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Dynamic Analysis of a SI System with Periodic Biological and Chemical Control[1]

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Abstract

In this paper, we consider a SI system with spraying microbial pesticide and releasing the infected pests, the infected pests have the function similar to the microbial pesticide and can infect the healthy pests, further weaken or disable their prey function till death. By applying the Floquet theorem of linear periodic impulsive equations and the comparison theorem, we show that there exists a globally asymptotically stable pest eradication periodic solution when the impulsive period is less than the critical value τ_{max}, we further prove that the system is uniformly permanent if the impulsive period $\tau > \tau_{max}$. Thus, we can use the stability of the positive periodic solution and its period to control insect pests at acceptably low levels.

Mathematics Subject Classification: 34C05, 92D25

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1. Introduction

Integrated Pest Management (IPM) is a safer, and usually less costly option for effective pest management. IPM is an approach to solving pest problems by applying knowledge about the pest to prevent them from damaging crops. Under an IPM approach, actions are taken to control insects, disease or weed problems only when their numbers exceed acceptable levels. The concept of

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IPM was widely practised during the 1970s and 1980s [1,2,3]. IPM is a long term management strategy that uses a combination of biological, cultural, and chemical tactics that reduce pests to tolerable levels, with little cost to grower and minimal effect on the environment.

However, it is inevitable that IPM may cause pollution to the environment more or less due to the use of chemical pesticide. Therefore, in the paper we propose a biological control strategy-controlling the pest by introducing microbial pesticide and infected pests simultaneously. The microbial pesticide comes from the insect pathology in these years and becomes an important part in biological control, which mainly includes virus pesticide, bacteria pesticide, protozoa pesticide and antibiotic. Among these pesticide, the virus pesticide and microsporidium have the most remarkable effect for pest control. Research shows that in the control of Laphygma exigua and Pieris rapae in the lettuce, tomato, capsicum and chrysanthemum and so on, the virus pesticide is even better than the bacteria pesticide and chemical pesticide[4]. Compared with the chemical pesticide, the microbial pesticide has many advantages in the pest control including: high specificity; high selectivity; no pollution; industrialization available; effective protection of the pest's natural enemies. The application shows that the microbial pesticide is an effective, highly infectious and safe bio-pesticide which can be used in both short-term and long-term controls and plays an important role in pest management.

2. Model formulation and Natations

For IPM strategy, we combine the biological control and chemical control. The infectious pests are released periodically every time period τ, meanwhile periodic spraying the microbial pesticide for susceptible pests. The infected pests have the function similar to the microbial pesticide and can infect the healthy pests, further weaken or disable their prey function till death. So we consider the following impulsive differential equation:

$$\begin{cases} \left.\begin{array}{l} \dot{S} = rS(1-\frac{S+\theta I}{k}) - \beta S I^2, \\ \dot{I} = \beta S I^2 - dI, \end{array}\right\} & t \neq n\tau, \\ \left.\begin{array}{l} \Delta S = -(\mu_1+\mu_2)S, \\ \Delta I = \mu_1 S - \mu_3 I + p, \end{array}\right\} & t = n\tau, \end{cases} \tag{2.1}$$

where $S(t)$ and $I(t)$ are densities of the susceptible and infectious, respectively, $\beta > 0$ is called the transmission coefficient, $d > 0$ is the death rate of the infectious pests. $\Delta S(t) = S(t^+) - S(t), \Delta I(t) = I(t^+) - I(t)$. $S(t)$ in the absence of $I(t)$ grows logistically with carrying capacity k, and with an intrinsic birth rate constant r, $0 \leq \mu_1 < 1$ represents the fraction from susceptible to infectious due to spraying the microbial pesticide at $t = n\tau$, $0 \leq \mu_2 < 1, 0 \leq \mu_3 < 1$ which represents the fraction of susceptible and infective pests due to spraying pesticides at $t = n\tau$, respectively, and $\mu_1 + \mu_2 < 1$, $p > 0$ is the release amount of the infected pests at $t = n\tau, n \in N, N = \{0, 1, 2, \cdots\}$, τ is the period of the impulsive effect. That is, we can use a combination of biological and chemical tactics to eradicate pests or keep the pest population below the damage level.

In the following, we agree on some notations which will prove useful and give some definitions.

