Applied Mathematical Sciences, Vol. 1, 2007, no. 14, 647 - 663

A New Approach for Efficiency Measures by Fuzzy Linear Programming and Application in Insurance Organization

F. Hosseinzadeh Lotfi[1]

Dept. of Math., Science and Research Branch,
Islamic Azad University, Tehran 14515-775, Iran

G. R. Jahanshahloo

Dept. of Math., Science and Research Branch,
Islamic Azad University, Tehran 14515-775, Iran

M. Alimardani

Dept. of Math., Science and Research Branch,
Islamic Azad University, Tehran 14515-775, Iran

Abstract

Data envelopment analysis is widely applied approach for measuring the relative efficiencies of a set of decision making units(DMUs) using various inputs to produce various output are limited to crisp data.In this paper, the focus will be on the CCR model because the CCR model was the original DEA model. All other models are extensions of the CCR model obtained by either modifying the production possibility set of the CCR model or adding slack variables in the objective function. However, in real-world problems inputs and outputs are often imprecise.This paper develops DEA models using imprecise data represented by fuzzy sets.By use of a ranking function we introduce the approach to solving

[1]Corresponding author: F. Hosseinzadeh Lotfi, E-mail: hosseinzadeh_lotfi@yahoo.com

the fuzzy CCR model (FCCR) and fuzzy DCCR model (FDCCR) with fuzzy data.

Mathematics Subject Classification: Operations Research, No. 90

Keywords: fuzzy linear programming problem, Data envelopment analysis DEA

1 Introduction

One approach for measuring the relative efficiencies of decision making units (DMUs) which consume multiple inputs to produce multiple outputs is data envelopment analysis (DEA) developed by Charnes et al.(1978,1979).This approach is essentially the counterpart of the pare to optimality in economics which states that a DMU is said to reduce the consumption of one input, another input will necessarily be raised in producing the same amount of output, or when it tries to raise the production of one output, another output will necessarily be reduced in consuming the same amount of input (see, for example, Ferguson and Gould,1975). To deal quantitatively with impression in decision process, Bellman and Zadeh (1970) introduce the notion of fuzziness.In the conventional DEA approach a set of weighs which satisfies a set of constraints is selected to give the highest possible efficiency measure for each DMU. when some observations are fuzzy the goal and constraints in the decision process become fuzzy as well. since the DEA model is essentially a linear program, one straightforward idea is to apply the existing fuzzy linear programming (FLP) to the fuzzy DEA problems. In this paper we use a new method for fuzzy linear programming to treat fuzzy DEA models.

2 Preliminarie

Definition 2.1. *We represent an arbitrary fuzzy number by an ordered pair of functions* $(\underline{u}(r), \overline{u}(r))$, $0 \leq r \leq 1$, *which satisfy the following requirements:*

- $\underline{u}(r)$ *is a bounded left continuous nondecreasing function over [0,1].*
- $\overline{u}(r)$ *is a bounded left continuous nonincreasing function over [0,1].*

- *$\underline{u}(r)$ and $\overline{u}(r)$ are right continuous in 0*
- *$\underline{u}(r) \leq \overline{u}(r)$,$0 \leq r \leq 1$.*

A crisp number α is simply represented by $\underline{u}(r) = \overline{u}(r) = \alpha$, $0 \leq r \leq 1$.

Theorem 2.1. *for arbitrary fuzzy numbers $x = (\underline{x}, \overline{x})$, $y = (\underline{y}, \overline{y})$ and real number k,*

1. *$x = y$ if and only if $\underline{x}(r) = \underline{y}(r)$ and $\overline{x}(r) = \overline{y}(r)$.*
2. *$x + y = (\underline{x}(r) + \underline{y}(r), \overline{x}(r) + \overline{y}(r))$.*
3.

$$kx = \begin{cases} (k\underline{x}, k\overline{x}), & k \geq 0 \\ (k\overline{x}, k\underline{x}), & k < 0 \end{cases} \tag{2.1}$$

Definition 2.2. *The suport of a fuzzy set $\tilde{A}$ is a set of elements in X for which $\mu_{\tilde{A}}(x)$is positive.*

Definition 2.3. *A convex fuzzy set $\tilde{A}$ on $\Re$ is a fuzzy number if the following conditions hold:*

- *Its membership function is piecewise continuous.*
- *There exist three intervals $[a, b]$, $[b, c]$ and $[c, d]$ such that $\mu_{\tilde{A}}(x)$ is increasing on $[a, b]$, equal to 1 on $[b, c]$, decreasing on $[c, d]$ and equal to 0 elsewhere.*

