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Numerical Method for Fuzzy Partial Differential Equations

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Abstract

In this paper a numerical method for solving "fuzzy partial differential equation" (FPDE) is considered. We present difference method to solve the FPDEs such as fuzzy hyperbolic equation and fuzzy parabolic equation , then see if stability of this method exist, and conditions for stability are given. Examples are presented showing the Hausdorff distance between exact solution and approximate solution is too small.

Keywords: fuzzy partial differential equation, difference method

1 Introduction

Knowledge about dynamical systems modeled by differential equations is often incomplete or vague. For example, for parametric quantities, functional relationships, or initial conditions, the well-known methods of solving Fuzzy Partial Differential Equations (FPDE) analytically or numerically can only be used for finding the selected system behavior, e.g., by fixing unknown parameters to some plausible values. Here, we are going to "operationalize" our approach, i.e., we are going to propose a method for computing approximate of the solution for a fuzzy partial differential equation using numerical methods. Since finding this set of solutions analytically does only work with trivial examples, a numerical approach seems to be the only way of "solving" such problems. In [4], J. Buckley and T. Feuring proposed a method to solutions of elementary fuzzy partial differential equations. In [5] T. Allahviranloo used a numerical method to solve FPDE, that was based on the Seikala derivative.
The method proposed here is to use a difference method for solving the fuzzy parabolic equations. The paper is organized as follows:

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In section 2 we bring some basic definitions of fuzzy numbers and fuzzy derivative which have been discussed by S. Seikkala and we will use in the paper. In section 3 we define a FPDE, in particular cases, the fuzzy heat equation and also use difference methods for it. The necessary conditions for stability of proposed method will discuss in section 4. The difference methods are illustrated by solving one example in section 5 and conclusions are drown in section 6.

2 Preliminaries

We begin this section with defining the notation we will use in the paper. We place a ~ sign over a letter to denote a fuzzy subset of the real numbers. We write $\widetilde{A}(x)$, a number in $[0,1]$, for the membership function of $\widetilde{A}$ evaluated at x. An $\alpha-$cut of $\widetilde{A}$, written $\widetilde{A}[\alpha]$, is defined as $\{x|\widetilde{A}(x) \geq \alpha\}$, for $0 < \alpha \leq 1$.

We represent an arbitrary fuzzy number by an ordered pair of functions $(\underline{u}(r), \overline{u}(r)),\ \ 0 \leq r \leq 1$, which satisfies the following requirements:

1. $\underline{u}(r)$ is a bounded left continuous non decreasing function over $[0,1]$.
2. $\overline{u}(r)$ is a bounded left continuous non increasing function over $[0,1]$.
3. $\underline{u}(r) \leq \overline{u}(r),\ 0 \leq r \leq 1$.

A crisp number α is simply represented by $\underline{u}(r) = \overline{u}(r) = \alpha,\ \ 0 \leq r \leq 1$. For arbitrary fuzzy numbers $x = (\underline{x}, \overline{x})$, $y = (\underline{y}, \overline{y})$ and real number k,

1. $x = y$ if and only if $\underline{x}(r) = \underline{y}(r)$ and $\overline{x}(r) = \overline{y}(r)$.
2. $x + y = (\underline{x}(r) + \underline{y}(r), \overline{x}(r) + \overline{y}(r))$
3.

$$kx = \begin{cases} (k\underline{x}, k\overline{x}) & k \geq 0, \\ \\ (k\overline{x}, k\underline{x}) & k < 0. \end{cases} \tag{2.1}$$

