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τ^c Prefilters in Intuitionistic Fuzzy Sets

N. Pankajam

Dr.Mahalingam College of Engineering & Technology
Coimbatore, India
pankajam_rangarj@yahoo.com

Abstract

The notion of fuzzy sets was introduced by L.A. Zadeh[9] and was extended to intuitionistic fuzzy subsets by K.Atanassov[1].The notions of fuzzy and intuitionistic fuzzy topological spaces were introduced and studied by C.L.Chang[5], D.Coker[6]. In this paper intuitionistic fuzzy τ^c prefilter is introduced and the relation between the τ^c prefilters and ultra filters are studied.

Keywords: Intuitionistic fuzzy topology; Prefilters, Ultra filters.

1 Introduction

After the introduction of the concept of fuzzy sets by Zadeh[9] in 1983, Atanassov proposed a generalization of the notion of fuzzy set: the concept of intuitionistic fuzzy set[1].Some basic results on intuitionistic fuzzy sets were published in [2,3].The notions of fuzzy and intuitionistic fuzzy topological spaces were introduced and studied in[5,6].Blasco Mardones et al [4] introduced a new process of compactification for a fuzzy topological space by using δ^c prefilters. In this paper we define τ^c prefilter in intuitionistic fuzzy sets and obtain some of its properties.

Definition1.1 [1]

Let X be a nonempty set. An intuitionistic fuzzy set(IFS for short) A is an object having the form $A = \{\langle x, \mu(x), \gamma(x) : x \in X \rangle\}$ where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership(namely $\mu_A(x)$) and the degree of non membership (namely $\gamma_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x) + \gamma_A(x) \le 1$ for each $x \in X$.

Definition1.2 [3]

Let X be a nonempty set and let A and B be two IFSs of X. Then

(a) $A \subseteq B$ iff $\mu_A(x) \le \mu_B(x)$ and $\gamma_A(x) \ge \gamma_B(x)$ for all $x \in X$.
(b) $A = B$ iff $A \subseteq B$ and $B \subseteq A$
(c) $A \cup B = \{\langle\ x, \mu_A(x) \vee \mu_B(x), \gamma_A(x) \wedge \gamma_B(x)\ \rangle : x \in X\}$
(d) $A \cap B = \{\langle\ x, \mu_A(x) \wedge \mu_B(x), \gamma_A(x) \vee \gamma_B(x)\ \rangle : x \in X\}$
e) $\overline{A} = \{\langle x.\gamma_A(x), \mu_A(x)\rangle : x \in X\}$

Definition1.3 [6]

Let $\{A_i : i \in J\}$ be an arbitrary family of IFSs in X.Then
(a) $\bigcap A_i = \{\langle x, \wedge \mu_{A_i}, \vee \gamma_{A_i} \rangle : x \in X\}$
(b) $\bigcup A_i = \{\langle x, \vee \mu_{A_i}, \wedge \gamma_{A_i} \rangle : x \in X\}$

Definition1.4 [6]

$0_{\sim} = \{\langle\ x, 0, 1\ \rangle : x \in X\}$ and $1_{\sim} = \{\langle\ x, 1, 0\ \rangle : x \in X\}$

Definition1.5 [6]

An intuitionistic fuzzy topology (IFT for short) on a nonempty set X is a family τ of IFSs in X satisfying the following axioms:

(a) $0_{\sim}, 1_{\sim} \in \tau$
(b) $A_1 \cap A_2 \in \tau$ for any $A_1, A_2 \in \tau$
(c) $\bigcup A_i \in \tau$ for any arbitrary family $\{A_i : i \in J\}$

In this case the pair (X, τ) is called an intuitionistic fuzzy topological spaces (IFTS for short) and any IFS in τ is known as an intuitionistic fuzzy open sets (IFOS for short) in X**.**

Definition1.6 [6]

An intuitionistic fuzzy topological space in the sense of Lowen [8] is a pair (X, τ) where (X, τ) is an IFTS and each IFS in the form $C_{\alpha,\beta} = \{(x, \alpha, \beta) : x \in X\}$ where $\alpha, \beta \in I$ are arbitrary and $\alpha + \beta \le 1$, belongs to τ .