Remark 2.1. *In the above defintion, we say interval $[b, c]$ is the modal set of fuzzy number $\tilde{A}$.*

Definition 2.4. *Let $\tilde{A} = (a^m, a^u, a^l)$ denote the triangular fuzzy number, where $[a^l, a^u]$ is the suport of $\tilde{A}$ and a^m its modal set.*

Remark 2.2. *We denote the set of all trapezoidal fuzzy numbers by $F(\Re)$. we obtain a triangular fuzzy number, and we show it with:*

$$\tilde{a} = (\underline{a}(r), \overline{a}(r)) = ((a^m - a^l)r + a^l, (a^m - a^u)r + a^u)$$

2.1 Ranking functions and Fuzzy linear programing problems

Several methods for solving fuzzy linear programing problems can be Fang (1999),Lai and Hwang (1992),Maleki et al (2000).one of the most convenient of these methods is based on the concept of comparison of fuzzy numbers by use of ranking functions.In fact,an efficient approach for ordering the elements of $F(\Re)$ is to define a ranking function $\tau : F(\Re) \to \Re$ which maps each fuzzy number into the line, where a natural order exists. We define orders on $F(\Re)$ by

1. $\tilde{a} \succeq \tilde{b}$ if and only if $\tau(\tilde{a}) \geq \tau(\tilde{b})$.
2. $\tilde{a} \succ \tilde{b}$ if and only if $\tau(\tilde{a}) > \tau(\tilde{b})$.
3. $\tilde{a} \simeq \tilde{b}$ if and only if $\tau(\tilde{a}) = \tau(\tilde{b})$.

where $\tilde{a}$ and $\tilde{b}$ are in $F(\Re)$.
The following lemma is now immediate.

Lemma 2.1. *Let τ be any linear ranking function.Then*

1. $\tilde{a} \succeq \tilde{b}$ *iff* $\tilde{a} - \tilde{b} \succeq 0$ *iff* $-\tilde{b} \succeq -\tilde{a}$.
2. $\tilde{a} \succeq \tilde{b}$ *and* $\tilde{c} \succeq \tilde{d}$ *iff* $\tilde{a} + \tilde{c} \succeq \tilde{b} + \tilde{d}$.

We restrict our attention to linear ranking function, that is, a ranking function τ such that

$$\tau(k\tilde{a} + \tilde{b}) = k\tau(\tilde{a}) + \tau(\tilde{b})$$

for any $\tilde{a}$ and $\tilde{b}$ belonging to $\tau(\Re)$ and any $k \in \Re$.

Here, we introduce a linear ranking function that is similar to the ranking function adopted by Maleki (FJMS)(2002). For a fuzzy number $\tilde{a} = (\underline{a}(r), \overline{a}(r))$,we use ranking function as follows:

$$\tau(\tilde{a}) = 1/2 \int_0^1 (\underline{a}(r) + \overline{a}(r))dr. \qquad (2.2)$$

This reduces to

$$\tau(\tilde{a}) = 1/2(a^m + 1/2(a^l + a^u)). \tag{2.3}$$

Then, for triangular fuzzy numbers $\tilde{a} == (a^m, a^u, a^l)$ and $\tilde{b} = (b^m, b^u, b^l)$,we have

$$[\tilde{a} \succeq \tilde{b}] \Longleftrightarrow [(a^m + 1/2(a^l + a^u)) \geq (b^m + 1/2(b^l + a^u))]. \tag{2.4}$$

Authors who use ranking function for comparison of fuzzy linear programming

problems usually define a crisp model which is equivalent to the Fuzzy linear programing problem and then use optimal solution of this model as the optimal solution of fuzzy linear programming problems. We now define fuzzy linear programming problems and the corresponding crisp model.

Definition 2.5. *A fuzzy linear programing problem (FLP) is defined as follows:*

$$\begin{aligned} \min \ \ & \tilde{z} \simeq \tilde{c}x \\ s.t. \ \ & \tilde{A}x \succeq \tilde{b} \\ & x \geq 0, \end{aligned} \tag{2.5}$$

where "$\simeq$" and "$\preceq$" mean equality and inequality with respect to the ranking function τ, $\tilde{A} = [\tilde{a_{ij}}]_{m\times n}$, $\tilde{c} = (\tilde{c_1}, ..., \tilde{c_n})$,
$\tilde{b} = (\tilde{b_1}, ..., \tilde{b_m})^T$,$x = (x_1, ..., x_n)$, and $\tilde{a_{ij}}, \tilde{b_i}, \tilde{c_j} \in F(\Re)$ and $x_j \in \Re$ for $i = 1, ..., m; j = 1, ..., n$.

