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A Method for Judging Decay or Growth of the Magnetic Field of Pulsar

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Abstract. This paper provides a method for judging growth or decay of the magnetic field of pulsar by using pulse period P, or frequency ν, and its first and second derivatives $\dot{P}$, $\ddot{P}$ or $\dot{\nu}$, $\ddot{\nu}$. The author uses this method to judge the growth or decay of the magnetic field of Crab pulsar. The judged result for Crab pulsar is that the magnetic field of Crab pulsar is growing now, but it is not decaying. The result corresponds with the actual case of Crab pulsar.

Key words. Pulsar—decay or growth of magnetic field—method of judgement.

1. Introduction

Usually the magnetic field of pulsars decay with time. Such is the case for a lot of pulsars, but the magnetic field for a distinct pulsar is growing. The young pulsar is possibly such a case when it is born. Afterward its magnetic field is decaying successively. When it arrive at old pulsar, its magnetic field is not decaying or nearly without variation or become a weak field. The magnetic field of Crab pulsar should be growing due to young pulsar. This point corresponds with the result for the research of some authors (Blandford & Romaru 1988; Lyne 2004). Lyne (2004) gave a formula for judging increase or decrease of the magnetic field of pulsar by using the braking index, and concluded that the magnetic field of Crab pulsar is increasing. The present paper provides a method for judging the decay or growth of the magnetic field of pulsar by using the observable data of the frequency or period and its first and second derivatives, and checked the result given by Lyne for the increase of the magnetic field of Crab pulsar.

2. The formulas for judging decay or growth of magnetic field of pulsar

We adopt the magnetic dipole model of pulsar to research this problem. We assume that the magnetic field can be decaying or growing as in the following exponential form:

$$B_p^2 = B_i^2 \exp(\pm\xi t). \qquad (1)$$

where B_p is the magnetic field at magnetic pole of pulsar and B_i is the field strength at $t = 0$. ξ is the coefficient of the magnetic decay or growth. If ξ is positive, the magnetic field of pulsar is growing; if it is negative, the magnetic field is decaying.

The pulsar radiating energy at a rate can be written from the magnetic dipole model (Shapiro & Teukolsky 1983):

$$\dot{E} = -\frac{2|\ddot{m}|^2}{3c^3} = -\frac{B_p^2 R^6 \Omega^4 \sin^2\alpha}{6c^3}. \tag{2}$$

The energy carried away by the radiation from the rotational energy of pulsar is

$$E = \frac{1}{2} I\Omega^2, \qquad \dot{E} = I\Omega\dot{\Omega}. \tag{3}$$

2.1 *The situation for the inclination without variation*

The equation of the magnetic dipole radiation of pulsar can be derived from the formulas (2)–(3) assuming the magnetic inclination $\alpha = \text{const}$.

$$\dot{\Omega} = -\frac{B_p^2 R^6 \Omega^3 \sin^2\alpha}{6c^3 I}, \tag{4}$$

where R, I and Ω denote radius, moment inertia and angular velocity of pulsar respectively.

Let $\Omega = 2\pi/P$ (P: pulse period). Inserting it into the equation (4), we get

$$B_p^2 = \frac{3c^3 I}{2\pi^2 R^6 \sin^2\alpha} P\dot{P}. \tag{5}$$

Differentiating equation (5) with respect to time, we get

$$\frac{dB_p^2}{dt} = \frac{3c^3 I}{2\pi^2 R^6 \sin^2\alpha}(\dot{P}^2 + P\ddot{P}). \tag{6}$$

where $\dot{P} = (dP/dt)$, $\ddot{P} = (d^2P/dt^2)$.

Combining equation (6) with equation (5) or the two-hand sides of the equation (6) is divided by the two-hand sides of the equation (5), we get

$$\frac{1}{B_p^2}\frac{dB_p^2}{dt} = \frac{\dot{P}}{P} + \frac{\ddot{P}}{\dot{P}}. \tag{7}$$

Let ν be the pulsar frequency,

$$\begin{aligned} &\dot{\nu} = \frac{d\nu}{dt}, \quad \ddot{\nu} = \frac{d^2\nu}{dt^2}, \\ &P = \frac{1}{\nu}, \quad \dot{P} = \frac{d}{dt}\left(\frac{1}{\nu}\right) = \frac{-\dot{\nu}}{\nu^2}, \quad \ddot{P} = \frac{d}{dt}\left(\frac{-\dot{\nu}}{\nu^2}\right) = \frac{2\dot{\nu}^2}{\nu^3} - \frac{\ddot{\nu}}{\nu^2}. \end{aligned} \tag{8}$$

