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A Self-similar Flow Behind a Shock Wave in a Gravitating or Non-gravitating Gas with Heat Conduction and Radiation Heat-flux

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Abstract. The propagation of a spherical shock wave in an ideal gas with heat conduction and radiation heat-flux, and with or without self-gravitational effects, is investigated. The initial density of the gas is assumed to obey a power law. The heat conduction is expressed in terms of Fourier's law and the radiation is considered to be of the diffusion type for an optically thick grey gas model. The thermal conductivity and the absorption coefficient are assumed to vary with temperature and density, and the total energy of the wave to vary with time. Similarity solutions are obtained and the effects of variation of the heat transfer parameters, the variation of initial density and the presence of self-gravitational field are investigated.

Key words. Shock wave—self-similar flow—self-gravitational effects—heat transfer effects—variable initial density—variable total energy.

1. Introduction

The explanation and analysis for the internal motion in stars is one of the basic problem in astrophysics. According to the observational data, the unsteady motion of large mass of the gas followed by sudden release of energy results flare-ups in novae and supernovae. A qualitative behaviour of the gaseous mass may be discussed with the help of the equations of motion and equilibrium taking gravitational forces into account. Numerical solutions for self-similar adiabatic flows in self-gravitating gas were obtained by Sedov (1959) and Carrus *et al.* (1951), independently. Purohit (1974) and Singh & Vishwakarma (1983) have discussed homothermal flows behind a spherical shock wave in a self-gravitating gas using similarity method. Nath *et al.* (1991) have studied the above problem assuming the flow to be adiabatic and self-similar and obtained the effects of the presence of a magnetic field. Shock wave through a variable density medium have been treated by Sedov (1959), Sakurai (1956), Rogers (1957), Rosenau & Frankenthal (1976a), Nath *et al.* (1991), Vishwakarma & Yadav (2003) and others. Their results are more applicable to the shock formed in the deep interior of stars.

Marshak (1958) studied the effects of radiation on the shock propagation by introducing the radiation diffusion approximation. Using the same mode of radiation, Elliott (1960) discussed the conditions leading to self-similarity with a specified functional form of the mean free path of radiation and obtained a solution for self-similar spherical explosions. Wang (1964), Helliwell (1969) and Nicastro (1970) treated the problems of radiating walls, either stationary or moving, generating shocks at the head of self-similar flow-fields. The non-similar problem of a blast wave associated with diffusive radiation was analysed by Kim *et al.* (1975), using matched expansions upon the assumption that the radiation and conduction effects are significant only in a boundary-layer around the centre of explosion. Gretler & Wehle (1993) studied the propagation of blast waves with exponential heat release by taking internal heat conduction and thermal radiation in a detonating medium. Also, Abdel-Raouf & Gretler (1991) obtained the non-self-similar solution for the blast waves with internal heat transfer effects. Ghoniem *et al.* (1982) obtained a self-similar solution for spherical explosions taking into account the effects of both conduction and radiation in the two limits of Rosseland radiative diffusion and Plank radiative emission. In these works, where both the radiation and conduction effects are considered, the density of the medium ahead of the shock is taken to be uniform and effects of self-gravitation of the medium are not taken into consideration.

The purpose of this study is, therefore, to obtain self-similar solutions for the shock propagation in a non-uniform gas with or without self-gravitational effects, in the presence of heat conduction and radiation heat flux. The mediums ahead and behind the shock front are assumed to be inviscid and to behave as thermally perfect gases. The initial density of gas is assumed to vary as some power of distance. The heat transfer fluxes are expressed in terms of Fourier's law for heat-conduction and a diffusion radiation mode for an optically thick grey gas, which is typical of large-scale explosions. The thermal conductivity and absorption coefficient of the gas are assumed to be proportional to appropriate powers of temperature and density (Ghoniem *et al.* 1982). Also, it is assumed that the gas is grey and opaque, and the shock is isothermal. The assumption that the shock is isothermal is a result of the mathematical approximation in which the heat flux is taken to be proportional to the temperature gradient; this excludes the possibility of temperature jump (Zel'dovich & Raizer 1967; Rosenau & Frankenthal (1976b, 1978); Bhowmick 1981; Singh & Srivastava 1982). The counter pressure (the pressure ahead of the shock) is taken into account. The radiation pressure and radiation energy are neglected (Elliott 1960; Wang 1964; Ghoniem *et al.* 1982; Abdel-Raouf & Gretler 1991). The assumption of an optically thick grey gas is physically consistent with the neglect of radiation pressure and radiation energy (Nicastro 1970). The total energy of the flow-field behind the shock is assumed to be increasing with time due to pressure exerted by a piston or inner expanding surface. The gas ahead of the shock is assumed to be at rest. Effects of viscosity and magnetic field are not taken into account. The results of numerical calculations were shown in the form of graphs and tables. A comparative study was made between the results with and without self-gravitation. Also, the effects of variation of heat transfer parameters and the initial density exponent on the flow-field behind the shock and the shock velocity were investigated.

2. Equations of motion and boundary conditions

The fundamental equations governing the unsteady and spherically symmetric motion of an inviscid, ideal and self-gravitating gas, with heat conduction and radiation heat flux taken into account, may be written as (Carrus *et al.* 1951; Ghoniem *et al.* 1982):

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{2\rho u}{r} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{Gm}{r^2} = 0, \tag{2}$$

$$\frac{\partial m}{\partial r} - 4\pi\rho r^2 = 0, \tag{3}$$

$$\frac{\partial e}{\partial t} + u\frac{\partial e}{\partial r} - \frac{p}{\rho^2}\left(\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\right) + \frac{1}{\rho r^2}\frac{\partial}{\partial r}(r^2 q) = 0, \tag{4}$$

where r and t are independent space and time co-ordinates, ρ is the density, p the pressure, u the fluid velocity, m the mass of the gas contained in the sphere of radius r, G the gravitational constant, e the internal energy and q the heat flux. In the non-gravitating case, the equation (3) and the term Gm/r^2 in the equation (2) do not occur.

