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Some Results of the NBUL Class of Life Distributions

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Abstract

In this paper, the new better than used in the Laplace transform order (NBUL) aging class is studied further. This class is defined based on comparing the residual life time to the whole life in the Laplace transform order. Some new results of this class are given including some closure properties and characterizations. Testing exponentiality against the NBUL class is addressed. The asymptotic normality of the proposed statistic is presented. Finally, the Pitman asymptotic efficacy, the power and the critical values of the proposed statistic are calculated.

Keywords: Life distributions, NBUL aging class, characterization of life distributions, life testing, efficiency

1 Introduction and motivation

Aging notions have been the subject of investigation for more than three decades. There are a number of classes that have been suggested in the literature to categorize distributions based on their aging properties or their dual. Such aging classes are derived via several notions of comparison between random variables. In this paper we study a new aging notion derived from the Laplace transform order. Before we go into the details, let us quickly review some common notions of stochastic orderings and aging notions considered in this paper (see, *Shaked* and *Shanthikumar* (1994) and *Barlow* and *Proschan* (1981)).

Formally, if X and Y are two random variables with distributions F and G (survivals $\overline{F}$ and $\overline{G}$), respectively, then we say that X is smaller than Y in the

(a) stochastic order sense (denoted by $X \leq_{st} Y$) if, and only, if

$$\overline{F}(x) \leq \overline{G}(x), \text{ for all } x;$$

(b) increasing concave order (denoted by $X \leq_{icv} Y$) if, and only, if

$$\int_0^x \overline{F}(u)du \leq \int_0^x \overline{G}(u)du, \text{ for all } x.$$

Another importing ordering that has come to use in reliability and life testing is the following: A random variable X is smaller than a random variable Y with respect to Laplace transform order (denoted by $X \leq_{Lt} Y$) if, and only, if

$$\int_0^\infty e^{-sx} dF(x) \geq \int_0^\infty e^{-sx} dG(x), \quad s \geq 0. \tag{1.1}$$

It is easy to check that (1.1) is equivalent to

$$\int_0^\infty e^{-sx}\overline{F}(x)dx \leq \int_0^\infty e^{-sx}\overline{G}(x)dx, \quad s \geq 0.$$

Applications, properties and interpretations of the Laplace transform order in the statistical theory of reliability, and in economics can be found in *Alzaid et al.* (1991), *Denuit* (2001), *Muller* and *Stoyan* (2002), *Klefsjo* (1983), *Shaked* and *Shanthikumar* (1994) and *Ahmed* and *Kayid* (2004).

On the other hand, in the context of lifetime distributions, some of the above orderings have been used to give characterizations and new definitions of aging classes. By aging, we mean the phenomenon whereby an older system has a shorter remaining lifetime, in some statistical sense, than a younger one (*Bryson* and *Siddiqui* (1969)). One of the most important approaches to the study of aging is based on the concept of the residual life. For any random variable X, let

$$X_t = [X - t \mid X > t], \quad t \in \{x : F(x) < 1\},$$

denote a random variable whose distribution is the same as the conditional distribution of $X - t$ given that $X > t$ and has survival function $\overline{F}_t(x) = \overline{F}(x+t)/\overline{F}(t)$. When X is the lifetime of a device, X_t can be regarded as the residual lifetime of the device at time t, given that the device has survived up to time t. We say that a non-negative random variable X , or its distribution F, is (*Barlow* and *Proschan* (1981) and *Deshpande et al.* (1986)).

(a) new better than used (denoted by $X \in NBU$) if

$$\overline{F}(x+t) \leq \overline{F}(t)\overline{F}(x), \quad \text{for all} \quad t \geq 0;$$

(b) new better than used in the increasing concave order (denoted by $X \in NBU(2)$)
if

$$\int_0^x \overline{F}(u+t)du \leq \overline{F}(t) \int_0^x \overline{F}(u)du, \quad \text{for all} \quad x,\, t \geq 0.$$

Recently, based on the Laplace transform order, *Wang* (1996) introduced a new aging class of life distributions. It's definition is also recalled here.