Let E be the set of all upper semicontinuous normal convex fuzzy numbers with bounded $\alpha-$level sets. Since the $\alpha-$cuts of fuzzy numbers are always closed and bounded, the intervals we write $\widetilde{N}[\alpha] = [\underline{N}(\alpha), \overline{N}(\alpha)]$, for all α. Consider the FPDE

$$\varphi(D_x, D_y)\widetilde{U}(x, y) = \widetilde{F}(x, y, \widetilde{K}), \tag{2.2}$$

subject to certain boundary conditions where the operator $\varphi(D_x, D_y)$ will be a polynomial, with a constant coefficient, in D_x and D_y, where $D_x(D_y)$ stands for the partial differential with respect to $x(y)$. The boundary conditions can be of the form $\widetilde{U}(0,y)=\widetilde{C}_1$, $\widetilde{U}(x,0)=\widetilde{C}_2$, $\widetilde{U}(M_1,y)=\widetilde{C}_3$, …, $\widetilde{U}(0,y)=\widetilde{C}_1$, $\widetilde{U}(0,y)=\widetilde{g}_1(y;\widetilde{C}_4)$, $\widetilde{U}(x,0)=\widetilde{f}_1(x;\widetilde{C}_5)$, …. $\widetilde{F}(x,y,\widetilde{K})$ is the fuzzy function which has $\widetilde{K}=(\widetilde{k}_1,\ldots,\widetilde{k}_n)$ for $\widetilde{k}_i$ a triangular fuzzy number in $J_i,\ 1\le i\le n$. Let $I_1=[0,M_1]$, $I_2=[0,M_2]$. The fuzzy function $\widetilde{U}$ maps $I_1\times I_2$ into fuzzy numbers. Also let $\widetilde{C}=(\widetilde{c}_1,\ldots,\widetilde{c}_m)$ with $\widetilde{c}_i$ being triangular fuzzy numbers in the intervals $L_i,\ 1\le i\le m$. Let

$$\widetilde{K}[\alpha]=\prod_{i=1}^{n}\widetilde{k}_i[\alpha],\quad \widetilde{C}[\alpha]=\prod_{i=1}^{m}\widetilde{c}_i[\alpha].$$

Let $\widetilde{U}(x,y)[\alpha]=[\underline{U}(x,y;\alpha),\overline{U}(x,y;\alpha)]$. We assume that the $\underline{U}(x,y;\alpha)$ and $\overline{U}(x,y;\alpha)$ have continuous partial so that $\varphi(D_x,D_y)\underline{U}(x,y;\alpha)$ and $\varphi(D_x,D_y)\overline{U}(x,y;\alpha)$ are continuous for all $(x,y)\in I_1\times I_2$, all α. Define

$$\Gamma(x,y;\alpha)=\varphi(D_x,D_y)\widetilde{U}(x,y)[\alpha]=[\varphi(D_x,D_y)\underline{U}(x,y;\alpha),\varphi(D_x,D_y)\overline{U}(x,y;\alpha)], \tag{2.3}$$

for all $(x,y)\in I_1\times I_2$, and all α.

Definition 2.1 *If for each fixed $(x,y)\in I_1\times I_2$, $\Gamma(x,y;\alpha)$ defines the $\alpha-$cut of a fuzzy number, then we will say that $\widetilde{U}(x,y)$ is differentiable.*

Sufficient conditions for $\Gamma(x,y;\alpha)$ to define $\alpha-$cuts of a fuzzy number are:

1. $\varphi(D_x,D_y)\underline{U}(x,y;\alpha)$ is an increasing function of α for each $(x,y)\in I_1\times I_2$;
2. $\varphi(D_x,D_y)\overline{U}(x,y;\alpha)$ is an decreasing function of α for each $(x,y)\in I_1\times I_2$; and
3. $\varphi(D_x,D_y)\underline{U}(x,y;1)\le\varphi(D_x,D_y)\overline{U}(x,y;1)$ for all $(x,y)\in I_1\times I_2$.

Consider the system of partial differential equations

$$\varphi(D_x,D_y)\underline{U}(x,y;\alpha)=\underline{F}(x,y;\alpha), \tag{2.4}$$

$$\varphi(D_x,D_y)\overline{U}(x,y;\alpha)=\overline{F}(x,y;\alpha), \tag{2.5}$$

for all $(x,y)\in I_1\times I_2$ and all $\alpha\in[0,1]$, where

$$\underline{F}(x,y;\alpha)=min\{F(x,y,k)|k\in\widetilde{K}[\alpha]\}, \tag{2.6}$$

$$\overline{F}(x,y;\alpha)=max\{F(x,y,k)|k\in\widetilde{K}[\alpha]\}. \tag{2.7}$$