The total heat flux q, which appears in the energy equation may be decomposed as:

$$q = q_C + q_R, \tag{5}$$

where q_C is the conduction heat flux, and q_R the radiation heat flux.

According to Fourier's law of heat conduction

$$q_C = -K\frac{\partial T}{\partial r}, \tag{6}$$

where K is the coefficient of thermal conductivity of the gas and T is the absolute temperature.

Assuming local thermodynamic equilibrium and using the radiative diffusion model for an optically thick grey gas (Pomraning 1973), the term q_R, which represents radiative heat flux, may be obtained from the differential approximation of the radiation-transport equation in the diffusion limit as:

$$q_R = -\frac{4}{3}\left(\frac{\sigma}{\alpha_R}\right)\frac{\partial T^4}{\partial r}, \tag{7}$$

where σ is the Stefan–Boltzmann constant and α_R is the Rosseland mean absorption coefficient.

The above system of equations should be supplemented with an equation of state. A perfect gas behaviour of the medium is assumed, so that

$$p = \Gamma\rho T, \qquad e = \frac{p}{\rho(\gamma - 1)}, \tag{8a, b}$$

where Γ is the gas constant and γ is the ratio of specific heats.

The thermal conductivity K and the absorption coefficient α_R are assumed to vary with temperature and density. These can be written in the form of power laws, namely (Ghoniem *et al.* 1982)

$$K = K_0\left(\frac{T}{T_0}\right)^{\beta_C}\left(\frac{\rho}{\rho_0}\right)^{\delta_C}, \qquad \alpha_R = \alpha_{R_0}\left(\frac{T}{T_0}\right)^{\beta_R}\left(\frac{\rho}{\rho_0}\right)^{\delta_R}, \tag{9a, b}$$

where subscript '0' denotes a reference state. The exponents in the above equations should be compatible with the conditions of the problem and the form of the required solution.

We have also assumed that the total energy of the explosion is non-constant and increasing with time as (Rogers 1958; Freeman 1968; Director & Dabora 1977)

$$E = E_0 t^s, \tag{10}$$

where E_0 and s are constants. This increase of energy may be achieved by the pressure exerted on the gas by a piston. The piston may be, physically, the surface of the stellar corona or the condensed explosives or the diaphragm containing a very high pressure driver gas, at $t = 0$. By sudden expansion of the stellar corona or the detonation products or the driver gas into the ambient gas, a shock wave is produced in the ambient gas, in an infinitesimal time interval t_0 (say). The shocked gas is separated from the expanding surface which is a contact discontinuity. This contact surface acts as a 'piston' for the shock wave in the ambient medium.

A shock (spherical) is supposed to be propagating in the undisturbed ideal gas with variable density $\rho = Ar^{-w}$, where A and w are constants.

The flow variables immediately ahead of the shock front are:

$$u_1 = 0, \tag{11a}$$

$$\rho_1 = AR^{-w}, \tag{11b}$$

$$p_1 = \frac{2\pi G A^2}{(w-1)(3-w)} R^{2-2w}$$

$$\text{in the case when the gas is self-gravitating,} \tag{11c}$$

$$p_1 = \text{constant in the non-gravitating case,} \tag{11d}$$

$$m_1 = \frac{4\pi A}{3-w} R^{3-w}, \tag{11e}$$

$$q_1 = 0 \text{ (Laumbach \& Probstein 1970),} \tag{11f}$$

where R is the shock radius and the subscript '1' denotes the conditions immediately ahead of the shock.

The shock is assumed to be isothermal (the formation of the isothermal shock is a result of the mathematical approximation in which the flux is taken to be proportional to the temperature gradient. This excludes the possibility of a temperature jump, see for example, Zel'dovich & Raizer 1967; Rosenau & Frankenthal 1976b, 1978), and the conditions across it are:

$$\rho_1 V = \rho_2 (V - u_2), \tag{12a}$$

$$p_1 + \rho_1 V^2 = p_2 + \rho_2 (V - u_2)^2, \tag{12b}$$

$$e_1 + \frac{p_1}{\rho_1} + \frac{V^2}{2} + \frac{q_2}{\rho_1 V} = e_2 + \frac{p_2}{\rho_2} + \frac{1}{2}(V - u_2)^2, \tag{12c}$$

$$T_1 = T_2, \tag{12d}$$

$$m_1 = m_2, \tag{12e}$$

where subscript '2' denotes condition immediately behind the shock front, and $V = dR/dt$ denotes the velocity of the shock front. From equations (12), we get:

$$u_2 = \left(1 - \frac{1}{\gamma M^2}\right) V, \tag{13a}$$

$$\rho_2 = \gamma \rho_1 M^2, \tag{13b}$$

$$p_2 = \rho_1 V^2, \tag{13c}$$

$$m_2 = \frac{4\pi A}{3 - w} R^{3-w}, \tag{13d}$$

$$q_2 = \frac{1}{2}\left(\frac{1}{\gamma^2 M^4} - 1\right) \rho_1 V^3, \tag{13e}$$

where $M = (\rho_1 V^2/\gamma p_1)^{\frac{1}{2}}$ is the shock-Mach number.