Substituting the expressions (8) into the equation (7), we obtain the formula (9)

$$\frac{1}{B_p^2}\frac{dB_p^2}{dt} = \frac{\ddot{\nu}}{\dot{\nu}} - 3\frac{\dot{\nu}}{\nu}. \tag{9}$$

We substitute the formula (1) into the formulas (7) and (9), we obtain

$$\pm\xi = \frac{\dot{P}}{P} + \frac{\ddot{P}}{\dot{P}} = \frac{\ddot{\nu}}{\dot{\nu}} - \frac{3\dot{\nu}}{\nu}. \tag{10}$$

The formula (10) is a formula for judging growth or decay of magnetic field of pulsar. If ξ is positive, the magnetic field of pulsar is growing; if it is negative, the magnetic field is decaying.

2.2 *The situation for the inclination with variation*

We research that the pulsar inclination α varies with time for the situation of the formula (10).

Differentiating the formula (5) with respect to time, we get

$$\frac{dB_P^2}{dt} = \frac{3c^3 I}{2\pi^2 R^6 \sin^2\alpha}(\dot{P}^2 + P\ddot{P}) - \frac{3c^3}{\pi^2 R^6}(IP\dot{P})\cot\alpha\csc^2\alpha\frac{d\alpha}{dt}. \tag{11}$$

For giving the expression for $d\alpha/dt$, we use the formula (Davis & Goldstein 1970)

$$I\frac{d(\Omega\cos\alpha)}{dt} = -\vec{N}\cdot\frac{\vec{M}}{M} = 0,$$

where M is the magnetic moment, N is the magnetic torque.

Let $\Omega = 2\pi/P$, and inserting it into the above formula, and then differentiating it, we get

$$\frac{d\alpha}{dt} = -\frac{\dot{P}}{P}\cot\alpha. \tag{12}$$

Substituting the expression (12) into the expression (11), then

$$\frac{dB_p^2}{dt} = \frac{3c^3 I}{2\pi^2 R^6 \sin^2\alpha}(\dot{P}^2 + P\ddot{P}) + \frac{3c^3 I}{\pi^2 R^6}\dot{P}^2\cot^2\alpha\csc^2\alpha. \tag{13}$$

when the two-hand sides of the equation (13) is divided by the two-hand sides of equation (5), we get

$$\frac{1}{B_p^2}\frac{dB_p^2}{dt} = \frac{\dot{P}}{P} + \frac{\ddot{P}}{\dot{P}} + 2\cot^2\alpha\left(\frac{\dot{P}}{P}\right) = (1 + 2\cot^2\alpha)\frac{\dot{P}}{P} + \frac{\ddot{P}}{\dot{P}}. \tag{14}$$

Substituting the expressions (8) into the above formula, we get

$$\frac{1}{B_p^2}\frac{dB_p^2}{dt} = \frac{\ddot{\nu}}{\dot{\nu}} - (3 + 2\cot^2\alpha)\frac{\dot{\nu}}{\nu}. \tag{15}$$

Substituting the formula (1) into the left side for the formulas (14) and (15), we get

$$\pm\xi = (1 + 2\cot^2\alpha)\frac{\dot{P}}{P} + \frac{\ddot{P}}{\dot{P}} = \frac{\ddot{\nu}}{\dot{\nu}} - (3 + 2\cot^2\alpha)\frac{\dot{\nu}}{\nu}. \tag{16}$$

The formulas (10) and (16) are the formulas for judging the growth or decay of the magnetic field of pulsar.

3. Application to crab pulsar

We use the formulas (10) and (16) to judge the growth or decay of the magnetic field of Crab pulsar. We use the following data to judge it.