Definition 1.1.

A non-negative random variable X is said to be new better than used in the Laplace transform order (denoted by $X \in NBUL$) if, and only, if

$$\int_0^\infty e^{-sx}\overline{F}(x+t)dx \leq \overline{F}(t) \int_0^\infty e^{-sx}\overline{F}(x)dx, \quad s \geq 0. \tag{1.2}$$

It is obvious that (1.2) is equivalent to

$$X_t \leq_{Lt} X, \quad \text{for all } t \geq 0.$$

To introduce the definition of the discrete $NBUL$, let X be a discrete non-negative random variable such that $P(X=k) = p_k$, $k = 0, 1, 2, \ldots$. Let $\overline{P}_k = P(X > k)$, $k \geq 1$, $\overline{P}_0 = 1$ denote the corresponding survival function. The discrete non-negative random variable X is said to be *discrete* new better than used in Laplace transform order (discrete $NBUL$) if, and only, if

$$\sum_{k=0}^{\infty} \overline{P}_{k+i}\, z^k \leq \overline{P}_i \sum_{k=0}^{\infty} \overline{P}_k\, z^k,$$

for all $\quad 0 \leq z \leq 1$ and $i = 0, 1, \ldots$.

The relations between the above stochastic orderings and aging classes are as follows:

$$X \leq_{st} Y \;\Rightarrow X \leq_{icv} Y \;\Rightarrow\; X \leq_{Lt} Y, \tag{1.3}$$

and

$$NBU \;\subset NBU(2) \;\subset NBUL.$$

Some interpretations for the $NBUL$ class are as follows:

(a) One simple interpretation of $\int_0^\infty e^{-sx}\overline{F}(x)dx$ is that the mean life of a series system of two statistically independent components, one having exponential survival function and the other having survival function $\overline{F}$. Consider now two such series systems, say system A and system B. System A has a used component of age t with survival function $\overline{F}_t$ while system B has a new component with survival function $\overline{F}$. Thus, $F \in NBUL$ implies that the mean life of a system A is not larger than that of system B.

(b) A machine has survival function $\overline{F}$ and produces one unit of output per hour when functioning. The present value of one unit produced at time t is $1.e^{-st}$, where s is the discount rate. Then the expected present value of total output produced during the life of the machine is

$$\int_0^\infty e^{-st}\overline{F}(t)dt.$$

Thus, $F \in NBUL$ implies that a used machine of age t governed by survival function $\overline{F}_t$ produces a smaller expected total present value than does a new machine governed by survival function $\overline{F}$. See further interpretations for the $NBUL$ by *Yue* and *Cao* (2001).

Some properties of the $NBUL$ class including some characterizations and closure properties have been discussed by *Yue* and *Cao* (2001) and *Gao et al.* (2002), while *Belzunce et al.* (1999) showed that the $NBUL$ class is preserved under both the pure birth shock model and Poisson shock model.

In the current investigation, we further develop the $NBUL$ class. In *Section* 2, we provide some new characterizations and closure properties for the $NBUL$ aging notion. In *Section* 3, we present a procedure to test that X is exponential against that it is $NBUL$ and not exponential. The asymptotic normality of the proposed statistic is presented. Finally, in *Section* 4, the Pitman asymptotic efficacy, the power and the critical values of the proposed statistic are also calculated.

2 Characterizations results

In this section, we provide some new preservation and characterization results concerning the $NBUL$ class. First, we recall the definition of completely monotone functions. A function $h : \Re_+ \to \Re$ is said to be completely monotone $(c.m.)$ if all its derivatives $h^{(n)}$ exist and satisfy $(-1)^n\ h^{(n)}(t) \geq 0$, for $t > 0$ and $n = 1, 2, ...,$ where $h^{(0)} \equiv h$.

The next theorem states that the $NBUL$ aging notion is preserved under $c.m.$ transformations.

Theorem 2.1.

