1+\frac{(1-\alpha_n)[k_n(s_n+w_nk_n)+(r_n+t_nk_n)k_n-1]}{1-(1-\alpha_n)k_n(s_n+w_nk_n)}]\|x_{n-1}-p\| \\
&\le [1+\frac{k_n(s_n+w_nk_n)+(r_n+t_nk_n)k_n-1}{1-(1-\alpha_n)k_n(s_n+w_nk_n)}]\|x_{n-1}-p\| \\
&\le [1+\frac{s_nk_n^2+w_nk_n^2+r_nk_n^2+t_nk_n^2-1}{1-(1-\alpha_n)k_n(s_n+w_nk_n)}]\|x_{n-1}-p\| \\
&= [1+\frac{k_n^2-1}{\frac{a}{2}}]\|x_{n-1}-p\| = [1+\frac{2}{a}(k_n^2-1)]\|x_{n-1}-p\|
\end{aligned}$$

Since condition (i) is equivalent to $\sum_{n=1}^{\infty}(k_n^2-1) < +\infty$, by lemma1.2 we know the limit $\lim_{n\to\infty}\|x_n-p\|$ exists and so let

$$\lim_{n\to\infty}\|x_n-p\| = d. \tag{2.3}$$

Hence $\{x_n\}$ is bounded sequence. Therefore, it follows from (1) and (3) that

$$\limsup_{n\to\infty}\|y_n-p\| \le d,$$

which implies that

$$\limsup_{n\to\infty}\|T_n^ny_n-p\| \ \le \limsup_{n\to\infty}k_n\|y_n-p\| \le d. \tag{2.4}$$

From general composite implicit iteration scheme (1.3), we have

$$\lim_{n\to\infty}\|\alpha_n(x_{n-1}-p)+(1-\alpha_n)(T_n^ny_n-p)\| = \lim_{n\to\infty}\|x_n-p\| = d. \tag{2.5}$$

By lemma1.3 and (2.3)(2.4)(2.5), we get

$$\lim_{n\to\infty}\|T_n^ny_n-x_{n-1}\| = 0. \tag{2.6}$$

Again, form the general composite modified implicit iteration scheme (1.3) and (2.6), we have

$$\lim_{n\to\infty}\|x_n-x_{n-1}\| = \lim_{n\to\infty}(1-\alpha_n)\|T_n^ny_n-x_{n-1}\| = 0,$$

so, for any $i=1,2,3,\cdots,N$, that

$$\lim_{n\to\infty}\|x_n-x_{n+i}\| = 0. \tag{2.7}$$

On the other hand

$$\|T_n^nx_n-x_n\| \le \|x_n-x_{n-1}\|+\|T_n^ny_n-x_{n-1}\|+\|T_n^ny_n-T_n^nx_n\| \tag{2.8}$$

$$\begin{aligned}
&\le \|x_n - x_{n-1}\| + \|T_n^n y_n - x_{n-1}\| + k_n\|y_n - x_n\| \\
&= \|x_n - x_{n-1}\| + \|T_n^n y_n - x_{n-1}\| + k_n\|y_n - x_{n-1}\| + k_n\|x_n - x_{n-1}\| \\
&= (1+k_n)\|x_n - x_{n-1}\| + \|T_n^n y_n - x_{n-1}\| + k_n\|y_n - x_{n-1}\|.
\end{aligned}$$

Now, we consider the third term on right of inequality (2.8), by the general composite modified iteration scheme (1.3) we have

$$\begin{aligned}
(2.9)\quad \|y_n - x_{n-1}\| &= \|t_n T_n^n x_{n-1} + w_n T_n^n x_n + s_n x_n - (1-r_n)x_{n-1}\| \\
&= \|t_n T_n^n x_{n-1} + w_n T_n^n x_n + s_n x_n - (s_n + t_n + w_n)x_{n-1}\| \\
&= \|t_n T_n^n x_{n-1} - t_n x_n + w_n T_n^n x_n - w_n x_n + (s_n + t_n + w_n)(x_n - x_{n-1})\| \\
&\le (s_n + t_n + w_n)\|x_n - x_{n-1}\| + t_n\|T_n^n x_{n-1} - x_n\| + w_n\|T_n^n x_n - x_n\| \\
&\le (s_n + t_n + w_n)\|x_n - x_{n-1}\| + t_n\|T_n^n x_{n-1} - T_n^n x_n\| + (t_n + w_n)\|T_n^n x_n - x_n\| \\
&\le (s_n + t_n + w_n)\|x_n - x_{n-1}\| + t_n k_n\|x_{n-1} - x_n\| + (t_n + w_n)\|T_n^n x_n - x_n\| \\
&\le (s_n + t_n + w_n + t_n k_n)\|x_n - x_{n-1}\| + (t_n + w_n)\|T_n^n x_n - x_n\|.
\end{aligned}$$