The data for the first set (Bonazzola & Schneider 1974) are:

$$\left.\begin{aligned} \nu &= 30.2137051\,\mathrm{HZ} \\ \dot{\nu} &= -0.38594 \times 10^{-9}\,\mathrm{HZs}^{-1} \\ \ddot{\nu} &= 1.1 \times 10^{-20}\,\mathrm{HZs}^{-2} \end{aligned}\right\}. \tag{17}$$

The data for the second set (Wang *et al.* 2001) are:

$$\left.\begin{aligned} \nu &= 29.836059670\,\mathrm{HZ} \\ \dot{\nu} &= -3.743460(3) \times 10^{-10}\,\mathrm{HZs}^{-1} \\ \ddot{\nu} &= 1.17(2) \times 10^{-20}\,\mathrm{HZs}^{-2} \end{aligned}\right\}. \tag{18}$$

The data for the third set (ATNF pulsar catalogue) are:

$$\left.\begin{aligned} \nu &= 30.225\,\mathrm{HZ} \\ \dot{\nu} &= -3.86 \times 10^{-10}\,\mathrm{HZs}^{-1} \\ \ddot{\nu} &= 1.240 \times 10^{-20}\,\mathrm{HZs}^{-2} \end{aligned}\right\}. \tag{19}$$

Substituting the data for the first set into the formula (10), we get

$$\pm\xi = \frac{\ddot{\nu}}{\dot{\nu}} - \frac{3\dot{\nu}}{\nu} = +0.098191804 \times 10^{-10} > 0,$$

so ξ should be taken as positive.

Substituting the data for the second set into the formula (10), we get

$$\pm\xi = \frac{\ddot{\nu}}{\dot{\nu}} - \frac{3\dot{\nu}}{\nu} = +0.063323602 \times 10^{-10} > 0,$$

so ξ should be taken as positive.

Substituting the data for the third set into the formula (10), we get

$$\pm\xi = \frac{\ddot{\nu}}{\dot{\nu}} - \frac{3\dot{\nu}}{\nu} = +0.061883027 \times 10^{-10} > 0,$$

so ξ should be taken as positive.

If we consider the magnetic inclination, α, we need to use formula (16) for the calculation.

We consider: $0^\circ \leq \alpha \leq 90^\circ$, then $+\infty \leq \cot\alpha \leq 0$. So when $0^\circ \leq \alpha \leq 90^\circ$, and $\dot{\nu} < 0$ (negative), substituting the data of three sets into the formula (16), we still obtain $\xi > 0$ (positive). Hence the magnetic field of Crab pulsar is still growing.

If we consider: $90^\circ \leq \alpha \leq 180^\circ$, and then $0 \leq \cot\alpha \leq -\infty$, we get

$$(3 + 2\cot^2\alpha)\frac{\dot{\nu}}{\nu} \leq 3\frac{\dot{\nu}}{\nu}.$$

Substituting the data of three sets into the formula (16), because $\dot{\nu} < 0$, we get $\xi < 0$ (negative).

So the magnetic field of Crab pulsar is decaying, only when its inclination is that of $90^\circ \leq \alpha \leq 180^\circ$. But for Crab pulsar its magnetic inclination α is (Davis & Goldstein 1970)

$$\alpha = 59^\circ.2.$$

So $0^\circ \leq \alpha \leq 90^\circ$, and $\xi > 0$ (positive). It is not negative. Hence when we consider magnetic inclination, α, the magnetic field of Crab pulsar is still growing with time.

Next we calculate the increase of magnetic field with time for the Crab pulsar. We take the average value of three sets for ξ in the expressions (17)–(19), we get

$$\xi = 0.0744682 \times 10^{-10}(c,\ g,\ s). \tag{20}$$

We use the present strength of the magnetic field of Crab pulsar $B_0 = 5.2 \times 10^{12} G$ ($\sin\alpha = 1$) (Shapiro & Teukolsky 1983). Substituting the values for ξ and B_0 into the formula (1) and taking ξ as the positive symbol, we get

$$B_p(t) = 5.2006 \times 10^{12} G,$$

$$\therefore\ \Delta B_p(t) = B_p(t) - B_p(0) = (5.2006 - 5.2000) \times 10^{12}$$

$$= +0.0006 \times 10^{12}\ G/\text{yr}. \tag{21}$$

i.e., the magnetic filed is increasing (growing). The increment is 6×10^8 G per year for Crab pulsar.

4. Discussion

(1) The formula given by Lyne and the correction for the formula,
Lyne (2004) derived a formula for judging the increase or decrease of the magnetic field of pulsar. He uses the magnetic dipole model

$$B = -\sqrt{\frac{3c^3 I}{8\pi^2 R^6 \sin^2 a} P\dot{P}} = 3.2 \times 10^{19}\sqrt{P\dot{P}}\ \text{Gauss}, \tag{22}$$

$$P = \frac{1}{\nu}, \quad n = \nu\bar{\nu}/\dot{\nu}^2, \quad \tau = \frac{P}{2\dot{P}}. \tag{23}$$

And then, he obtained from the expressions (22) and (23)

$$\frac{dB}{dt} = \frac{B}{\tau}\{3 - n\}. \tag{24}$$

He infers from the above formula that the magnetic field is increasing, if $n < 3$ and the magnetic field is decreasing, if $n > 3$.