Let X be non-negative random variable. Then X is $NBUL$ if, and only, if $h(X_t) \leq_{Lt} h(X)$, for all non-negative function h with $c.m.$ derivative and $t \geq 0$.

Proof.

We give the proof of the necessity only. Let h be any non-negative function with a $c.m.$ derivative, and suppose that X is $NBUL$. We have to prove that

$$[h(X_t) \mid h(X_t) > s] \leq_{Lt} [h(X) \mid h(X) > s], \text{ for all } s < min(h(u_{X_t}), h(u_X)). \tag{2.1}$$

From the assumption, it follows that $(X_t)_{h^{-1}(s)} \leq_{Lt} (X)_{h^{-1}(s)}$ or, equivalently,

$$[X_t \mid X_t > h^{-1}(s)] \leq_{Lt} [X \mid X > h^{-1}(s)], \text{ for each } s. \tag{2.2}$$

Here the inverse h^{-1} of h is taken to be the right continuous version of it defined by $h^{-1}(u) = sup\{x : h(x) \leq u\}$ for $u \in \Re$. From the definition of h^{-1} and the continuity of h, it is easy to check that $x > h^{-1}(s)$ if, and only, if $h(x) > s$. Thus (2.2) can be rewritten as

$$[X_t \mid h(X_t) > s] \leq_{Lt} [X \mid h(X) > s], \text{ for each } s \text{ and } t \geq 0.$$

By *Corollary* 3.2 of *Alzaid et al.* (1991), we get that

$$h([X_t \mid h(X_t) > s]) \leq_{Lt} h([X \mid h(X) > s]),$$

implying (2.1). This completes the proof. ∎

Suppose now that $X_1, X_2, ...$ be a sequence of independent and identical distributed ($i.i.d$) random variables and N be a positive integer-valued random variable which is independent of the X_i. Put

$$X_{(1:N)} \equiv \min\{X_1, X_2, ..., X_N\}.$$

The random variables $X_{(1:N)}$ arise naturally in reliability theory as the lifetimes of a series systems, with the random number N of identical components with lifetimes $X_1, X_2, ..., X_N$. In life-testing, if a random censoring is adopted, then the completely observed data constitute a sample of random size, say $X_1, X_2, ..., X_N$, where $N > 0$ is a random variable of integer value. In survival analysis, $X_{(1:N)}$ arises naturally as the minimal survival time of a transplant operation, where N of them are defective and hence may cause death.

To state and prove the closure of $NBUL$ class under series systems which are composed of a random number of *i.i.d.* components, we need the following preliminary results, which is due to *Shaked* and *Shanthikuma*r (1994).

Lemma 2.1.

Let the independent non-negative random variables X_1, X_2,...,X_n, $Y_1, Y_2, ..., Y_n$ have the survival functions $\overline{F}_1, \overline{F}_2, ..., \overline{F}_n$, $\overline{G}_1, \overline{G}_2, ..., \overline{G}_n$, respectively, and $\overline{F}_i$ and $\overline{G}_i$ are all *c.m.* If $X_i \leq_{Lt} Y_i$, $i = 1, 2, ..., n$, then

$$\min\{X_1, ..., X_n\} \leq_{Lt} \min\{Y_1, ..., Y_n\}.$$

Lemma 2.2.

Let X, Y, and N be random variables such that $[X \mid N = n] \leq_{Lt} [Y \mid N = n]$ for all n in the support of N. Then $X \leq_{Lt} Y$. That is, the Laplace transform order is closed under mixtures.