3. Similarity solutions

Following the general similarity analysis we define the two characteristic parameters 'a' and 'b' with independent dimensions as:

$$[a] = [A], \tag{14a}$$

and

$$[b] = \left[\frac{E_0}{A}\right]. \tag{14b}$$

The single dimensionless independent variable in this case will be:

$$\eta = \left[\frac{\nu E_0}{A}\right]^{-1/(5-w)} r t^{-\delta}, \tag{15a}$$

where

$$\delta = \frac{2 + s}{5 - w}, \tag{15b}$$

and ν is a constant such that η assumes the value '1' at the shock front. This gives the shock propagation law in the explicit form as:

$$R = \left[\frac{\nu E_0}{A}\right]^{1/(5-w)} t^{\delta}. \tag{16}$$

This gives the shock propagation law as:

$$\frac{V}{V_0} = \left(\frac{R}{R_0}\right)^{(\delta-1)/\delta}, \tag{17}$$

where V_0 and R_0 are the velocity and radius of the shock at the instant of its generation.

We express the fluid velocity u, density ρ, pressure p, mass m and total heat flux q as:

$$u = VU(\eta), \qquad \rho = \rho_1 D(\eta), \qquad p = V^2 \rho_1 P(\eta), \tag{18a–c}$$

$$m = \rho_1 R^3 N(\eta), \qquad q = V^3 \rho_1 Q(\eta), \tag{18d–e}$$

where U, D, P, N and Q are functions of η only.

For the existence of similarity solutions the shock-Mach number M should be constant. Therefore, in the gravitating case

$$\delta = \frac{2}{w} \quad \text{and} \quad w = \frac{10}{s+4}, \tag{19a, b}$$

where $1 < s < 6$ $(1 < w < 2)$ or $0 \leq s < 1$ $(2 < w \leq 5/2)$.

In the non-gravitating case

$$\delta = \frac{2}{2-w} \quad \text{and} \quad s = \frac{6}{2-w}, \tag{20a, b}$$

where $0 < w < 2$ $(3 < s < \infty)$.

The conservation equations (1) to (4) can be transformed into a system of ordinary differential equations

$$(U - \eta)\frac{dD}{d\eta} + D\frac{dU}{d\eta} + \frac{2DU}{\eta} - wD = 0, \tag{21}$$

$$(U - \eta)\frac{dU}{d\eta} + \frac{1}{D}\frac{dP}{d\eta} + \left(\frac{\delta - 1}{\delta}\right)U + \frac{G_0 N}{\eta^2} = 0, \tag{22}$$

$$\frac{dN}{d\eta} = 4\pi D\eta^2, \tag{23}$$

$$\begin{aligned}(U - \eta)D\frac{dP}{d\eta} &- \gamma P(U - \eta)\frac{dD}{d\eta} + (\gamma - 1)D\frac{dQ}{d\eta} \\ &+ 2\left(\frac{\delta - 1}{\delta}\right)PD + D(\gamma - 1)\left(\frac{2Q}{\eta} + wP\right) = 0,\end{aligned} \tag{24}$$

where

$$G_0 = \frac{(w-1)(3-w)}{2\pi\gamma M^2}. \tag{25}$$

By using equations (6), (7) and (9) in (5) we get:

$$q = -\left[\frac{K_0}{T_0^{\beta_C}\rho_0^{\delta_C}}T^{\beta_C}\rho^{\delta_C} + \frac{16}{3}\frac{\sigma T_0^{\beta_R}\rho_0^{\delta_R}}{\alpha_{R_0}}T^{3-\beta_R}\rho^{-\delta_R}\right]\frac{\partial T}{\partial r}. \tag{26}$$

Using the equations (8) and (18) in (26) we get:

$$Q = -\frac{d}{d\eta}\left(\frac{P}{D}\right)\left[\frac{K_0 A^{\delta_C - 1} P^{\beta_C} \delta^{\frac{\delta}{\delta-1}\{w(\delta_C-1)+1\}} V^{2\beta_C - 1 - \frac{\delta}{\delta-1}\{w(\delta_C-1)+1\}}}{T_0^{\beta_C} \rho_0^{\delta_C} \Gamma^{\beta_C+1} D^{\beta_C - \delta_C}}\right.$$

$$\times \left\{\frac{\nu E_0}{A}\right\}^{\frac{w(\delta_C-1)+1}{(\delta-1)(5-w)}} + \frac{16\sigma T_0^{\beta_R} \rho_0^{\delta_R} P^{3-\beta_R} A^{-\delta_R-1} \delta^{\frac{\delta}{\delta-1}\{1-w(\delta_R+1)\}} V^{5-2\beta_R+\frac{\delta}{\delta-1}\{w(\delta_R+1)-1\}}}{3\alpha_{R_0} \Gamma^{4-\beta_R} D^{3-\beta_R+\delta_R}}$$

$$\left.\times \left\{\frac{\nu E_0}{A}\right\}^{\frac{1-w(\delta_R+1)}{(\delta-1)(5-w)}}\right]. \tag{27}$$

Equation (27) shows that the similarity solution of the present problem exists only when

$$\beta_C = 1 + \frac{1}{2\delta - 2}\left[1 + w\delta(\delta_C - 1)\right], \tag{28a}$$

and

$$\beta_R = 2 - \frac{1}{2\delta - 2}\left[1 - w\delta(\delta_R + 1)\right]. \tag{28b}$$