We append to equations (2.4) and (2.5) any boundary conditions, for example, if they were $\widetilde{U}(0,y)=\widetilde{C}_1$ and $\widetilde{U}(M_1,y)=\widetilde{C}_2$, then we add

$$\underline{U}(0,y;\alpha)=\underline{C}_1(\alpha),\quad \underline{U}(M_1,y;\alpha)=\underline{C}_2(\alpha) \tag{2.8}$$

to equation (2.4) and

$$\overline{U}(0,y;\alpha)=\overline{C}_1(\alpha), \quad \overline{U}(M_1,y;\alpha)=\overline{C}_2(\alpha) \tag{2.9}$$

to equation (2.5) where $\widetilde{C}_i[\alpha]=[\underline{C}_i(\alpha),\overline{C}_i(\alpha)]$, $i=1,2$. Let $\underline{U}(x,y;\alpha)$ and $\overline{U}(x,y;\alpha)$ solves equations (2.4) and (2.5), plus the boundary equations, respectively.

Definition 2.2 *If $\widetilde{U}(x,y)[\alpha]=[\underline{U}(x,y;\alpha),\overline{U}(x,y;\alpha)]$, defines the $\alpha-$cut of a fuzzy number, for all $(x,y)\in I_1\times I_2$, then $\widetilde{U}(x,y)$ is the solution for (2.2), See [4].*

3 A fuzzy partial differential equation

In this section we solve the two types of FPDE as numerically. **Fuzzy parabolic equation**
Consider the fuzzy heat equation which is illustrated below with the parabolic equation:

$$(D_t-\beta^2 D_x D_x)\widetilde{U}(x,t)=0, \quad 0<x<l, \quad t>0, \tag{3.10}$$

where

$$\widetilde{U}(0,t)=\widetilde{K_1}, \quad \widetilde{U}(l,t)=\widetilde{K_2}, \qquad t>0,$$

$$\widetilde{U}(x,0)=\widetilde{f}(x), \qquad 0<x<l.$$

If $(\frac{\partial^2\widetilde{U}}{\partial x^2})\in E$ and $(\frac{\partial\widetilde{U}}{\partial t})\in E$ then by (2.4) and (2.5) we have

$$\begin{aligned}&(D_t-\beta^2 D_x D_x)\underline{U}(x,t;\alpha)=0,\\&(D_t-\beta^2 D_x D_x)\overline{U}(x,t;\alpha)=0, \quad 0<x<l, \quad t>0, \quad \alpha\in[0.1]\end{aligned} \tag{3.11}$$

where

$$\underline{U}(0,t;\alpha)=\underline{K_1}(\alpha), \quad \underline{U}(l,t;\alpha)=\underline{K_2}(\alpha), \qquad t>0, \quad \alpha\in[0.1],$$

$$\overline{U}(0,t;\alpha)=\overline{K_1}(\alpha), \quad \overline{U}(l,t;\alpha)=\overline{K_2}(\alpha), \qquad t>0, \quad \alpha\in[0.1],$$

$$\underline{U}(x,0;\alpha)=\underline{f}(x;\alpha),\overline{U}(x,0;\alpha)=\overline{f}(x;\alpha) \quad 0<x<l, \quad \alpha\in[0.1].$$