Definition 2.6. *Any x which satisfies the set of constraints of (FLP)is called afeasible solution. Let $\tilde{S}$ be the set of all feasible solution of (FLP).We say that $x^* \in \tilde{S}$ is an optimal feasible solution for FLP iff $\tilde{c}x^* \preceq \tilde{c}x$ for all $X \in \tilde{S}$.*

Definition 2.7. *We say that the real number a corresponds to the fuzzy number $\tilde{a}$, with respect to a given linear ranking function τ, if $a = \tau(\tilde{a})$.*

However the following theorem shows that any FLP can be reduced to a linear programing problem.

Theorem 2.2. *The following linear programing problem (LP) and the FLP in* (2.5) *are equivalent:*

$$\begin{aligned} \min\ & z = cx \\ s.t.\ & Ax \geq b \\ & x \geq 0, \end{aligned} \tag{2.6}$$

where a_{ij}, b_i, c_j *are real numbers corresponding to the fuzzy numbers* $\tilde{a_{ij}}, \tilde{b_i}, \tilde{c_j}$ *with respect to a given linear ranking function* τ*, respectively.*

Proof: By considerate the ranking function and definition (2.6) it is to see that every optimal feasible solution of FLP(2.6) is an optimal feasible solution of LP(2.7), on the other hand, every optimal feasible solution of LP(2.7) is an optimal feasible solution of FLP(2.6).

Remark 2.3. *The above theorem shows that the sets of all feasible solutions of FLP and LP are the same. Also if* $\bar{x}$*is an optimal solution for FLP, then* $\bar{x}$ *is an optimal solution for LP.*

Corollary 2.1. *If problem LP does not have a optimal solution then FLP does not have a optimal solution either.*

Similar to the duality theory in linear programing (see for example,Bazaraa et al.), for every FLP, there is an associated problem which satisfies some important properties.
For the FLP

$$\begin{aligned} \min\ & \tilde{c}x \\ s.t.\ & \tilde{A}x \succeq \tilde{b} \\ & x \geq 0, \end{aligned} \tag{2.7}$$

define the dual fuzzy linear programming problem (DFLP)as

$$\begin{aligned} \max\ & w\tilde{b} \\ s.t.\ & w\tilde{A} \preceq \tilde{c} \\ & w \geq 0, \end{aligned} \tag{2.8}$$

Theorem 2.3. *(Fundamental theorem of duality). For any FLP and its corresponding DFLP, exactly one of the following statements is true.*

1. *Both have optimal solutions* x^* *and* w^* *with* $\tilde{c}x^* \simeq w^*\tilde{b}$.
2. *One problem has unbounded objective value, in witch case the other infeasible.*
3. *Both problems are infeasible.*

3 DEA and fuzzy DEA models

The most frequently used DEA model is the CCR model, name after Charnes, Cooper and Rhodes (1978). Suppose that there are n DMUs, each of which consumers the same type of inputs and produces the same type of outputs. Let m be the number of inputs and let r be the number of outputs.All inputs and outputs are assumed to be nonnegative, but at least one input and one output are positive. The following notation will be used throughout this paper.

Notation

- DMU_j is the jth DMU.
- DMU_o is the target DMU.
- $X_j \in R^{m\times 1}$ is the column vector of inputs consumed by DMU_j.
- $X_o \in R^{m\times 1}$ is the column vector of inputs consumed by DMU_o.
- $X \in R^{m\times n}$ is the matrix of inputs of all DMUs.
- $Y_j \in R^{s\times 1}$ is the column vector of outputs consumed by DMU_j.
- $Y_o \in R^{s\times 1}$ is the column vector of outputs consumed by DMU_o.
- $Y \in R^{s\times n}$ is the matrix of outputs of all DMUs.
- $\lambda = (\lambda_j)_{n\times 1}, \lambda \in R^n$ is the column vector of a linear combination of n DMUs.
- θ is the objective value (efficiency) of the CCR model.
- $V \in R^{m\times 1}$ is the column vector of input weights.
- $U \in R^{s\times 1}$ is the column vector of output weights.