Therefore, equation (27) becomes:

$$Q = -X\left[\frac{1}{D}\frac{dP}{d\eta} - \frac{P}{D^2}\frac{dD}{d\eta}\right], \tag{29}$$

where

$$X = \left(\frac{P}{D}\right)^{(2\delta-1)/(2\delta-2)}\left[\Gamma_C \delta^{\frac{\delta}{\delta-1}\{1+w(\delta_C-1)\}} P^{\frac{w\delta(\delta_C-1)}{2\delta-2}} D^{\delta_C - \frac{w\delta(\delta_C-1)}{2\delta-2}}\right.$$

$$\left. + \Gamma_R \delta^{\frac{\delta}{\delta-1}\{1-w(\delta_R+1)\}} P^{\frac{-w\delta(\delta_R+1)}{2\delta-2}} D^{\frac{w\delta(\delta_R+1)}{2\delta-2} - \delta_R}\right]. \tag{30}$$

Here, Γ_C and Γ_R are the conductive and radiative non-dimensional heat transfer parameters, respectively. The parameters Γ_C and Γ_R depend on the thermal conductivity K and the mean free path of radiation $1/\alpha_R$, respectively, and also on the exponents δ and w, and they are given by:

$$\Gamma_C = \frac{K_0 A^{\delta_C - 1}}{T_0 \Gamma^2 \rho_0^{\delta_C}} (T_0 \Gamma)^{-\frac{1+w\delta(\delta_C-1)}{2\delta-2}} \left(\frac{\nu E_0}{A}\right)^{\frac{1+w(\delta_C-1)}{(\delta-1)(5-w)}}, \tag{31a}$$

and

$$\Gamma_R = \frac{16\sigma A^{-\delta_R - 1} \rho_0^{\delta_R} T_0^2}{3\alpha_{R_0} \Gamma^2} (T_0 \Gamma)^{\frac{w\delta(\delta_R+1)-1}{2\delta-2}} \left(\frac{\nu E_0}{A}\right)^{\frac{1-w(\delta_R+1)}{(\delta-1)(5-w)}}. \tag{31b}$$

Using the similarity transformations (18) and the equation (16), equations (13) can be written as:

$$U(1) = \left(1 - \frac{1}{\gamma M^2}\right), \tag{32a}$$

$$D(1) = \gamma M^2, \tag{32b}$$

$$P(1) = 1, \tag{32c}$$

$$N(1) = \frac{4\pi}{3-w}, \tag{32d}$$

$$Q(1) = \frac{1}{2}\left(\frac{1}{\gamma^2 M^4} - 1\right). \tag{32e}$$

By solving equations (21), (22), (24) and (29) for $dD/d\eta$, $dP/d\eta$, $dQ/d\eta$, $dU/d\eta$, we have:

$$\frac{dD}{d\eta} = -\frac{D}{U-\eta}\left[\frac{dU}{d\eta} + \frac{2U}{\eta} - w\right] \tag{33}$$

$$\frac{dP}{d\eta} = -D\left[(U-\eta)\frac{dU}{d\eta} + \left(\frac{\delta-1}{\delta}\right)U + \frac{G_0 N}{\eta^2}\right], \tag{34}$$

$$\begin{aligned}\frac{dQ}{d\eta} &= \frac{(U-\eta)^2 D - \gamma P}{\gamma - 1}\frac{dU}{d\eta} + \frac{D(U-\eta)}{\gamma-1}\left[\left(\frac{\delta-1}{\delta}\right)U + \frac{G_0 N}{\eta^2}\right] \\ &\quad - \frac{\gamma P}{\gamma-1}\left(\frac{2U}{\eta} - w\right) - \frac{2Q}{\eta} - wP - \frac{2P}{\gamma-1}\left(\frac{\delta-1}{\delta}\right),\end{aligned} \tag{35}$$

$$\begin{aligned}\frac{dU}{d\eta} &= \frac{D(U-\eta)}{P - D(U-\eta)^2}\left[\left(\frac{\delta-1}{\delta}\right)U + \frac{G_0 N}{\eta^2}\right. \\ &\quad \left. - \frac{2PU}{D\eta(U-\eta)} + \frac{wP}{D(U-\eta)} - \frac{Q}{X}\right].\end{aligned} \tag{36}$$

The condition to be satisfied at the inner expanding surface is that the velocity of the fluid is equal to the velocity of the surface itself. The kinematic condition, from equations (15) and (18), can be written as:

$$U(\overline{\eta}) = \overline{\eta}, \tag{37}$$

where $\overline{\eta}$ is the value of η at the inner expanding surface.

For exhibiting the numerical solutions it is convenient to write the flow variables in the non-dimensional form as:

$$\frac{u}{u_2} = \frac{U(\eta)}{U(1)}, \qquad \frac{\rho}{\rho_2} = \frac{D(\eta)}{D(1)}, \qquad \frac{p}{p_2} = \frac{P(\eta)}{P(1)}, \tag{38a–c}$$

$$\frac{m}{m_2} = \frac{N(\eta)}{N(1)}, \qquad \frac{q}{q_2} = \frac{Q(\eta)}{Q(1)}. \tag{38d–e}$$

Numerical integration of the differential equations (23), (33), (34), (35), (36) with the boundary conditions (32) give the solution in the gravitating case and of the differential equations (33), (34), (35), (36) with $G_0 = 0$ give the solution in the non-gravitating case.