Substituting (2.9) into (2.8), we have

$$\begin{aligned}
\|T_n^n x_n - x_n\| &\le (1+k_n)\|x_n - x_{n-1}\| + \|T_n^n y_n - x_{n-1}\| + k_n\|y_n - x_{n-1}\| \\
&\le (1+k_n)\|x_n - x_{n-1}\| + \|T_n^n y_n - x_{n-1}\| \\
&\quad + k_n[(s_n + t_n + w_n + t_n k_n)\|x_n - x_{n-1}\| + (t_n + w_n)\|T_n^n x_n - x_n\|].
\end{aligned}$$

By removing the terms of above inequality, we get

$$(2.10)\qquad [1 - k_n(t_n + w_n)]\|T_n^n x_n - x_n\| \le M\|x_n - x_{n-1}\| + \|T_n^n y_n - x_{n-1}\|,$$

where $M \ge 1 + k_n + k_n(s_n + t_n + w_n + t_n k_n)$ is a constant. From the condition (ii) we know that, if n sufficient large then $1 - k_n(t_n + w_n) \ge \frac{a}{2} > 0$, hence, it follows from (2.6)(2.7) and (2.10) that

$$(2.11)\qquad \lim_{n\to\infty} \|T_n^n x_n - x_n\| = 0.$$

Therefore, we have

$$\begin{aligned}
(2.12)\qquad \|T_n x_n - x_n\| &\le \|x_n - T_n^n x_n\| + \|T_n^n x_n - T_n x_n\| \\
&\le \|x_n - T_n^n x_n\| + k_1\|T_n^{n-1} x_n - x_n\| \\
&\le \|x_n - T_n^n x_n\| + k_1\|T_n^{n-1} x_n - x_{n-1}\| + k_1\|x_{n-1} - x_n\| \\
&\le \|x_n - T_n^n x_n\| + k_1\|T_n^{n-1} x_n - T_{n-1}^{n-1} x_n\| \\
&\quad + \|T_{n-1}^{n-1} x_n - T_{n-1}^{n-1} x_{n-1}\| + k_1\|T_{n-1}^{n-1} x_{n-1} - x_{n-1}\| + k_1\|x_{n-1} - x_n\|. \\
&\le \|x_n - T_n^n x_n\| + k_1\|T_n^{n-1} x_n - T_{n-1}^{n-1} x_n\| \\
&\quad + k_1\|T_{n-1}^{n-1} x_{n-1} - x_{n-1}\| + (k_{n-1} + k_1)\|x_{n-1} - x_n\|.
\end{aligned}$$

Therefore, combining (2.7)(2.11)(2.12) and condition (iii), we obtain

$$(2.13)\qquad \lim_{n\to\infty} \|T_n x_n - x_n\| = 0.$$

Because for any $i = 1, 2, 3, \cdots, N$, we also have

$$\begin{aligned}
\|x_n - T_{n+i} x_n\| &\le \|x_n - x_{n+i}\| + \|x_{n+i} - T_{n+i} x_{n+i}\| + \|T_{n+i} x_{n+i} - T_{n+i} x_n\| \\
&\le \|x_n - x_{n+i}\| + \|x_{n+i} - T_{n+i} x_{n+i}\| + k_1\|x_{n+i} - x_n\|.
\end{aligned}$$

Thus, it follows from (2.7) and (2.13) that

$$\lim_{n\to\infty} \|T_{n+i}x_n - x_n\| = 0,\ i = 1, 2, 3, \cdots, N.$$

Because $T_n = T_{nmodN}$, it is easy to see, for any $l = 1, 2, 3, \cdots, N$, that

$$\lim_{n\to+\infty} \|T_l x_n - x_n\| = 0. \tag{2.14}$$

Since E is uniformly convex, every bounded subset of E is weakly compact, so that there exists a subsequence $\{x_{n_k}\}$ of bounded sequence $\{x_n\}$ such that $\{x_{n_k}\}$ converges weakly to a point $q \in K$. Therefore, it follows from (2.14) that

$$\lim_{k\to\infty} \|T_l x_{n_k} - x_{n_k}\| = 0, \quad \forall\ l = 1, 2, 3, \cdots, N \tag{2.15}$$

By lemma1.1, we know $I - T_l$ is demi-closed, it is esay to see that $q \in F(T_l)$, so that $q \in F = \bigcap_{l=1}^{N} F(T_l)$, where $F(T_l)$ denote the fixed points set of T_l.