In the CCR model, the multiple input and multiple output of each DMU are aggregated into a single virtual input and a single virtual output, respectively. The CCR model and its dual are formulated as the following linear programming models:
(CCR)

$$\begin{aligned}
\max \quad & \sum_{r=1}^{s} u_r y_{ro} \\
\text{s.t.} \quad & \sum_{i=1}^{m} v_i x_{io} = 1; \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \quad \text{j=1,...,n} \\
& u_r, v_i \geq 0, \qquad \text{r=1,...,s,} \quad \text{i=1,...,m}
\end{aligned} \tag{3.9}$$

$(DCCR)$

$$\begin{aligned}
\text{Min} \quad & \theta \\
\text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{io}, \quad \text{i=1,...,m} \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}, \quad \text{r=1,...,s} \\
& \lambda_j \geq 0, \qquad \text{j=1,...,n}
\end{aligned} \tag{3.10}$$

From the dualty theorem of linear programming, the optimal objective values of the CCR and DCCR model are equal. Let θ^* be the optimal objective value (efficiency value). Using the constraints $\sum_{i=1}^{m} v_i x_{io} = 1$ and $\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0$ in (CCR),an efficiency value θ^* of the target DMU falls in the rann of (0,1]. To determine which DMUs are efficient, we introduce the definition of Pareto-Koopmans efficiency (1978,1985) as follws:

Definition 3.1. *(Pareto-Koopmans Efficiecy).A DMU is fully efficient if and only if it is impossible to improve any input or output without worsening some other input or outputs.*

From definition, at the optimal solution,the DMUo with $\theta^* = 1$ may not be Pareto-Koopman efficient if it is possible to make additional improvement(lower input or higher output) without worsening any other input or output.Therefore, we introduce vectors of input excesses and output shortfall as follows:

$$S^- = \theta X_o - X\lambda,$$

$$S^+ = Y\lambda - Y_o$$

where $S^{-} \geq 0$,$S^{+} \geq 0$ are defined as slack vector for any feasible solution (θ, λ) of the DCCR model.Then a DMU is Pareto-Koopman efficient if $\theta^{*} = 1$ and all opimal slack values are zroe.

For the CCR model, Pareto-Koopmans efficiency. A DMU with $\theta^{*} = 1$ but with an excess in inputs and/or a shortage in outputs is "technically" efficient but is "mix" inefficient owing to the fact that there are some inputs or outputs that can be improved.

In addition to the CCR model, other well-known DEA models include the "BCC" model, named after Banker, Charnes and Coopers (1984), the "additive" model, the "free disposal hull" (FDH) model, and "slack-based measure of efficiency" (SBM) model.

In this paper, the focus will be on the CCR model because the CCR model was the original DEA model. All other models are extensions of the CCR model obtained by either modifying the production possibility set of the CCR model or adding slack variables in the objective function. Hence, an approach developed for solving the CCR model can be adapted for other DEA models.

3.1 Data envelopment analysis with fuzzy data

In recent years, fuzzy set theory has been proposed as a way to quantify imprecise and vague data in DEA models. fuzzy DEA models take the form of fuzzy linear programming model. The CCR model with fuzzy coefficients and its dual are formulated as the following linear programming models:

$(FCCR)$

$$
\begin{aligned}
\max \quad & \sum_{r=1}^{s} u_r \tilde{y}_{ro} \\
\text{s.t.} \quad & \sum_{i=1}^{m} v_i \tilde{x}_{io} \simeq 1; \\
& \sum_{r=1}^{s} u_r \tilde{y}_{rj} - \sum_{i=1}^{m} v_i \tilde{x}_{ij} \preceq 0 \quad \text{j=1,...,n} \\
& u_r, v_i \geq 0, \qquad \text{r=1,...,s,} \quad \text{i=1,...,m}
\end{aligned}
\tag{3.11}
$$

$(FDCCR)$

$$\begin{aligned}
\text{Min} \quad & \theta \\
\text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j \tilde{x}_{ij} \preceq \theta \tilde{x}_{io}, && \text{i=1,...,m} \\
& \sum_{j=1}^{n} \lambda_j \tilde{y}_{rj} \succeq \tilde{y}_{ro}, && \text{r=1,...,s} \\
& \lambda_j \geq 0, && \text{j=1,...,n}
\end{aligned} \tag{3.12}$$

where $\tilde{X}_o$ is the column vector of fuzzy inputs consumed by the target DMU (DMUo), $\tilde{X}$ is the matrix of fuzzy inputs of all DMUs, $\tilde{Y}_o$ is the column vector of fuzzy outputs consumed by the target DMU (DMUo), $\tilde{Y}$ is the matrix of fuzzy outputs of all DMUs.

The fuzzy CCR models cannot be solved by a standard LP solver like a crisp CCR model because coefficients in the fuzzy CCR model are fuzzy sets. With the fuzzy inputs and fuzzy outputs, the optimality conditions for the crisp DEA model need to be clarified and generalized. The few papers that have been published on solving fuzzy DEA problems can be categorized into four distinct approaches: tolerance approach, defuzzification approach, α-level based approach, and fuzzy ranking approach.