4. Results and discussion

Distributions of the flow variables in the flow-field behind the shock front are obtained by numerical integration of the equations (23) and (33) to (36) with the boundary conditions (32) in the gravitating case and of the equations (33)–(36) in the non-gravitating case. The expressions for the gravitational parameter G_0 and the exponent in the shock propagation law δ are, in the gravitating case,

$$G_0 = \frac{(w-1)(3-w)}{2\pi\gamma M^2}, \qquad \delta = \frac{2}{w},$$

and in the non-gravitating case,

$$G_0 = 0, \qquad \delta = \frac{2}{2-w},$$

where w is the exponent in the law of variation (decrease) of the initial density. Also, the exponent in the law of variation of the total energy behind the shock 's' is related with w by $s = (10-4w)/w$ and $s = 6/(2-w)$ in the two cases, respectively. For the purpose of numerical integration, values of the constant parameters are taken to be (Ghoniem *et al.* 1982) $\gamma = 1.4$; $M = 5$; $\delta_C = 1$, $\delta_R = 2$; $\Gamma_C = 1, 10, 100$; $\Gamma_R = 1, 100, 1000$; $w = 1.1, 1.2$.

Figures 1–10 show the variation of the flow variables u/u_2, ρ/ρ_2, p/p_2, m/m_2, q/q_2 with η at various values of the parameters Γ_C, Γ_R, w, and Fig. 11 shows the

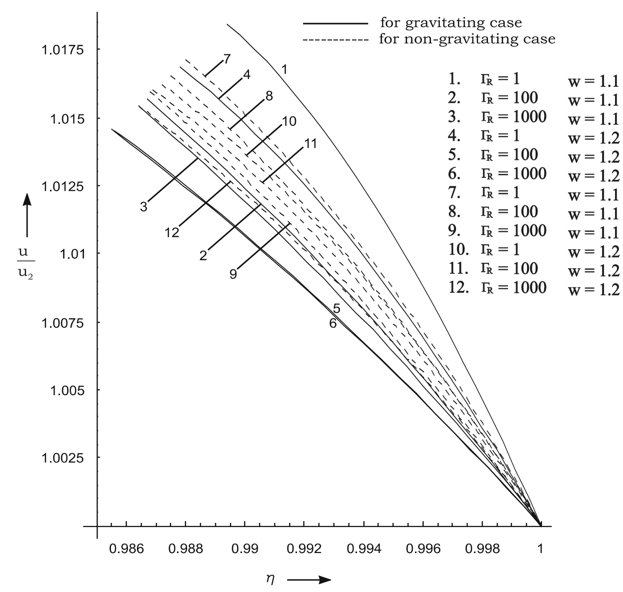

Figure 1. Variation of reduced velocity in the region behind the shock front with $\Gamma_C = 1$.

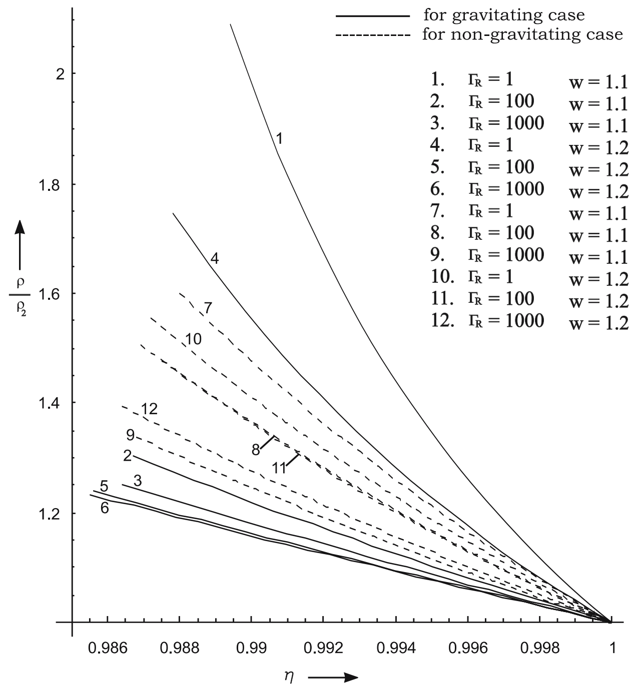

Figure 2. Variation of reduced density in the region behind the shock front with $\Gamma_C = 1$.

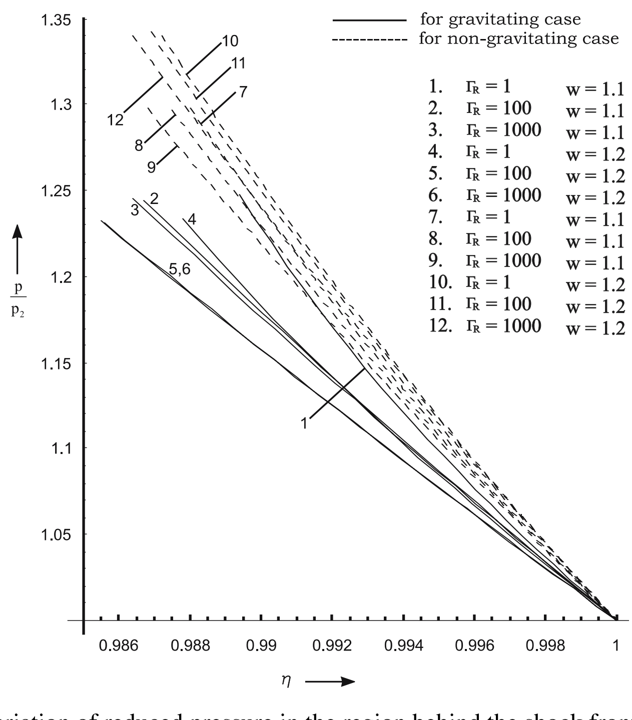

Figure 3. Variation of reduced pressure in the region behind the shock front with $\Gamma_C = 1$.