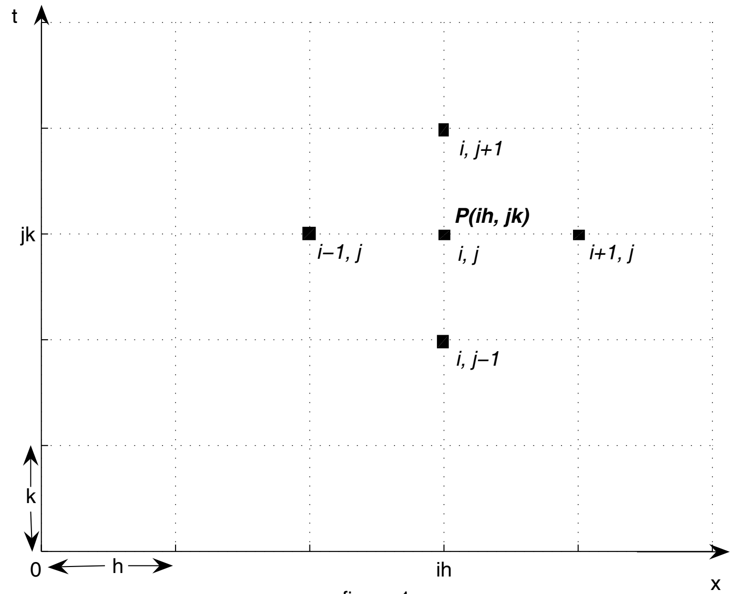

figure 1

Assume $\widetilde{U}$ is a fuzzy function of the independent crisp variables x and t. Subdivide the x-t plane into sets of equal rectangles of sides $\delta x = h$, $\delta t = k$, by equally spaced grid lines parallel to O_y, defined by $x_i = ih$, $i = 0, 1, 2, \ldots$ and equally spaced grid lines parallel to O_x, defined by $y_j = jk$, $j = 0, 1, 2, \ldots$ as shown in (fig 1).

Denote the value of $\widetilde{U}$ at the representative mesh point $p(ih, jk)$ by

$$\widetilde{U}_p = \widetilde{U}(ih, jk) = \widetilde{U}_{i,j} \tag{3.12}$$

and also denote the parametric form of fuzzy number, $\widetilde{U}_{i,j}$ as follow

$$\widetilde{U}_{i,j} = (\underline{u}_{i,j}, \overline{u}_{i,j}). \tag{3.13}$$

Then by Taylor's theorem and definition of standard difference

$$(D_xD_x)\widetilde{U}_{i,j} = (\underline{(D_xD_x)\widetilde{U}_{i,j}}, \overline{(D_xD_x)\widetilde{U}_{i,j}}),$$

where

$$\underline{(D_xD_x)\widetilde{U}_{i,j}} \simeq \frac{\underline{u}\{(i+1)h, jk\} - 2\overline{u}\{ih, jk\} + \underline{u}\{(i-1)h, jk\}}{h^2} ,$$

$$\overline{(D_xD_x)\widetilde{U}_{i,j}} \simeq \frac{\overline{u}\{(i+1)h, jk\} - 2\underline{u}\{ih, jk\} + \overline{u}\{(i-1)h, jk\}}{h^2} .$$

By (3.12) and (3.13) we have

$$\begin{aligned} \underline{(D_xD_x)\widetilde{U}_{i,j}} &\simeq \tfrac{\underline{u}_{i+1,j} - 2\overline{u}_{i,j} + \underline{u}_{i-1,j}}{h^2} , \\ \overline{(D_xD_x)\widetilde{U}_{i,j}} &\simeq \tfrac{\overline{u}_{i+1,j} - 2\underline{u}_{i,j} + \overline{u}_{i-1,j}}{h^2} , \end{aligned} \tag{3.14}$$

with a leading error of order h^2. With this notation the forward - difference approximation for $(D_t)\widetilde{U}$ at P is

$$\underline{(D_t)\widetilde{U}_{i,j}} \simeq \frac{\underline{u}_{i,j+1}-\overline{u}_{i,j}}{k} \ , \qquad \overline{(D_t)\widetilde{U}_{i,j}} \simeq \frac{\overline{u}_{i,j+1}-\underline{u}_{i,j}}{k} \ , \tag{3.15}$$

with a leading error of $O(k)$. By (3.14) and (3.15) and definition of standard difference one finite-difference approximation to

$$(D_t)\widetilde{U} - \beta^2(D_xD_x)\widetilde{U} = \widetilde{0} \tag{3.16}$$

is

$$\underline{(D_t)\widetilde{U}} - \beta^2\overline{(D_xD_x)\widetilde{U}} = \varepsilon(r-1) \ ,$$