In the next sections we present a new possibility approach to solving fuzzy DEA models.This approach applies the ranking function (3.2) to solve the above FCCR and FDCCR.

Theorem 3.1. *The following linear programing problem (CCR) and the FCCR are equivalent:*

$(FCCR)$

$$\begin{aligned}
max \quad & \sum_{r=1}^{s} u_r \tilde{y}_{ro} \\
s.t. \quad & \sum_{i=1}^{m} v_i \tilde{x}_{io} \simeq 1; \\
& \sum_{r=1}^{s} u_r \tilde{y}_{rj} - \sum_{i=1}^{m} v_i \tilde{x}_{ij} \preceq 0 && j=1,\dots,n \\
& u_r, v_i \geq 0, && r=1,\dots,s, \quad i=1,\dots,m
\end{aligned} \tag{3.13}$$

(CCR)

$$\begin{aligned} max \quad & \sum_{r=1}^{s} u_r y_{ro} \\ s.t. \quad & \sum_{i=1}^{m} v_i x_{io} = 1; \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \quad j=1,\dots,n \\ & u_r, v_i \geq 0, \qquad r=1,\dots,s, \quad i=1,\dots,m \end{aligned} \tag{3.14}$$

where x_{ij}, y_{rj} are real numbers corresponding to the fuzzy numbers $\tilde{x}_{ij}, \tilde{y}_{rj}$ with respect to a given linear ranking function τ, respectively.

Proof: By considerate the ranking function and definition (2.6) it is to see that every optimal feasible solution of FCCR is an optimal feasible solution of CCR, on the other hand, every optimal feasible solution of CCR is an optimal feasible solution of FCCR.

Theorem 3.2. *The above theorem shows that the sets of all feasible solutions of FCCR and CCR are the same. Also if $(\bar{U}, \bar{V})$is an optimal solution for FCCR, then $(\bar{U}, \bar{V})$ is an optimal solution for CCR.*

Theorem 3.3. *If problem CCR does not have a optimal solution then FCCR does not have a optimal solution either.*

Similar to the duality theory in linear programing (see for example,Bazaraa et al.),for every FCCR, there is an associated problem which satisfies some important properties.
For the (FCCR)

$$\begin{aligned} \max \quad & \sum_{r=1}^{s} u_r \tilde{y}_{ro} \\ \text{s.t.} \quad & \sum_{i=1}^{m} v_i \tilde{x}_{io} \simeq 1; \\ & \sum_{r=1}^{s} u_r \tilde{y}_{rj} - \sum_{i=1}^{m} v_i \tilde{x}_{ij} \preceq 0 \quad \text{j=1,...,n} \\ & u_r, v_i \geq 0, \qquad \text{r=1,...,s,} \quad \text{i=1,...,m} \end{aligned} \tag{3.15}$$

define the dual fuzzy linear programing problem (FDCCR)as

$$
\begin{aligned}
\text{Min} \quad & \theta \\
\text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j \tilde{x}_{ij} \preceq \theta \tilde{x}_{io}, \quad && \text{i=1,...,m} \\
& \sum_{j=1}^{n} \lambda_j \tilde{y}_{rj} \succeq \tilde{y}_{ro}, \quad && \text{r=1,...,s} \\
& \lambda_j \geq 0, && \text{j=1,...,n}
\end{aligned} \tag{3.16}
$$

Theorem 3.4. *(Fundamental theorem of duality). For any FCCR and its corresponding FDCCR, exactly one of the following statements is true.*

1. *Both have optimal solutions (U^*, V^*) and θ^* with ${V^*}^T \tilde{Y}_o \simeq \theta^*$.*

2. *If one problem is unbounded then the other must be infeasible.*

3. *Both problems are infeasible.*

Theorem 3.5. *Let (U^*, V^*) be an optimal solution for FCCR then there exists at least one binding constraint such as $\sum_{r=1}^{s} u_r^* \tilde{y}_{rj} - \sum_{i=1}^{m} v_i^* \tilde{x}_{ij} \simeq 0$.*