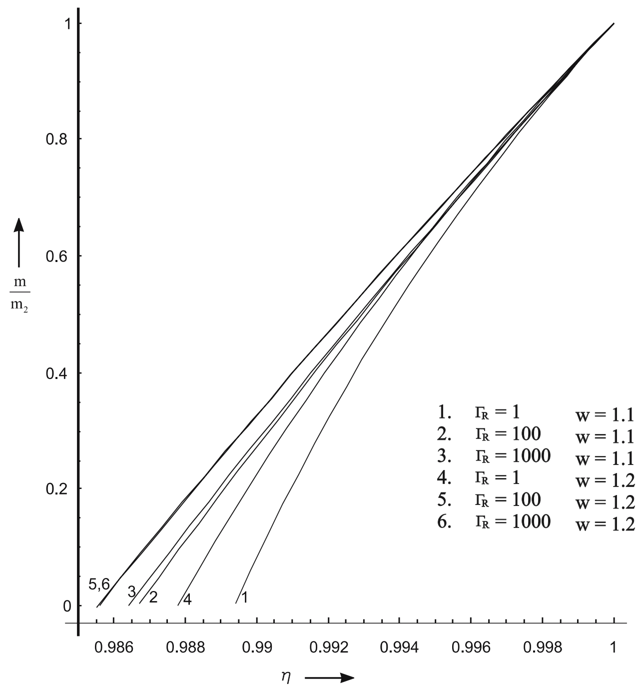

Figure 4. Variation of reduced mass in the region behind the shock front with $\Gamma_C = 1$.

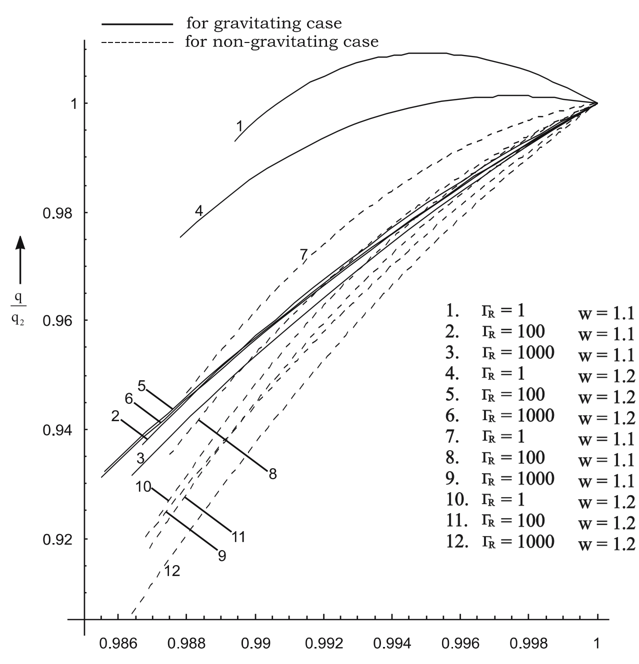

Figure 5. Variation of reduced total heat flux in the region behind the shock front with $\Gamma_C = 1$.

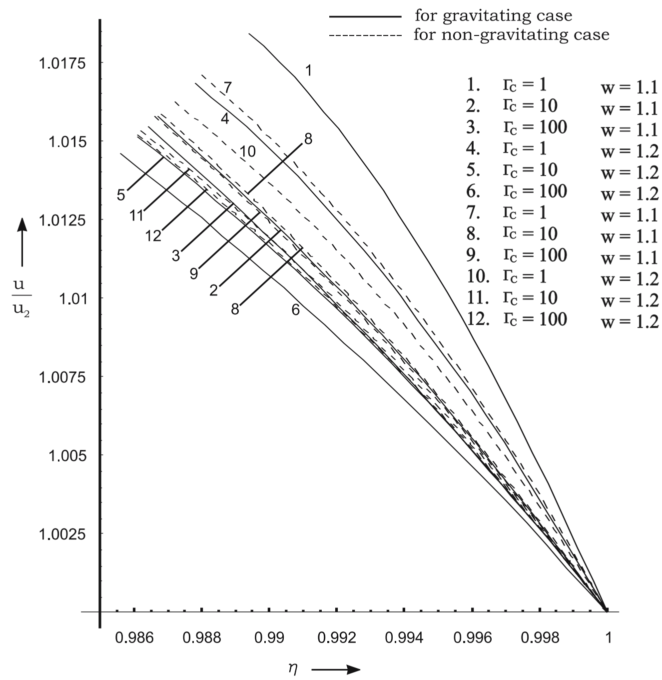

Figure 6. Variation of reduced velocity in the region behind the shock front with $\Gamma_R = 1$.

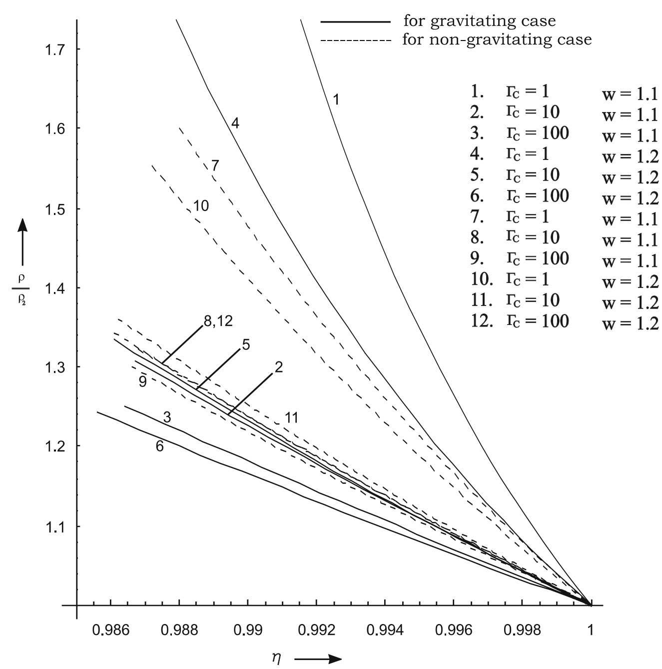

Figure 7. Variation of reduced density in the region behind the shock front with $\Gamma_R = 1$.