Suppose now that X_1, X_2,...,X_N and $Y_1, Y_2, ..., Y_N$ have the survival functions $\overline{F}_1, \overline{F}_2, ..., \overline{F}_N$ and $\overline{G}_1, \overline{G}_2, ..., \overline{G}_N$, respectively and N is independent of X_i's and Y_i's. Let $\overline{F}_i$ and $\overline{G}_i$ are all *c.m.* Then by *Lemmas* 2.1 and 2.2, $X_i \leq_{Lt} Y_i$ for $i = 1, 2, ...$ implies

$$\min\{X_1, X_2, ..., X_N\} \leq_{Lt} \min\{Y_1, Y_2, ..., Y_N\}. \tag{2.3}$$

We also observe that, given a set X_1, X_2,...,X_N of independent components and letting $T_N = \tau(X_1, X_2, ..., X_N)$ be the random lifetime of coherent system with components X_1, X_2,...,X_N, we have

$$[T_N - t \mid T_N > t] \leq_{st} \tau\left\{[X_1 - t \mid X_1 > t], ..., [X_N - t \mid X_N > t]\right\}. \tag{2.4}$$

Theorem 2.2.

Let X_1, X_2,...,X_N, be a set of $NBUL$ independent components, with *c.m.* survival functions and N be a positive integer-valued random variable. Then $X_{(1:N)} \in NBUL$.

Proof.

By (2.4) and (1.3) and we have that

$$\left[X_{(1:N)} - t \mid X_{(1:N)} > t\right] \leq_{Lt} \min\left\{[X_1 - t \mid X_1 > t, ..., X_N - t \mid X_N > t]\right\},$$

Let $\overline{F}_{i|t}$ be the survival function of $[X_i - t \mid X_i > t]$. Then $\overline{F}_{i|t}$ is *c.m.* if $\overline{F}_i$ is *c.m.* (see, *Gao et al.* (2002)). Now by the assumption and (2.3) we get

$$\min\left\{[X_1 - t \mid X_1 > t], ..., [X_N - t \mid X_N > t]\right\} \leq_{Lt} \min\{X_1, X_2, ..., X_N\}.$$

Hence the result follows. ∎

3 Testing against NBUL alternatives

In the context of reliability and life testing, the hazard rate of a life distribution plays an important role for stochastic modeling and classification. Being a ratio of probability density function and the corresponding survival function, it uniquely determines the underlying distribution and exhibits different monotonic behaviors. The concept of the *ageless* notion is equivalent to the phenomenon that age has no effect on the hazard rate. Thus the *ageless* property is equal to constant hazard rate, that is, the distribution is exponential. Hence testing non-parametric classes is done by testing exponentially versus some kind of classes. This applies to many non-parametric classes such as NBU, $NBU(2)$, $NBUT$ and $IMIT$, among many others. For a recent literature on testing the above classes as well as others we refer the readers to *Ahmad* (2001), *Ahmad et al.* (2001), *Ahmad* and *Mugdadi* (2004), *Kayid* and *Ahmad* (2004), *Ahmad, Kayid* and *Pellerey* (2005) and *Ahmad*, *Kayid* and *Li* (2005). Much of the earlier literature is cited in those papers where definitions, inter-relations and discussion of above classes are presented.

This section is divided into two main subsections. The first one is concerned with the construction of the proposed statistic as a U- statistic, discussing its asymptotic normality and explains how one can use it as application of testing of hypotheses. In the second subsection, the simulated upper percentile values for 90, 95 and 99 of the proposed statistic are presented and some applications are provided.

3.1 The U-statistic test procedure

The test presented here depends on a sample $X_1, X_2, ..., X_n$ from a population with distribution F. We wish to test the null hypothesis $H_0 : F$ is exponential with mean μ against $H_1 : F$ is $NBUL$ and is not exponential. According to (1.2) we may use the following as a measure of departure from H_0 in favor of H_1 :

$$\delta(s) = \int_0^\infty \overline{F}(t)dt \int_0^\infty e^{-su}\overline{F}(u)du - \int_0^\infty \int_0^\infty e^{-su}\overline{F}(u+t)dudt.$$

Lemma 3.1.

If $\phi(s) = \int_0^\infty e^{-sx}dF(x)$ and $\mu = \int_0^\infty \overline{F}(x)dx$, then

$$\delta(s) = \frac{1}{s^2}\left[1 - (1+\mu s)\,\phi(s)\right],$$

Proof.