Finally, we prove that the sequence $\{x_n\}$ converges weakly to q. In fact, suppose this is not true, then there must exists a subsequence $\{x_{n_i}\} \subset \{x_n\}$ such that $\{x_{n_i}\}$ converges weakly to another $q_1 \in K,\ q_1 \neq q$. Then, by the same method given above, we can also prove that $q_1 \in F = \bigcap_{l=1}^{N} F(T_l)$.

Because, we have proved that, for any $p \in F$, the limit $\lim_{n\to+\infty} \|x_n - p\|$ exists. Then we can let

$$\lim_{n\to\infty} \|x_n - q\| = d_1, \quad \lim_{n\to\infty} \|x_n - q_1\| = d_2,$$

by Opial's condition of E, we have

$$\begin{aligned} d_1 = \limsup_{i\to\infty} \|x_{n_i} - q\| &< \limsup_{i\to\infty} \|x_{n_i} - q_1\| = d_2, \\ = \limsup_{j\to\infty} \|x_{n_j} - q_1\| &< \limsup_{j\to\infty} \|x_{n_j} - q\| = d_1. \end{aligned}$$

This is a contradiction, hence $q = q_1$. This implies that $\{x_n\}$ converges weakly to a common fixed point q of $\{T_l\}_{l=1}^{N}$, this completes the proof.

From the proof of theorem2.1, we give the following strong convergence theorem:

Theorem 2.2 *Let E be a real uniformly convex Banach space , K be a nonempty closed convex subset of E, $\{T_i\}_{i=1}^{N} : K \to K$ be N asymptotically nonexpansive mappings with nonempty common fixed points set F, and at least there exists an $T_l, 1 \leq l \leq N$ is semi-compact. Let $\{\alpha_n\}, \{r_n\}, \{s_n\}, \{t_n\}, \{w_n\}$ be five real sequences in $[0, 1]$ and $\{k_n\}$ be the uniformly asymptotically nonexpansive coefficient of $\{T_i\}_{i=1}^{N}$ satisfying the following conditions*

(i) $\sum_{n=1}^{+\infty}(k_n - 1) < +\infty$,

(ii) there exists constants a, b such that $0 < a \leq \alpha_n \leq b < 1$ and $0 < a \leq r_n$,

(iii) For any bounded subset $C \subset K$, the limit

$$\lim_{n\to\infty} \|T_i^n x - T_{i+1}^n x\| = 0,\ i = 1, 2, 3, \cdots, N$$

holds uniformly for all $x \in C$. Then there exists sufficient large positive n_1, such that for any initial $x_{n_1} \in K$, the general composite modified implicit *iteration processes* $\{x_n\}_{n=n_1+1}^{\infty}$ *defined by* (1.3) *converges strongly to a common fixed point of* $\{T_i\}_{i=1}^{N}$.

Proof From the proof of theorem2.1, we know that there exists subsequence $\{x_{n_k}\} \subset \{x_n\}$ such that $\{x_{n_k}\}$ converges weakly to some $q \in K$ and satisfies (2.15). By the semi-compactness of T_l, there exists a subsequence of $\{x_{n_k}\}$ (we denote it still by $\{x_{n_k}\}$) such that

$$\lim_{n\to\infty} \|x_{n_k} - q\| = 0.$$

Because the limit $\lim_{n\to\infty} \|x_n - q\|$ exists, thus we get

$$\lim_{n\to\infty} \|x_n - q\| = 0.$$

This completes the proof.

Remark If $r_n \equiv 1$, the general composite modified implicit iteration scheme (1.3) become modified Mann iteration scheme, so from therom2.1 and theorem2.2, we obtain the convergence theorems of modified Mann iteration scheme. If $s_n \equiv 1$, the general composite modified implicit iteration scheme (1.3) become implicit iteration scheme of Xu and Ori, so from therom2.1 and theorem2.2, we obtain the convergence theorems of implicit iteration scheme. If $s_n + w_n \equiv 0$, the general composite modified implicit iteration scheme (1.3) become modified Ishikawa iteration scheme, so from therom2.1 and theorem2.2, we obtain the convergence theorems of modified Ishikawa iteration scheme. If $s_n + t_n \equiv 0$ or $r_n + t_n \equiv 0$, the general composite modified implicit iteration scheme (1.3) become composite modified implicit iteration scheme, so from therom2.1 and theorem2.2, we obtain the convergence theorems of composite modified implicit iteration scheme.

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