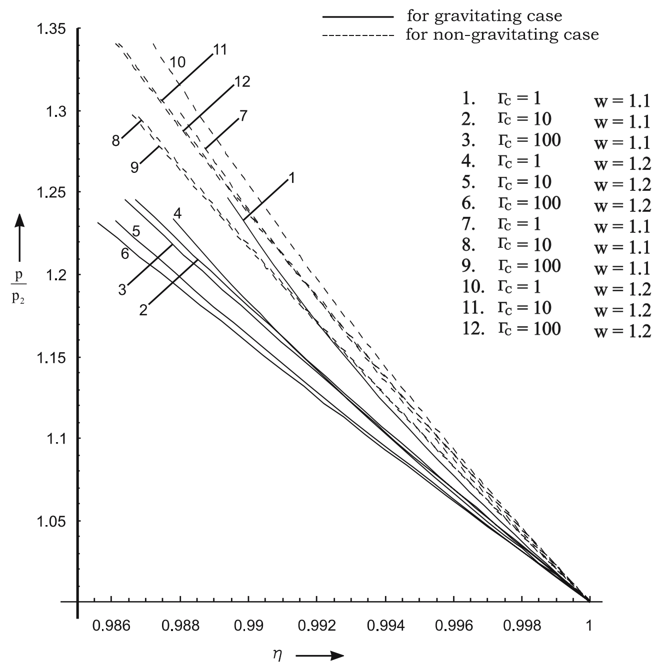

Figure 8. Variation of reduced pressure in the region behind the shock front with $\Gamma_R = 1$.

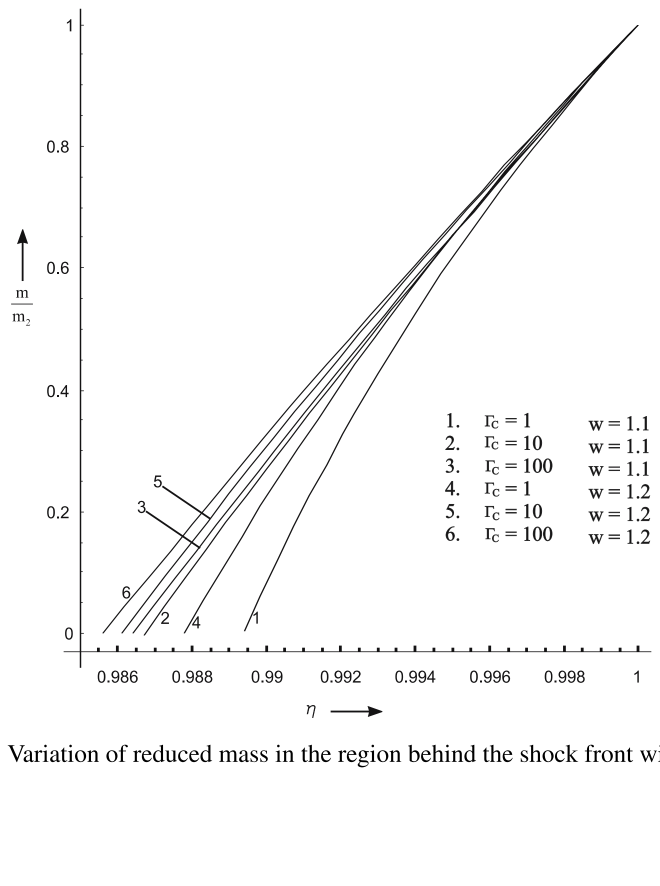

Figure 9. Variation of reduced mass in the region behind the shock front with $\Gamma_R = 1$.

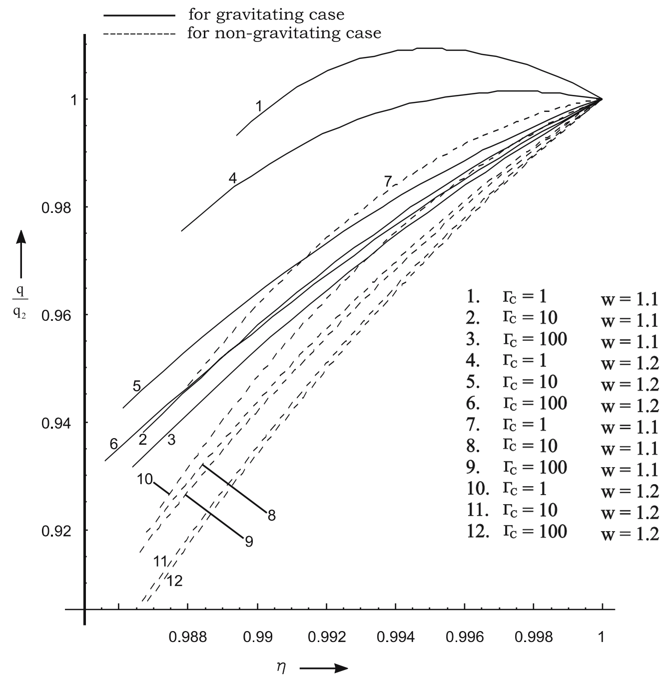

Figure 10. Variation of reduced total heat flux in the region behind the shock front with $\Gamma_R = 1$.