Definition1.7 [6]

The complement $\overline{A}$ of an IFOS A is an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS for short) in X.

Definition1.8 [7]

The support of a fuzzy set A is a crisp set that contains all the elements of X that have nonzero membership grades in A.

Notation

If τ is an IFT on X we let τ^c denotes the collection of all intuitionistic fuzzy closed sets.

2. τ^c Prefilters in Intuitionistic Fuzzy Sets

Definition 2.1

Let be a IFTS.Let $\mathcal{F} \subset \tau^c$ satisfies

(i) $\mathcal{F} \neq 0_\sim$ and $0_\sim \notin \mathcal{F}$

(ii) $A_1, A_2 \in \tau$ then $A_1 \cap A_2 \in \tau$

(iii) If $A \in \mathcal{F}$ and $B \subset \tau^c$ with $A \subseteq B$ then $B \in \mathcal{F}$.

$\mathcal{F}$ is called an IF closed filter or τ^c a -prefilter on X.

Definition 2.2

Let $\mathcal{F}$ be a τ^c prefilter and let B $\subset \mathcal{F}$. $\mathcal{B}$ is called a base for $\mathcal{F}$ if for each $A \in \mathcal{F}$ there is a $B \in \mathcal{B}$ such that $B \subseteq A$.

Definition 2.3

Let $\mathcal{H} \subset \tau^c$.$\mathcal{H}$ is a sub base for some τ^c prefilter if the collection $\{\bigcap A_i : i \in H\}$ is a base for some τ^c prefilter.

Theorem2.1

Let $\mathcal{B} \subset \tau^c$ **.** Equivalent statements are

(i) There is a unique τ^c prefilter $\mathcal{F}$ such that $\mathcal{B}$ is a base for it.

(ii) (a)$\mathcal{B} \neq 0_\sim$ and $0_\sim \notin \mathcal{B}$

(b)If $B_1, B_2 \in \mathcal{B}$, there is $B_3 \in \mathcal{B}$ with $B_3 \subseteq B_1 \cap B_2$

Proof follows from the definitions 2.1, 2.2, 2.3.

Remark2.1

If $\mathcal{B}$ satisfies (a) and (b) the generated τ^c prefilter $\mathcal{F}$ is $\mathcal{F} = \{ A \in \tau^c :$ there exists $B \in \mathcal{B}$ with $B \subseteq A \}$.

Definition2.4:

Let $\mathcal{G} \subset \tau^c$ with the property that the intersection of any finite sub collection from $\mathcal{G}$ is nonempty. There exists a unique τ^c prefilter containing $\mathcal{G}$ whose base is the set of all finite intersections of elements in $\mathcal{G}$. Such a τ^c prefilter is called the τ^c prefilter generated by $\mathcal{G}$.

As a consequence of the previous observations we get the following result.

Theorem 2.2

Let $\mathfrak{F}$ be τ^c prefilter and $A \in \tau^c$. The following statements are equivalent:

(i)$\mathfrak{F} \cup \{A\}$ is contained in a τ^c prefilter

(ii)For each $B \in \mathfrak{F}$ we have $A \cap B \neq 0_{\sim}$

Definition 2.5

Let $\mathfrak{F}$ be τ^c prefilter. $\mathfrak{F}$ is an intuitionistic fuzzy τ^c ultra filter if $\mathfrak{F}$ is a maximal element in the set of τ^c prefilters ordered by the inclusion relation.

Theorem 2.3

Every τ^c prefilter is contained in some intuitionistic fuzzy τ^c ultra filter.

The following result characterizes τ^c ultra filters.

Theorem 2.4

Let $\mathfrak{F}$ be τ^c prefilter on X.The following statements are equivalent:

(i)$\mathfrak{F}$ is an intuitionistic fuzzy τ^c ultra filter

(ii)If A is an element of τ^c such that $A \cap B \neq 0_{\sim}$ for each $B \in \mathfrak{F}$ then $A \in \mathfrak{F}$.