The formula (24) needs to be corrected.

Differentiating the equation (22), we obtain

$$\frac{dB}{dt} = \frac{B}{2}\left\{\frac{\dot{P}}{P} + \frac{\ddot{P}}{\dot{P}}\right\} \quad \text{or} \quad \frac{dB}{dt} = \left\{\frac{\ddot{\nu}}{\dot{\nu}} - 3\frac{\dot{\nu}}{\nu}\right\}. \tag{25}$$

We use

$$n = \frac{\nu\ddot{\nu}}{\dot{\nu}^2} = 2 - \frac{P\ddot{P}}{\dot{P}^2}. \tag{26}$$

In the above two equations, the equation (25) can be written as

$$\frac{dB}{dt} = \frac{B}{2}\frac{\dot{P}}{P}\{3 - n\} = \frac{B}{4\tau}\{3 - n\}. \tag{27}$$

This is the corrective formula for the formula (24) given by Lyne.

(2) This paper supports and has checked the result given by Lyne for the increase of the magnetic field of Crab pulsar.

It can be checked that the growth of the magnetic field of Crab pulsar can be used for the observable data of three sets $(\nu, \dot{\nu}, \ddot{\nu}\}$ provided by this paper.

Substitution of data of three sets (17)–(19) into the formula of the braking index (23) or (26), we get

$$n = 2.2313, \quad n = 2.4948, \quad n = 2.5154. \tag{28}$$

So, $n \prec 3$. Based on the formula (24) given by Lyne, the magnetic field of Crab pulsar is increasing by using the above three data of braking index. Hence this paper supports and has checked the result given by Lyne provided in this paper.

(3) The comparison of methods of this paper with Lyne's paper is given below:

Both papers start from and are based on the magnetic dipole model of pulsar, but the methods of derivation are different. The author uses the observational data for periods P, $\dot{P}$, $\ddot{P}$ or frequencies ν, $\dot{\nu}$, $\ddot{\nu}$ to judge the decay or growth of the magnetic field of pulsar. Lyne uses the braking index, n, to judge the increase or decrease of the magnetic field of pulsar. In addition, the results given by this paper consider the variation of the magnetic inclination with time. The results given by Lyne only consider the magnetic inclination as a constant.

5. Conclusion

(1) The magnetic field of a pulsar is denoted by

$$B^2 = B_i^2 \exp(\pm\xi t).$$

It is a decaying or a growing field, which is judged by the following formulas

$$\pm\xi = \frac{\dot{P}}{P} + \frac{\ddot{P}}{\dot{P}} = \frac{\ddot{\nu}}{\dot{\nu}} - \frac{3\dot{\nu}}{\nu}$$

or

$$\pm\xi = (1 + 2\cot^2\alpha)\frac{\dot{P}}{P} + \frac{\ddot{P}}{\dot{P}} = \frac{\ddot{\nu}}{\dot{\nu}} - (3 + 2\cot^2\alpha)\frac{\dot{\nu}}{\nu}.$$

If the value for $\xi > 0$, i.e., ξ is positive, then

$$B^2 = B_i^2 \exp(+\xi t).$$

The magnetic field is a growing field.

If the value for $\xi < 0$, i.e., ξ is negative, then

$$B^2 = B_i^2 \exp(-\xi t).$$

The magnetic field is a decaying field.

If the value $\xi = 0$ or $\xi \to 0$, the magnetic field

$$B^2 = B_i^2 \quad \text{or} \quad B^2 \to B_i^2,$$

i.e., the magnetic field is an invariable field or the field is a very weak field.

(2) The value of ξ suggests the evolutionary significance. When $\xi > 0$, the magnetic field is a growing field. This belongs to the situation for young pulsar. When $\xi < 0$, the magnetic field is a decaying field. This belongs to a pulsar between the young and old age. When $\xi = 0$ or $\xi \to 0$, the magnetic field is an invariable field or a weak field. This belongs to the situation for an old pulsar.

(3) We infer that the magnetic field of Crab pulsar is growing with time because the value of ξ is positive. The conclusion corresponds with the results of the research of the authors Blandford & Romaru (1988) and Lyne (2004).

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