$$\overline{(D_t)\widetilde{U}} - \beta^2\underline{(D_xD_x)\widetilde{U}} = \varepsilon(1-r) \ ,$$

or the following equations must be hold:

$$\frac{\underline{u}_{i,j+1} - \overline{u}_{i,j}}{k} = \frac{\beta^2(\overline{u}_{i+1,j} - 2\underline{u}_{i,j} + \overline{u}_{i-1,j})}{h^2} \tag{3.17}$$

$$\frac{\overline{u}_{i,j+1} - \underline{u}_{i,j}}{k} = \frac{\beta^2(\underline{u}_{i+1,j} - 2\overline{u}_{i,j} + \underline{u}_{i-1,j})}{h^2} \tag{3.18}$$

where $\widetilde{U} = (\underline{u}, \overline{u})$ is the exact solution of the approximating difference equations, $x_i = ih, \quad (i = 0, 1, 2, \ldots)$ and $t_j = jk, \quad (j = 0, 1, 2, \ldots)$. This can be written as

$$\underline{u}_{i,j+1} = r\overline{u}_{i-1,j} + (-2r)\underline{u}_{i,j} + r\overline{u}_{i+1,j} + \overline{u}_{i,j} \tag{3.19}$$

$$\overline{u}_{i,j+1} = r\underline{u}_{i-1,j} + (-2r)\overline{u}_{i,j} + r\underline{u}_{i+1,j} + \underline{u}_{i,j} \tag{3.20}$$

where $r = \frac{\beta^2 k}{h^2}$. Hence we can calculate the unknown pivotal values of u along the first time-row, $t = k$, in terms of known boundary and initial values along $t = 0$, then the unknown pivotal values along the second time-row in terms of the calculated pivotal values along the first, and so on.

4 A necessary condition for stability

Now we are going to consider the stability of the classical explicit equations (3.19), (3.20). If the boundary values at $i = 0$ and N, $j > 0$ are known, then

$2(N-1)$ equations can be written in matrix as

$$\begin{bmatrix} \underline{u}_{1,j+1} \\ \underline{u}_{2,j+1} \\ \vdots \\ \underline{u}_{N-1,j+1} \\ \overline{u}_{1,j+1} \\ \overline{u}_{2,j+1} \\ \vdots \\ \overline{u}_{N-1,j+1} \end{bmatrix} = \begin{bmatrix} -2r & & & & & 1 & r & & & \\ & . & & & & r & 1 & r & & \\ & & . & & & & \ddots & \ddots & \ddots & \\ & & & . & & & & r & 1 & r \\ & & & & -2r & & & & r & 1 \\ 1 & r & 0 & & & -2r & & & & \\ r & 1 & r & & & & . & & & \\ & \ddots & \ddots & \ddots & & & & . & & \\ & & r & 1 & r & & & & . & \\ & & & r & 1 & & & & & -2r \end{bmatrix} \begin{bmatrix} \underline{u}_{1,j} \\ \underline{u}_{2,j} \\ \vdots \\ \underline{u}_{N-1,j} \\ \overline{u}_{1,j} \\ \overline{u}_{2,j} \\ \vdots \\ \overline{u}_{N-1,j} \end{bmatrix} \tag{4.21}$$

i.e.

$$\begin{bmatrix} \underline{w}_{j+1} \\ \overline{w}_{j+1} \end{bmatrix} = P \begin{bmatrix} \underline{w}_{j} \\ \overline{w}_{j} \end{bmatrix} \tag{4.22}$$

where

$$P = \begin{bmatrix} A & B \\ B & A \end{bmatrix}, \qquad A = (-2r)I, \qquad B = I + r \begin{bmatrix} 0 & 1 & & & \\ 1 & . & \ddots & & \\ & \ddots & . & 1 \\ & & 1 & 0 \end{bmatrix} . \tag{4.23}$$

Before proving the stability of the method, below lemmas should be considered :