Lemma 2.1. Suppose $x(t)$ is a solution of (2.1) with $x(0^+) \geq 0$, then $x(t) \geq 0$ for $t \geq 0$, and further $x(t) > 0, t \geq 0$ for $x(t) > 0$.

For convenience, we give some basic properties of the following system:

$$\begin{cases} \dot{I} = -dI, & t \neq n\tau, \\ \triangle I = -\mu_3 I + p, & t = n\tau, \end{cases} \tag{2.2}$$

We have the following lemma:

Lemma 2.2. System (2.2) has a unique positive periodic solution $\widetilde{I}(t)$ with period τ and for every solution $I(t)$ of (2.2). $|I(t) - \widetilde{I}(t)| \to 0$ as $t \to \infty$, where $\widetilde{I}(t) = \frac{p\exp(-d(t-n\tau))}{1-(1-\mu_3)\exp(-d\tau)}, n\tau < t \leq (n+1)\tau, n \in N, \widetilde{I}(0^+) = \frac{p}{1-(1-\mu_3)\exp(-d\tau)}$ and $\widetilde{I}(t)$ is globally asymptotically stable. Hence the solution of (2.2) is

$$I(t) = (1 - \mu_3)(\widetilde{I}(0^+) - \frac{p}{1 - (1 - \mu_3)\exp(-d\tau)})\exp(-dt) + \widetilde{I}(t).$$

Lemma 2.3. There exists a constant $M > 0$, such that $S(t) \leq M, I(t) \leq M$, for each positive solution $x(t) = (S(t), I(t))$ of (2.1) with all t large enough.

Definition 3.1. System (2.1) is said to be permanent if there exist positive constants m, M such that each positive solution $(S(t), I(t))$ of the system satisfies $m \leq S(t) \leq M, \ m \leq I(t) \leq M$ for all t sufficiently large.

3. Stability of the pest-eradication periodic solution

In this section, we study the stability of the pest-eradication periodic solution as a solution of the full system (2.1).

Theorem 3.1. The pest-eradication periodic solution $(0, \widetilde{I}(t))$ is globally asymptotically stable provided

$$r\tau + \ln(1-\mu_1-\mu_2) - \frac{pr\theta(1 - \exp(-d\tau))}{dk[1 - (1 - \mu_3)\exp(-d\tau)]} - \frac{p^2\beta(1 - \exp(-2d\tau))}{2d[1 - (1 - \mu_3)\exp(-d\tau)]^2} < 0. \tag{3.1}$$

Proof. Similar to Theorem 4.1 of Zhang et al.(2003)[5], we can prove the pest-eradication periodic solution $(0, \widetilde{I}(t))$ is locally asymptotically, so we omit it.

In the following, we prove the global attractivity. Choose sufficiently small $\varepsilon > 0$ such that

$$\delta = (1-\mu_1-\mu_2)\exp(\int_0^\tau (r - \frac{r\theta}{k}(\widetilde{I}(t)-\varepsilon) - \beta(\widetilde{I}(t)-\varepsilon)^2)dt) < 1.$$

Noting that $\dot{I}(t) \geq -dI(t)$ as $t \neq n\tau$ and $\Delta I(t) \geq -\mu_3 I(t) + p$ as $t = n\tau$, consider the following impulsive differential equation:

$$\begin{cases} \dot{x}(t) = -dx(t), & t \neq n\tau, \\ \Delta x(t) = -\mu_3 x(t) + p, & t = n\tau. \end{cases} \tag{3.2}$$

By lemma 2.2, system (3.2) has a globally asymptotically stable positive periodic solution

$$\widetilde{x}(t) = \frac{p\exp(-u(t-n\tau))}{1-\exp(-\mu\tau)} = \widetilde{I}(t), \quad n\tau < t \leq (n+1)\tau.$$

So by the comparison theorem and lemma 2.2, we get

$$I(t) \geq x(t) > \widetilde{I}(t) - \varepsilon, \tag{3.3}$$

From system (2.1), we obtain that

$$\begin{cases} \dot{S}(t) \leq S(t)(r - \frac{r\theta}{k}(\widetilde{I}(t)-\varepsilon) - \beta(\widetilde{I}(t)-\varepsilon)^2), & t \neq n\tau, \\ \Delta S(t) = -(\mu_1+\mu_2)S(t), & t = n\tau. \end{cases} \tag{3.4}$$