Proof: Consider FCCR model:

$$
\begin{aligned}
\max \quad & \sum_{r=1}^{s} u_r \tilde{y}_{ro} \\
\text{s.t.} \quad & \sum_{i=1}^{m} v_i \tilde{x}_{io} \simeq 1; \\
& \sum_{r=1}^{s} u_r \tilde{y}_{rj} - \sum_{i=1}^{m} v_i \tilde{x}_{ij} \preceq 0 \quad && \text{j=1,...,n} \\
& u_r, v_i \geq 0, && \text{r=1,...,s,} \quad \text{i=1,...,m}
\end{aligned} \tag{3.17}
$$

When we have $(\sum_{r=1}^{s} u_r^* \tilde{y}_{rj} - \sum_{i=1}^{m} v_i^* \tilde{x}_{ij} \prec 0;\ j = 1, ..., n)$ then $(\sum_{r=1}^{s} u_r^* y_{rj} - \sum_{i=1}^{m} v_i^* x_{ij} < 0;\ j = 1, ..., n)$ where x_{ij}, y_{rj} are real numbers corresponding to the fuzzy numbers $\tilde{x}_{ij}, \tilde{y}_{rj}$ with respect to a given linear ranking function τ, respectively and in CCR model equivalent to FCCR we have:
$\sum_{r=1}^{s} u_r^* y_{rj} - \sum_{i=1}^{m} v_i^* x_{ij} < 0;\ j = 1, ..., n$ with usual theorems in DEA it is not true.

4 Methodology and examples

We evaluate thirty branches of Tehran Social Security Insurance Organization at this section.Each branch uses of four inputs in order to produce four outputs. The labels of inputs and outputs are presented in under table.

	Input	Output
1	The number of personals	The total number of insured persons
2	The total number of computers	The number of insured persons'agreements
3	The area of the branch	The total number of life-pension receivers
4	Administrative expenses	The receipt total sum (Incom)

Table1.The labels of inputs and outputs.

The total dates are related to a chronological sections of 2003(A-D). The total triangular Fuzzy dates has been viewed in tables (2).It is considered that "M" as number middle,"U" as number up and "L" as number low.For example if $(\tilde{I}1 = (I^M1, I^U1, I^L1)$ denote a triangular fuzzy number then we use ranking function as follows:

$$\tau(\tilde{I}1) = 1/2(I^M1 + 1/2(I^U1 + I^L1)). \tag{4.18}$$

After using of ranking function τ , the dates are given as crisp and then with applying explained method on the essay the results are presented in the table (4).

	I^M1	I^U1	I^L1	I^M2	I^U2	I^L2	I^M3	I^U3	I^L3	I^M4	I^U4	I^L4
1	98.83	100	96	86.5	87	86	4000	4000	4000	62895029.33	103656656	14730450
2	78.66	81	75	88.83	90	88	2565	2565	2565	71650228.17	95701909	41144517
3	78.5	80	77	87	89	85	1343	1343	1343	42813134.5	60301920	28792550
4	92.33	94	91	94.5	96	93	1500	1500	1500	72295517.33	94569518	24277018
5	90.66	92	89	83	83	83	1680	1680	1680	69200351.17	80729482	43355800
6	103	105	102	97	97	97	3750	3750	3750	77762043	116268609	45571800
7	97.33	100	96	91	92	90	3313	3313	3313	114537397.8	171152176	78011675
8	87	90	85	92	92	92	1500	1500	1500	75597743	135858469	14969393
9	108.5	112	106	88.33	92	84	1600	1600	1600	101976164.3	182951858	61024310
10	108.83	111	107	95	95	95	1725	1725	1725	71223781.17	94511449	21625962
11	97.16	101	94	78	78	78	1920	1920	1920	90515357	124984300	39664990
12	78.83	79	78	89	89	89	4433	4433	4433	74994666.67	139888000	23106000
13	102	102	102	107.66	111	107	2500	2500	2500	77815934.67	144462162	17756770
14	85	88	82	93.16	94	92	2800	2800	2800	89785320.5	168961589	30082208
15	80.16	82	77	92.83	94	92	1630	1630	1630	76900120.67	158581843	48034155
16	89.5	91	89	85	85	85	1127	1127	1127	190788340.2	827138457	35631132
17	88.5	90	84	104	104	104	3400	3400	3400	100948330.3	214346133	33983780
18	101.83	108	94	91.66	92	91	1304	1304	1304	205857963.5	389185592	37568447
19	101	103	97	95.16	96	95	4206	4206	4206	65746035.5	100855390	5504769
20	84	87	82	100.16	101	100	1340	1340	1340	95179649.67	136748402	52060934
21	72	73	71	89.33	90	88	1393	1393	1393	45477995.83	62663680	19453875
22	115.66	118	112	121.83	123	120	2191	2191	2191	91465848.33	190535604	45229750
23	82.83	86	80	100	100	100	2140	2140	2140	78629283.5	96578148	45992927
24	90.66	93	87	91.5	93	91	1231	1231	1231	78946229.17	113530688	47909750
25	99.33	103	97	90	90	90	1960	1960	1960	210592149.3	757564117	52110247
26	81	83	79	81	81	81	3375	3375	3375	65799659.17	109022822	33088649
27	108	110	107	101	101	101	2540	2540	2540	67435472.67	98528957	21300223
28	98.16	102	96	91.5	97	87	1603	1603	1603	178472638.7	537844764	45363108
29	67.66	69	67	83	86	81	2300	2300	2300	59116363.33	94148887	10617400
30	91	93	88	92.16	94	90	2930	2930	2930	87029553.67	113930230	66736398