Lemma 4.1 *Let x be an eigenvector of the matrix A corresponding to the eigenvalue λ . Then $Ax = \lambda x$. Hence $A(Ax) = A^2x = \lambda Ax = \lambda^2 x$ showing that the matrix A^2 has an eigenvalue λ^2 corresponding to the eigenvector x. Similarly $A^p x = \lambda^p x \quad p = 3, 4, \ldots$. [**?**]*

Lemma 4.2 *If $f(A) = a_p A^p + a_{p-1}A^{p-1} + \ldots + a_0 I$ is a polynomial then $F(A)x = (a_p\lambda^p + \ldots + a_0)x = f(\lambda)x$, showing that $f(A)$has an eigenvalue $f(\lambda)$ corresponding to the eigenvector x. [**?**]*

Lemma 4.3 *The eigenvalues of a common tridiagonal matrix the eigenvalue of the $N \times N$ matrix*

$$\begin{bmatrix} a & b & & & \\ c & a & b & & \\ & \ddots & \ddots & \ddots & \\ & & c & a & b \\ & & & c & a \end{bmatrix}$$

are

$$\lambda_k = a + 2\{\sqrt{bc}\}\cos\frac{k\pi}{N+1} \qquad k = 1, 2, \ldots, N$$

where a, b and c may be real or complex.[6]

Theorem 4.1 *Let matrix P has special structure as follow*

$$\begin{bmatrix} A & B \\ B & A \end{bmatrix}$$

then the eigenvalues of P are union of eigenvalues of $A+B$ and eigenvalues of $A-B$.[7]

Now we prove the stability of this method in the following theorem.

Theorem 4.2 *If $r = \frac{k}{h^2} < \frac{1}{2}$ difference equations (3.19) and (3.20) are stable.*

Proof:
It is sufficient to show in (4.22) $\rho(P) < 1$, thus by theorem (4.1) it is enough to find eigenvalues of

$$A + B = \begin{bmatrix} 1-2r & r & & & \\ r & 1-2r & r & & \\ & \ddots & \ddots & \ddots & \\ & & r & 1-2r & r \\ & & & r & 1-2r \end{bmatrix}$$

and

$$A - B = \begin{bmatrix} -2r-1 & -r & & & \\ -r & -2r-1 & -r & & \\ & \ddots & \ddots & \ddots & \\ & & -r & -2r-1 & -r \\ & & & -r & -2r-1 \end{bmatrix}.$$

Let matrices $(N-1)\times(N-1)$ T and T' as follow

$$T = \begin{bmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 2 & -1 \\ & & & -1 & 2 \end{bmatrix}, T' = \begin{bmatrix} 2 & 1 & & & \\ 1 & 2 & 1 & & \\ & \ddots & \ddots & \ddots & \\ & & 1 & 2 & 1 \\ & & & 1 & 2 \end{bmatrix}$$

thus

$$A + B = I - rT,$$

$$A - B = -rT' - I,$$

where $I_{2(N-1)}$ is the unit matrix of order $2(N-1)$ and T_{N-1} an $(N-1)\times(N-1)$ matrix whose eigenvalues λ_T are given by

$$\lambda_T = \lambda_{T'} = 4\cos^2\frac{k\pi}{2(N+1)} \qquad k = 1, 2, \ldots, N-1.$$

Hence the eigenvalues of $A - B$ and $A + B$ as shown later in *Lemma 4.2* are

$$\lambda_{A-B} = -1 - 4r\cos^2\frac{k\pi}{2(N+1)},$$

$$\lambda_{A+B} = 1 - 4r\cos^2\frac{k\pi}{2(N+1)}.$$

Therefore the equations will be stable when

$$\rho(A-B) = \max_k |-4r\cos^2\frac{k\pi}{2(N+1)} - 1| < 1 \quad k = 1, 2, \ldots, N-1$$

$$\rho(A+B) = \max_k |1 - 4r\cos^2\frac{k\pi}{2(N+1)}| < 1 \quad k = 1, 2, \ldots, N-1$$

i.e.

$$-1 < -1 - 4r\cos^2\frac{k\pi}{2(N+1)} < 1 \quad k = 1, 2, \ldots, N-1,$$

$$-1 < 1 - 4r\cos^2\frac{k\pi}{2(N+1)} < 1 \quad k = 1, 2, \ldots, N-1.$$

As $h \to 0$, $N \to \infty$ and $\cos^2\frac{k\pi}{2(N+1)} \to 1$ hence $|r| < \frac{1}{2}$.
This is the necessary and sufficient condition for the difference equations to be stable when the solution of the FPDE dose not increase as t increases.