Integrating (3.4) on $(n\tau, (n+1)\tau]$, which yields

$$\begin{array}{l} S((n+1)\tau^+) = S(n\tau^+)\exp(\int_{n\tau}^{(n+1)\tau}(r - \frac{r\theta}{k}(\widetilde{I}(t)-\varepsilon) - \beta(\widetilde{I}(t)-\varepsilon)^2)dt) \\ = (1-\mu_1-\mu_2)S(n\tau)\exp(\int_{n\tau}^{(n+1)\tau}(r - \frac{r\theta}{k}(\widetilde{I}(t)-\varepsilon) - \beta(\widetilde{I}(t)-\varepsilon)^2)dt) \\ = S(n\tau)\delta. \end{array} \tag{3.5}$$

Thus, $S(n\tau) \leq S(0^+)\delta^n$ and $S(n\tau) \to 0$ as $n \to \infty$. Therefore, $S(t) \to 0$ as $n \to \infty$, since $0 < S(t) < (1-\mu_1-\mu_2)S(n\tau)\exp(r\tau)$ for $n\tau < t \leq (n+1)\tau$,

Next, we prove that $I(t) \to \widetilde{I}(t)$ as $t \to \infty$. for sufficiently small $0 < \varepsilon < \frac{d}{\beta M}$, there exists a $T_1 > 0$ such that $0 < S(t) < \varepsilon$ for all $t > T_1$. From system (2.1), we have

$$\begin{cases} \dot{I}(t) \leq (\beta\varepsilon M - d)I(t), & t \neq n\tau, \\ \Delta I(t) \leq \mu_1\varepsilon - \mu_3 I(t) + p, & t = n\tau, \end{cases} \tag{3.6}$$

considering the following comparison system

$$\begin{cases} \dot{y}(t) = (\beta\varepsilon M - d)y(t), & t \neq n\tau, \\ \Delta y(t) = \mu_1\varepsilon - \mu_3 y(t) + p, & t = n\tau. \end{cases} \tag{3.7}$$

By lemma 2.2, system (3.7) has a positive periodic solution

$$\widetilde{y}(t) = \frac{(\mu_1\varepsilon + p)\exp(-(d-\beta\varepsilon M)(t-n\tau))}{1-(1-\mu_3)\exp(-(d-\beta\varepsilon M)\tau)}, n\tau < t \leq (n+1)\tau,$$

which is globally asymptotically stable. Thus, for sufficiently small $\varepsilon_1 > 0$, there exists a $T_2 > T_1$ such that

$$I(t) \leq y(t) < \widetilde{y}(t) + \varepsilon_1. \tag{3.8}$$

Combining (3.3) and (3.8), we obtain $\widetilde{I}(t) - \varepsilon < I(t) < \widetilde{y}(t) + \varepsilon_1$ for t large enough, let $\varepsilon, \varepsilon_1 \to 0$, we get $\widetilde{y}(t) \to \widetilde{I}(t)$, then $I(t) \to \widetilde{I}(t)$ as $t \to \infty$. This completes the proof.

4. Permanence

Theorem 4.1. System (2.1) is uniformly permanent if

$$r\tau - \frac{pr\theta(1-\exp(-d\tau))}{dk[1-(1-\mu_3)\exp(-d\tau)]} - \frac{p^2\beta(1-\exp(-2d\tau))}{2d[1-(1-\mu_3)\exp(-d\tau)]^2} > -\ln(1-\mu_1-\mu_2). \tag{5.1}$$

Proof. Suppose $x(t) = (S(t), I(t))$ is a solution of (2.1) with $x(t) > 0$, from lemma 2.3, we may assume $S(t) \leq M, I(t) \leq M$ and $M > (\frac{r}{\beta})^{\frac{1}{2}}$, for t large enough. We may assume $S(t) \leq M, I(t) \leq M$ for $t \geq 0$.

Let $\zeta = \dfrac{p\exp(-d\tau)}{1-(1-\mu_3)\exp(-d\tau)} - \varepsilon_2 > 0$, where $\varepsilon_2 > 0$ sufficiently small.

According to lemma 2.2, we have $I(t) > \zeta$ for t large enough. So, if we can find positive number $\xi > 0$, such that $S(t) \geq \xi$ for t large enough, then our aim is obtained.