Table 2. The triangular fuzzy Inputs for 30 branches of Insurance Organization.

	O^M1	O^U1	O^L1	O^M2	O^U2	O^L2	O^M3	O^U3	O^L3	O^M4	O^U4	O^L4
1	56570.66	57318	55830	36.83	45	30	1336.33	1350	1307	169.83	192	145
2	36800.5	36852	36740	15.66	22	0	8463.33	8571	8385	336.83	486	175
3	38446.16	38783	38004	17.66	27	11	6594.16	6601	6588	181.83	276	113
4	35685.16	36017	35469	30.33	55	10	10820.66	10821	10820	237.66	316	128
5	53869	54817	52927	27.16	43	9	9605.83	9751	9493	201	263	101
6	72446.33	78574	70254	12.33	19	7	8066.16	8752	7536	305	615	82
7	35856.33	37443	32585	93.16	129	47	14557.16	14994	14118	225	392	154
8	45027.66	47270	42900	17.33	27	11	1648.66	1661	1634	146	220	54
9	86221.5	87220	85399	58.16	97	43	10492.66	10775	10206	243	289	179
10	47142.5	47316	46924	26.33	36	9	6697.66	6823	6608	200	342	117
11	40482.33	44298	36652	178.5	242	81	12099.83	12261	11996	194.83	286	37
12	39594.5	39620	39582	20.33	31	11	7515.5	7624	7422	153.83	184	124
13	57484.83	58816	56144	42	57	30	7565.66	7936	7380	241.66	430	185
14	88898.5	90250	87716	35.66	43	28	646	660	630	107	167	51
15	50400.33	50593	50210	11.83	16	6	10251.66	10256	10247	137.33	295	28
16	48577.83	49489	47727	22.33	30	15	7411.5	7542	7302	200.16	286	85
17	53099.5	53249	52923	20	28	15	4866.16	5058	4740	161.83	240	109
18	83235.5	89111	78550	20.66	25	13	4879.5	5151	4745	136	224	72
19	46496.83	46791	46154	16.16	21	13	1621.66	1636	1611	232.83	477	129
20	28973.83	32943	27978	83.83	325	29	14637.83	14820	14473	275.5	368	190
21	27568.5	27940	27128	12.5	20	0	936.66	973	921	119.16	179	55
22	102562.5	103047	102175	39.83	49	31	2438.83	3577	252	212.16	320	120
23	33616	35627	31819	19.16	32	12	2049.83	2147	1963	300.666	522	156
24	53183.66667	55163	51345	62	73	35	10185	10238	10157	151.33	205	85
25	73729	74633	72915	45	52	40	4400.5	4668	4193	233.16	427	112
26	43402.66	44363	42887	25.16	33	11	598.33	628	560	296.16	390	218
27	78840.83	79695	78068	38.5	46	26	9160.66	9338	8963	188.66	265	136
28	72152.33	72534	71743	70.66	92	50	11687.66	12569	8762	163.83	240	102
29	38516	38914	38054	21.66	33	13	1445.16	1477	1405	72.66	156	23
30	63973.33	64541	63182	23.33	32	10	11380.83	11609	11143	269.5	378	122

Table 3. The triangular fuzzy Outputs for 30 branches of Insurance Organization.