■

5 Examples

Example 5.1 *Consider the fuzzy parabolic equation*

$$\frac{\partial \widetilde{U}}{\partial t}(x,t) - \frac{\partial^2 \widetilde{U}}{\partial x^2}(x,t) = 0, \quad 0 < x < l, \quad t > 0, \tag{5.24}$$

with the boundary conditions

$$\widetilde{U}(0,t) = \widetilde{U}(l,t) = 0,\ \ t > 0,$$

and

$$\widetilde{U}(x,0) = \widetilde{f}(x) = \widetilde{k}cos(\pi x - \pi/2),\ \ 0 \leq x \leq 1.$$

and $\widetilde{k}[\alpha] = [\underline{k}(\alpha), \overline{k}(\alpha)] = [\alpha - 1, 1 - \alpha]$.

The exact solution for

$$\frac{\partial \underline{U}}{\partial t}(x,t;\alpha) - \frac{\partial^2 \underline{U}}{\partial x^2}(x,t;\alpha) = 0, \quad (5.25)$$

$$\frac{\partial \overline{U}}{\partial t}(x,t;\alpha) - \frac{\partial^2 \overline{U}}{\partial x^2}(x,t;\alpha) = 0, \quad (5.26)$$

for $0 < x < l,\ \ t > 0$ are $\underline{U}(x,y;\alpha) = \underline{k}(\alpha)e^{-\pi^2 t}cos(\pi x - \pi/2)$ and $\overline{U}(x,y;\alpha) = \overline{k}(\alpha)e^{-\pi^2 t}cos(\pi x - \pi/2)$. We use the equations (3.19) and (3.20) to approximate the exact solution with $h = 0.1$ and $k = 0.00001$, therefore $r = 0.001$. Fig. 3 shows the exact and approximate solution at the point $(0.1, 0.000001)$ for each $\alpha \in (0,1]$. The *Hausdorff distance* between the solutions is $\mathbf{1.2363e-004}$.

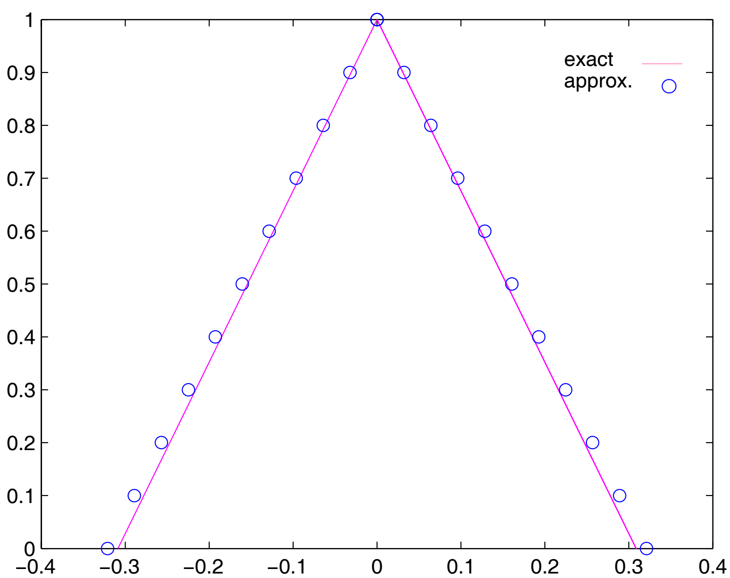

figure 2. h=0.1, l=0.0001

6 Conclusions

We presented difference methods for solving fuzzy partial differential equations. This numerical method based on the *seikkala derivative*. If all terms of FPDE belong to E then solutions of FPDE , would exist, which have been concluded from the numerical values. We presented necessary conditions for stability of this method.

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