Next, we focus on finding $\xi > 0$ following two steps.

Step I: If (4.1) holds true, we can choose $0 < m_1 < \frac{d}{\beta M}$ and $\varepsilon_3 > 0$ small enough such that $\delta_1 = (1-\mu_1-\mu_2)\exp(\int_{n\tau}^{(n+1)\tau}(r - \frac{rm_1}{k} - \frac{r\theta}{k}(\widetilde{I}(t)+\varepsilon_3) - \beta(\widetilde{I}(t)+\varepsilon_3)^2)dt) > 1$, we will prove there exist a $t_1 \in (0,\infty)$, such that $S(t_1) \geq m_1$. Otherwise $S(t) < m_1$ for all $t > 0$. From system (2.1), we obtain that

$$\begin{cases} \dot{I}(t) \leq (\beta m_1 M - d)I(t), & t \neq n\tau, \\ \triangle I(t) \leq \mu_1 m_1 - \mu_3 I(t) + p, & t = n\tau. \end{cases} \tag{4.2}$$

According the comparison theorem, there exists a $T_3 > 0$ such that

$$I(t) \leq z(t) < \widetilde{z}(t) + \varepsilon_3, \tag{4.3}$$

for $t > T_3$, where $\tilde{z}(t) = \frac{(\mu_1 m_1 + p)\exp(-(d-\beta m_1 M)(t-n\tau))}{1-(1-\mu_3)\exp(-(d-\beta m_1 M)\tau)}, n\tau < t \le (n+1)\tau$. Thus

$$\begin{cases} \dot{S}(t) \ge S(t)(r - \frac{rm_1}{k} - \frac{r\theta}{k}(\tilde{I}(t)+\varepsilon_3) - \beta(\tilde{I}(t)+\varepsilon_3)^2), & t \neq n\tau, \\ \Delta S(t) = -(\mu_1+\mu_2)S(t), & t = n\tau, \end{cases} \tag{4.4}$$

for $t > T_3$, integrating (4.4) on $(n\tau, (n+1)\tau]$, $n \ge N_1$, here, N_1 is a nonnegative integer and $N_1\tau \ge T_3$, then we obtain

$$\begin{aligned} S((n+1)\tau) &\ge S(n\tau)(1-\mu_1-\mu_2)\exp(\textstyle\int_{n\tau}^{(n+1)\tau}(r - \frac{rm_1}{k} - \frac{r\theta}{k}(\tilde{I}(t)+\varepsilon_3) \\ &\quad -\beta(\tilde{I}(t)+\varepsilon_3)^2)dt) = S(n\tau)\delta_1. \end{aligned}$$

Then $S((N_1+k)\tau) \ge S(N_1\tau)\delta_1^k \to \infty, \quad k \to \infty$, which is a contradiction to $S(t) < m_1$ for all $t > 0$. Hence there exists a $t_1 > 0$ such that $S(t_1) \ge m_1$.

Step II :

If $S(t) \ge m_1$ for all $t \ge t_1$, then our aim is obtained. Otherwise $S(t) < m_1$ for some $t \ge t_1$, setting $t^* = \inf\limits_{t>t_1}\{S(t) < m_1\}$, in this case $S(t) \ge m_1$ for $t \in [t_1, t^*)$ and $(1-\mu_1-\mu_2)m_1 \le S(t^{*^+}) = (1-\mu_1-\mu_2)S(t^*) < m_1$. Let $T_4 = n_2\tau + n_3\tau$, where $n_2 = n_2' + n_2''$, n_2', n_2'' and n_3 satisfy the following inequalities:

$$\begin{aligned} &n_2'\tau > -\frac{1}{d-\beta M m_1}\ln\frac{\varepsilon_3}{(M+p+\mu_1 m_1)(1-\mu_3)}, \\ &(1-\mu_1-\mu_2)^{n_2}\exp(\eta n_2\tau)\delta_1^{n_3} > 1, \end{aligned}$$

where $\eta = (r - \frac{rm_1}{k} - \frac{r\theta}{k}M - \beta M^2) < 0$. We claim that there must be exist a time $t_1' \in (t^*, t^*+T_4)$ such that $S(t_1') \ge m_1$, if it is not true, i.e. $S(t) < m_1, t \in (t^*, t^*+T_4)$, similar to the analysis before, we consider system (4.3) with initial value $z(t^{*^+}) = I(t^{*^+}) \ge 0$, by lemma 2.2, we have