	Input 1	Input 2	Input 3	Input 4	Output 1	Output 2	Output 3	Output 4	θ^*
1	94.91	85	4000	89300200.67	56572.33	37.16	1332.41	169.16	0.79
2	77	93	2565	85025580	36798.25	13.33	8470.66	333.66	1.00
3	76.5	87	1500	911447170.8	35714.08	31.416	10820.58	229.83	0.86
4	92.91	93	1500	911447170.8	35714.08	31.41	10820.58	229.83	0.87
5	90.16	85.33	1680	92167454.92	53870.5	26.58	8105.08	326.75	0.95
6	102.66	97	3750	78966919.67	73430.16	12.66	8105.08	326.75	1.00
7	94.5	90.5	3313	79493485.17	45056.33	18.16	1648.08	141.5	1.00
8	86.16	92.41	1500	79493485.17	45056.33	18.16	1648.08	141.5	0.71
9	103.41	92	1600	153439927.2	86265.5	64.08	12114.16	178.16	1.00
10	102.5	96.16	1725	77960013.08	47131.25	24.41	6706.58	214.75	0.89
11	95	79	1920	100704537.3	40478.66	170	12114.16	178.16	1.00
12	77.83	91	4433	92118083.33	39597.75	20.66	7519.25	153.91	0.78
13	104.83	104.16	2500	88756293.75	57482.41	42.75	7611.83	274.583	0.91
14	88.25	95	2800	59769138.42	88940.75	35.58	645.5	108	1.00
15	81.33	93.66	1127	71782212.67	48592.91	22.41	7416.75	192.83	1.00
16	89	85.41	1127	71782212.67	48592.91	22.41	7416.75	192.83	1.00
17	91.5	104	3400	147562525.8	83533	19.83	4913.75	142	0.72
18	114.16	93.91	1304	79763145.5	83533	19.83	4913.75	142	1.00
19	96.41	98	4206	132489092.7	46484.66	16.58	1622.58	267.91	0.78
20	86.58	101	1340	83499803.42	29717.16	130.41	14642.16	277.25	1.00
21	113.41	123	2191	108737313.3	102586.75	39.91	2176.66	216.08	0.60
22	113.41	123	2191	108737313.3	102586.75	39.91	2176.66	216.08	1.00
23	79	100	2140	60940728.17	53218.83	58	10191.25	148.16	1.00
25	98.41	90	1960	93199199.67	73751.5	45.5	4415.5	251.33	1.00
26	74.83	84.5	3375	75652227.83	43513.83	23.58	596.16	300.08	1.00
27	104.41	102.25	2540	108386428.6	78861.16	37.25	9155.58	194.58	0.97
28	99.33	97.25	1603	476709469.2	72145.41	70.83	11176.58	167.41	0.98
29	73.16	79	2300	84387893.42	38500	22.33	1443.08	81.08	0.58
30	62564.25	92	2930	80225938.08	63917.41	22.16	11378.41	259.75	1.00

Table 4. The crisp data for 30 branches of Insurance Organization.

the θ^* of all DMUs are denoted in the last column of table (4). As it is apparent in above table, we see the efficient in 16 branches and the inefficient in remained 14 branches. It is viewed the most inefficient in the branch 29.

5 Conclusion

The purpose of this study was to develop the DAE models to DMUs with fuzzy data that since the level of inputs and outputs for DMU_o are not know exactly, we respect to a given linear ranking function then, in this cases, the DAE models solve when inputs and outputs data are fuzzy , we try to use a fuzzy linear programing method to approach for efficiency measures by Fuzzy linear programing and applied to a numerical example.

References

[1] M.S. Bazaraa, J.J. Jarvis, H.D. Sherali, Linear Programming and Network Flows, John Wiley & Sons, New York, 1990.

[2] Delgado, M., Verdegay, J.L., and Vila, M. A., A general model for fuzzy linear programming, Fuzzy Sets and Systems 29 (1989) 21-29.

[3] Verdegay, J.L., A dual approach to solve the fuzzy linear programming problem, Fuzzy Sets and Systems 14 (1984) 131-141.

[4] Herrera, F. and Verdegay, J.L., Approaching fuzzy linear programming problems, Interactive fuzzy optimization, Springer-Verlag, Berlin, 1991.

[5] C.A. Arnade, using data envelopment analysis to measure international agricultural efficiecy and productivity, United States Department of Agriculture, Economic Research Service, Tech. Bull. 1831(1994) 1-30.

[6] Saowanee Lertworasirikul, Shu-Cherng Fang, Jeffrey A. Joines, Henry L.W. Nuttle, Fuzzy data envelopment analysis (DEA): a possibility approach, Fuzzy Sets and Systems (2002).

[7] N.Mahdavi-Amiri, S.H. Nasseri,Dualtiy in fuzzy number linear programming by use of a certain linear ranking function, Applied Mathematics and Computation (2006).

[8] G.R.Jahanshahloo, F.Hosseinzadeh Lotfi, M.Moradi, Sensitivity and stability analysis in DEA with interval data, Applied Mathematics and Computation, (2003).

Received: July 30, 2006