First, note that

$$\begin{aligned}\int_0^\infty \overline{F}(t)dt \int_0^\infty e^{-su}\overline{F}(u)du &= \mu\ E\int_0^X e^{-su}du \\ &= \frac{\mu}{s}\left[1-\phi(s)\right].\end{aligned}$$

Next,

$$\begin{aligned}\int_0^\infty \int_0^\infty e^{-su}\overline{F}(u+t)dudt &= \int_0^\infty \int_t^\infty e^{-s(w-t)}\overline{F}(w)dwdt \\ &= -\frac{1}{s^2}\left[1-\phi(s)\right]+\frac{\mu}{s}.\end{aligned}$$

Hence, the result follows. ∎

In the sequel, we will use $\delta_1(s)$ as a measure of departure from H_0, where

$$\delta_1(s) = s^2\ \delta(s).$$

To estimate $\delta_1(s)$, let $X_1, X_2, ..., X_n$ be a random sample from F. We estimate $\delta_1(s)$ by

$$\hat{\delta}_1(s) = \frac{1}{n(n-1)}\sum_i \sum_{\neq j}\left[1-(1+sX_i)\,e^{-sX_j}\right].$$

To find the limiting distribution of $\hat{\delta}_1(s)$ we resort to the U-statistics theory (cf. *Lee,* 1989). Set

$$\phi_s\left(X_1, X_2\right) = 1-(1-sX_1)\,e^{-sX_2}.$$

Thus

$$\begin{aligned}\phi_{1,s}\left(X_1\right) &= E\left[\phi_s\left(X_1, X_2\right)|X_1\right] \\ &= 1-(1+sX_1)\,\phi(s),\end{aligned}$$

and

$$\begin{aligned}\phi_{2,s}\left(X_1\right) &= E\left[\phi_s\left(X_1, X_2\right)|X_1\right] \\ &= 1-(1+s\mu)\,e^{-sX_1}.\end{aligned}$$

Thus, set

$$\begin{aligned}\psi_s(X_1) &= \phi_{1,s}\left(X_1\right)+\phi_{2,s}\left(X_1\right) \\ &= 2-(1+sX_1)\phi(s)-(1+s\mu)e^{-sX_1}.\end{aligned}$$

Thus, the variance of $\hat{\sigma}(s)$ is

$$\sigma_s^2 = Var\left(\psi_s\left(X_1\right)\right).$$

Under H_0, we get that

$$\sigma_{0,s}^2 = \frac{\mu^4 s^4}{\left(1+s\mu\right)^2\left(1+2s\mu\right)}.$$

According to *Lemma* 3.1, *Theorem* 3.1 below is immediate.

Theorem 3.1.

As $n \to \infty$, $\sqrt{n}\left(\hat{\delta}_1(s) - \delta_1(s)\right)$ is asymptotically normal with zero mean and variance σ_s^2. Under H_0, the variance is $\sigma_{0,s}^2$.

Note that $\sigma_{0,s}^2$ can be easily estimated by:

$$\sigma_{0,s}^2 = \frac{\overline{X}^4 s^4}{\left(1+s\overline{X}\right)^2\left(1+2s\overline{X}\right)},$$

where $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$.

Hence we reject H_0 if $\sqrt{n}\ \hat{\delta}_1(s)\ /\ \hat{\sigma}_{0,s} >> Z_\alpha$, the standard normal variate.

3.2 Monte Carlo null distribution critical values

In practice, simulated percentiles for small samples are commonly used by applied statisticians and reliability analyst. We have simulated the upper percentile values for 90, 95 and 99. Table (1) presents these percentile values of the statistic $\hat{\delta}_1(s)$ and the calculations are based on 5000 simulated samples of sizes $n = 5(5)100$.