$z(t) = (1-\mu_3)(z(t^{*^+}) - \frac{p+\mu_1 m_1}{1-(1-\mu_3)\exp(-(d-\beta M m_1)\tau)})\exp(-(d-\beta M m_1)(t-t^*)) + \tilde{z}_1(t)$ for $t \in (n\tau, (n+1)\tau], n_1 \le n \le n_1+n_2+n_3$. Then

$|z(t) - \tilde{z}(t)| < (1-\mu_3)(M+p+\mu_1 m_1)\exp(-(d-\beta M m_1))(t-n_1\tau)) < \varepsilon_3$, and $I(t) \le z(t) < \tilde{z}(t)+\varepsilon_3$ for $t^*+n_2'\tau \le t \le t^*+T_4$. which implies that system (4.4) holds for $[t^*+n_2\tau, t^*+T_4]$, integrating system (4.4) on this interval, we have

$$S((n_1+n_2+n_3)\tau) \ge S((n_1+n_2)\tau)\delta_1^{n_3}. \tag{4.5}$$

In addition, we have

$$\begin{cases} \dot{S}(t) \ge S(t)(r - \frac{rm_1}{k} - \frac{r\theta}{k}M - \beta M^2) = \eta S(t), \\ \Delta x_1(t) = -E_1 x_1(t). \end{cases} \tag{4.6}$$

Integrating system (4.6) on this interval $[t^*, (n_1+n_2)\tau]$, which yields

$$x_1((n_1+n_2)\tau) \ge m_1(1-\mu_1-\mu_2)^{n_2}\exp(\eta n_2\tau), \tag{4.7}$$

combining (4.5) and (4.7), we have

$$S((n_1+n_2+n_3)\tau) \geq m_1(1-\mu_1-\mu_2)^{n_2}\exp(\eta n_2\tau)\delta_1^{n_3} > m_1,$$

which is a contradiction, so there exists a time $t_1' \in [t^*, t^*+T_4]$ such that $S(t_1') \geq m_1$, let $\hat{t} = \inf_{t\geq t^*}\{S(t) \geq m_1\}$, since $0 < \mu_1+\mu_2 < 1, S(n\tau^+) = (1-\mu_1-\mu_2)S(n\tau) < S(n\tau)$ and $S(t) < m_1, t \in (t^*, \hat{t})$. Thus, $S(\hat{t}) = m_1$, suppose $t \in (t^*+(l-1)\tau, t^*+l\tau] \subset (t^*, \hat{t}], l$ is a positive integer and $l \leq n_2+n_3$, from system (4.6), we have

$$S(t) \geq (1-\mu_1-\mu_2)^l m_1 \exp(l\eta\tau) \geq (1-\mu_1-\mu_2)^{n_2+n_3} m_1 \exp((n_2+n_3)\eta\tau) \triangleq \xi$$

for $t > \hat{t}$. The same arguments can be continued since $S(\hat{t}) \geq m_1$. Hence $S(t) \geq \xi$ for all $t > t_1$. The proof is complete.

5. Discussion

In this paper, we have investigated the dynamic behavior of a pest management model with impulsive spraying microbial pesticides and releasing infected pests tactics. The infected pests have function similar to the microbial pesticide and can infect the healthy pests. We have shown that there exists an asymptotically stable the susceptible pest-eradication periodic solution if impulsive period is less than some threshold. When the stability of pest-eradication periodic solution is lost, system (2.1) is permanent, which is in line with reality from a biological point of view. Now we can compare validity of our impulsive control strategy with the classical methods (only biological control or chemical control). If $\mu_1 = \mu_2 = \mu_3 = 0$, that is, we only choose the biological control, we can obtain that τ_0 is the threshold and $\tau_{max} > \tau_0$, which implies that we must release more infected pest to eradicate the pests. If $p = 0$, that is, there is no periodic releasing infective pests, so we can easily obtain that $\tau_1 = -\frac{1}{r}\ln(1-\mu_1-\mu_2)$ is the threshold and $\tau_{max} > \tau_1$, it is obviously, impulsive releasing pests may lengthen the period of spraying pesticides and therefore reduce the cost of pests control.

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