(iii)If $A \in \tau^c$ and $A \notin \mathfrak{F}$, then there is $B \in \mathfrak{F}$ such that $\mathrm{supp}(B) \subseteq X - \mathrm{supp}(A)$.

Proof

(i) $\Rightarrow$(ii)

Suppose $A \in \tau^c$ and $A \cap B \neq 0_{\sim}$ for each $B \in \mathfrak{F}$.

By theorem2.2, there is τ^c prefilter $\mathfrak{F}^*$ generated by $\mathfrak{F} \cup \{A\}$.Then $\mathfrak{F} \subseteq \mathfrak{F}^*$

Since $\mathfrak{F}$ is a τ^c ultra filter the above inclusion must imply that $\mathfrak{F} = \mathfrak{F}^*$. Therefore $A \in \mathfrak{F}$.

(ii) $\Rightarrow$(iii)

Let $A \in \tau^c$ and $A \notin \mathfrak{F}$

By (ii) there exists at least one $B \in \mathfrak{F}$ such that $A \cap B = 0_{\sim}$. Take $x \in \mathrm{supp(B)}$,then $\mu_{\mathrm{B}}(x) > 0$ .Since $A \cap B = 0_{\sim}$ we get $\mu_{\mathrm{A}}(x) = 0$ .Therefore $x \notin \mathrm{supp(A)}$. That is $x \in X - \mathrm{supp(A)}$ .Hence $\mathrm{supp(B)} \subseteq X - \mathrm{supp(A)}$.

(iii) $\Rightarrow$(i)

Let $\mathfrak{G}$ be a τ^c prefilter with $\mathfrak{F} \subset \mathfrak{G}$ and $\mathfrak{F} \neq \mathfrak{G}$.

Then there exists $A \in \mathfrak{G}$ such that $A \notin \mathfrak{F}$.

By (iii) there exists $B \in \mathfrak{F}$ such that $\mathrm{supp(B)} \subseteq X - \mathrm{supp(A)}$.

Take $x \in X$ .Then $\mu_B(x) = 0$ or $\mu_B(x) > 0$.

If $\mu_B(x) = 0$ then $\mu_{A\cap B}(x) = 0$.
If $\mu_B(x) > 0$ then $x \in \text{supp(B)}$.
Hence $x \in X - \text{supp(A)}$ .That is $x \notin \text{supp(A)}$. Therefore $\mu_A(x) = 0$.
Hence $\mu_{A\cap B}(x) = 0$ .Therefore $A \cap B = 0_\sim$.
Since $A, B \in \mathcal{G} \Rightarrow A \cap B \in \mathcal{G}$ we get $0_\sim \in \mathcal{G}$ which is a contradiction.
Therefore $\mathcal{F} = \mathcal{G}$. Hence $\mathcal{F}$ is an intuitionistic fuzzy τ^c ultra filter.

Theorem 2.5

Let U_1 and U_2 be a pair of different intuitionistic fuzzy τ^c ultra filters on X. Then $\left(\bigcap_{A_1 \in U_1} \sup p(A_1)\right) \bigcap \left(\bigcap_{A_2 \in U_2} \sup p(A_2)\right) = 0_\sim$

Proof

Suppose there exists x belonging to the above intersection. Then for each $A_1 \in U_1$ and $A_2 \in U_2$, $\mu_{A_1}(x) \neq 0$, $\mu_{A_2}(x) \neq 0$. Therefore $A_1 \cap A_2 \neq 0_\sim$ for every $A_1 \in U_1$.
Hence by (i) $\Rightarrow$(ii) of theorem2.4 we get $A_2 \in U_2$.This is true for every $A_2 \in U_2$.
Therefore $U_2 \subseteq U_1$.
Since U_2 is an ultra filter we get $U_1 = U_2$.
This is a contradiction as U_1 and U_2 are given to be different intuitionistic fuzzy τ^c ultra filters. Hence our assumption is wrong. Therefore the given intersection is empty.

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