Table (1)

The upper percentile of $\hat{\delta}_1(s)$ with 25000 replications

n	90%	95%	99%
5	0.74256	1.34667	4.61977
10	0.63819	1.02126	2.57858
15	0.56908	0.86261	2.06033
20	0.51537	0.74802	1.66147
25	0.47643	0.67448	1.35911
30	0.44703	0.62258	1.18827
35	0.42684	0.57545	1.03522
40	0.41402	0.55222	0.93588
45	0.40102	0.52450	0.89542
50	0.38110	0.49556	0.82259
55	0.37621	0.48435	0.79845
60	0.36382	0.46074	0.73920
65	0.35501	0.44621	0.69728
70	0.34742	0.43265	0.67009
75	0.33948	0.42240	0.63321
80	0.33656	0.41688	0.62341
85	0.32801	0.40318	0.59381
90	0.32045	0.39388	0.57691
95	0.32157	0.38850	0.56357
100	0.31349	0.38257	0.55237

4 Asymptotic efficiency

To asses the quality of this procedure, we evaluate its *asymptotic Pittman efficacy* for three alternatives in the class (since they are in the new better than used in expectation class). These are:

(i) Weibull Family:

$$\overline{F}_1(x) = e^{-X^\theta}, \quad x \geq 0, \quad \theta \geq 0;$$

(ii) Linear Failure Rate Family:

$$\overline{F}_2(x) = e^{-x-\frac{\theta}{2}x^2}, \quad x \geq 0, \quad \theta \geq 0;$$

(iii) Makeham Family:

$$\overline{F}_3(x) = e^{-x-\theta(x+e^{-x}-1)}, \quad x \geq 0, \quad \theta \geq 0.$$

Note that the Pitman asymptotic efficacy (PAE) is defined by:

$$PAE\left(\delta_1(s)\right) = \frac{d}{d\theta}\delta_1(s)|_{\theta\to\theta_0} \;/\; \sigma_{0,s}.$$

In the above three cases we get the following PAE values:

(i) Weibull Family:

$$s^{-1}(1+2s)^{\frac{1}{2}}\ln(1+s);$$

(ii) Linear Failure Rate:

$$\frac{(1+2s)^{\frac{1}{2}}}{(1+s)};$$

(iii) Makeham Family:

$$\frac{(1+2s)^{\frac{1}{2}}}{2(2+s)}.$$

Note also that all these alternatives have efficacies decreasing in $s > 0$. Hence we should choose s small enough to get good efficacy of the test.

4.1 The power of the proposed test

The power of the proposed test at a significance level α with respect to the alternatives F_1, F_2 and F_3 is calculated based on simulation data. In such simulation, 25000 samples were generated with sizes n =10, 20 and 30 from the alternatives. Table (2) shows the power of test at different values of θ and the significance level $\alpha = 0.05$.

Table (2)

Alternative Distributions: Makeham, LFR, Weibull

n	θ	**Makeham**	LFR	Weibull
	2	0.84916	0.91756	0.99872
10	3	0.92136	0.97696	1.00000
	4	0.94876	0.99188	1.00000
	2	0.88712	0.94196	1.00000
20	3	0.94884	0.98740	1.00000
	4	0.97440	0.99688	1.00000
	2	0.90788	0.95376	1.00000
30	3	0.96784	0.99172	1.00000
	4	0.98572	0.99860	1.00000

From the above table, it is noted that the power of the test increases by increasing the values of the parameter θ and the sample size n as it was expected.

4.2 Numerical example

Consider the following data of *Bryson* and *Siddiqui* (1969), which represent the survival times of 43 patients suffering from chronic granulocytic leukemia and the ordered life times (in days) are:
13, 14, 19, 19, 20, 21, 23, 23, 25, 26, 26, 27, 27, 31, 32, 34, 34, 37, 38, 40, 46, 50, 53, 54, 57, 58, 59, 60, 65, 65, 66, 70, 85, 90, 98, 102, 103, 110, 118, 124, 130, 136, 138.

Calculating $\sqrt{n}\ \hat{\delta}_1(s)\ /\ \hat{\sigma}_{0,s}$, we get 0.0239, which is smaller than Z_α for any α. This value leads to the acceptance of H_0 agreeing with conclusion of *Bryson* and *Siddiqui* (1969).

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