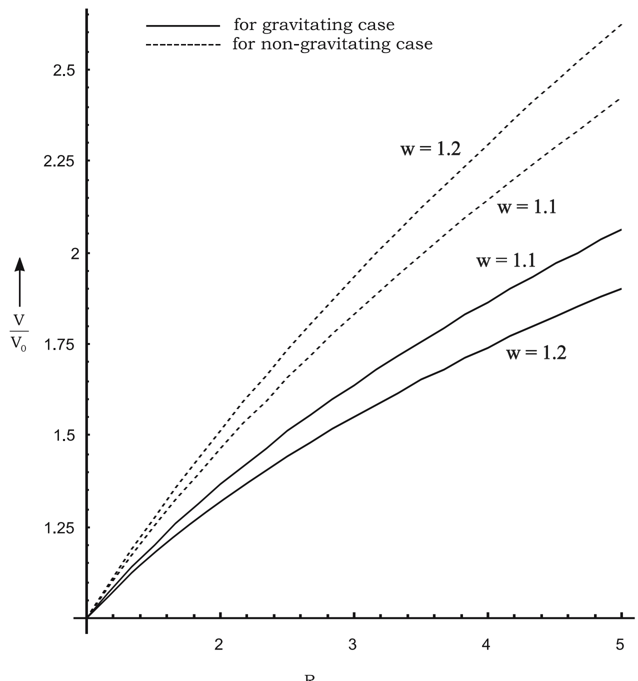

Figure 11. Variation of reduced shock velocity with reduced shock radius.

variation of the reduced shock velocity V/V_0 with reduced shock radius R/R_0 at various values of the parameter w. It is observed that, as we move inward from the shock front towards the inner expanding surface, the reduced fluid velocity u/u_2, reduced density ρ/ρ_2 and reduced pressure p/p_2 increase, and the reduced mass m/m_2 and reduced total heat flux q/q_2 decrease, in general. Also, the shock velocity V/V_0 increases with the shock radius R/R_0.

It is found that the effects of an increase in the value of radiation heat transfer parameter Γ_R are (from Figs. 1–5 and Table 1):

- to decrease the velocity u/u_2, density ρ/ρ_2, pressure p/p_2 and total heat flux q/q_2 at any point in the flow-field behind the shock;
- to increase the mass m/m_2;
- to increase the distance of the inner expanding surface from the shock front (see Table 1); and
- to decrease the slope of profiles of velocity, density, pressure and mass and to increase that of total heat flux.

The conduction heat transfer parameter Γ_C has similar effects on the flow-field behind the shock as the radiation heat transfer parameter Γ_R (see Figs. 6–10 and Table 2).

The effects of an increase in the density variation exponent w are (from Figs. 1–11 and Tables 1–2):

- to decrease the velocity u/u_2 and to increase the mass m/m_2 at any point in the flow-field behind the shock;
- to decrease the density ρ/ρ_2 in the gravitating case, in general; and in the non-gravitational case, to decrease ρ/ρ_2 for lower values of Γ_C and Γ_R and to increase that for higher values of Γ_C and Γ_R;
- to decrease the pressure p/p_2 in the gravitating case and to increase that in the non-gravitating case; and

Table 1. Position of the inner expanding surface $\overline{\eta}$ at different values of Γ_R for $\Gamma_C = 1$, $\gamma = 1.4$, $\delta_C = 1$, $\delta_R = 2$, $M = 5$ and $w = 1.1, 1.2$.

w	δ	s	G_0	Γ_R	$\overline{\eta}$
1.1	1.8181	5.0909	0.0008644	1	0.9894
				100	0.9867
				1000	0.9864
1.2	1.6666	4.3333	0.0016379	1	0.9878
				100	0.9856
				1000	0.9855
1.1	2.2222	6.6666	0	1	0.9880
				100	0.9875
				1000	0.9868
1.2	2.5	7.5	0	1	0.9872
				100	0.9869
				1000	0.9864

Table 2. Position of the inner expanding surface $\bar{\eta}$ at different values of Γ_C for $\Gamma_R = 1$, $\gamma = 1.4$, $\delta_C = 1$, $\delta_R = 2$, $M = 5$ and $w = 1.1$, 1.2.

w	δ	s	G_0	Γ_C	$\bar{\eta}$
1.1	1.8181	5.0909	0.0008644	1	0.9894
				10	0.9867
				100	0.9864
1.2	1.6666	4.3333	0.0016379	1	0.9878
				10	0.9861
				100	0.9856
1.1	2.2222	6.6666	0	1	0.9880
				10	0.9868
				100	0.9866
1.2	2.5	7.5	0	1	0.9872
				10	0.9862
				100	0.9861

- to decrease the total heat flux q/q_2 in the non-gravitating case; and in the gravitational case, to decrease q/q_2 for lower values of Γ_C and Γ_R ($\Gamma_C = 1$, $\Gamma_R = 1$) and to increase that for higher values of Γ_C and Γ_R;
- to increase the distance of the inner expanding surface from the shock front (see Tables 1 and 2);
- to decrease the shock velocity V/V_0 in the gravitating case, and to increase that in the non-gravitating case.

The effects of self-gravitational field (from Figs. 1, 2, 3, 5 and 6, 7, 8, 10, 11 and Tables 1–2) are:

- to increase the velocity u/u_2 and the density ρ/ρ_2 at lower value of Γ_R and Γ_C, and to decrease those at higher values of Γ_R and Γ_C, at any point in the flow-field behind the shock;
- to decrease the pressure p/p_2;
- to increase the total heat flux q/q_2; and
- to decrease the shock velocity V/V_0.

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