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On Testing Exponentiality against RNBRUE Alternatives

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Abstract

A moment inequality for the class of renewal new is better than renewal used in expectation (RNBRUE) of ageing distributions is derived. This class is defined based on comparing the residual equilibrium life at a certain age and its equilibrium (stationary) life in expectation. This inequality demonstrate that if the mean life is finite,then all higher order moments exist. A new test statistics for testing exponentiality against RNBRUE is investigating based on this inequality. The asymptotic normality of the proposed statistic is presented. Pitman's asymptotic efficiency of the test and critical values of the proposed statistic are calculated. It is shown that ,the proposed statistic has a high asymptotic relative efficiency with respect to tests of other classes for commonly used alternatives. The set of real data is used as a practical application of the proposed test in the medical science.

Keywords: Life distributions, RNBRUE, Moments inequalities, Testing Exponentiality, Asymptotic normality, Efficiency

1 Introduction and Motivation

In reliability theory, ageing life is usually characterized by a nonnegative random variable $X \geq 0$ with cumulative distribution function (cdf) F and survival function (sf) $\overline{F} = 1 - F$. For any random variable X, let

$$X_t = [X - t | X > t], \quad t \in \{x : F(x) < 1\},$$

denote a random variable whose distribution is the same as the conditional distribution of $X - t$ given that $X > t$. When X is the lifetime of a device, X_t can be regarded as the residual lifetime of the device at time t, given that the device has survived up to time t. Its survival function is (see, for instance, *Deshpand et al.* (1986))

$$\overline{F_t}(x) = \frac{\overline{F}(t+x)}{\overline{F}(t)}, \qquad \overline{F}(t) > 0,$$

where $\overline{F}(x)$ is the survival function of X. It is well-known fact that when $\overline{F}$ is an exponential distribution then $X_t \overset{st}{=} X$ or $\overline{F_t}(x) = \overline{F}(x)$. Comparing X and X_t in various forms and types create classes of ageing useful in many biomedical, engineering and statistical studies, cf. *Barlow* and *Proschan* (1981). It is well known that the relation $X_t \overset{st}{\leq} X$ or $\overline{F_t}(x) \overset{st}{\leq} \overline{F}(x)$ defines the class of new better than used (NBU).On the other hand, the relation $E(X_t) \leq E(X) = \mu$ defines the class of new better than used in expectation ($NBUE$). Another notion associated with X is the weak limit of X_t as $t \to \infty$. It is well known that, cf. *Ross* (2003), X_t converges weakly to a nonnegative random variable $\widetilde{X}$ with sf

$$\overline{W}_F(x) = \frac{1}{\mu}\,\overline{V}(x) \text{ where } \overline{V}(x) = \int_x^\infty \overline{F}(u)du\ , \quad x \geq 0.$$

Define $\widetilde{X}_t$ to be the random residual life of $\widetilde{X}$ at age t. Thus, the survival function of $\widetilde{X}_t$ is given by

$$\overline{W}_{F,t}(x) = \frac{\overline{W}_F(x+t)}{\overline{W}_F(t)} \quad , x, t \geq 0.$$

From the above discussion, we see that there are four random quantities related to life and these are the life itself X, the random residual life X_t, the equilibrium life $\widetilde{X}$, and the residual equilibrium life $\widetilde{X}_t$. It is also well known that stochastic or in moment comparisons between X and X_t define two of the most applicable ageing classes, namely, the NBU and the $NBUE$. These classes are useful to characterize ageing as well as in replacement policies. Hence it would be of interest to compare a life X to its equilibrium form $\widetilde{X}$ or to its residual equilibrium form $\widetilde{X}_t$ or to compare the equilibrium life $\widetilde{X}_t$ to the residual life X_t. This is precisely what we do in the current investigation. These comparisons produce new NBU type classes including "new better than renewal of used"($NBRU$), "renewal new is better than used " ($RNBU$),and ($RNBRUE$)" renewal new is better than renewal used" when comparing stochastically and comparing the residual equilibrium life at a certain age and its equilibrium (stationary) life in expectation,or similarly $NBRUE$ and $RNBUE$ when comparing in the mean. Other comparisons are also possible and some are addressed here. Some of the classes we discuss have been also developed by other authors including *Bhattacharjee* and *Sethuraman* (1990), *Bhattacharjee et al.* (2000), *Cao* and *Wang* (1991), *Franco et al.* (2001), *Kaur et al.* (1994), *Li et al.* (2000), *Muller* and *Stoyan* (2002), and *Shaked* and *Shanthikumar* (1994). Most of these authors address probabilistic properties of the ageing classes they study.

Abouammoh et al . (2000) introduced the $NRBU$, $RNBU$, $NRBUE$, $HNRBUE$ classes of life distributions and studied the relation between them. *Abouammoh* and *Khalique* (1998) investigated a test statistic of $NRBU$ based on total time of test (TTT)-transform impirically.*Mahmoud et al.*(2005) discussed a test statistic of $RNBU$ by using U-test. Testing exponentiality against $NRBU$ and $NRBUE$ where studied by *Abu-Youssef* (2003,2004). While testing against $RNBU$ investigated by *Mugdadi* and *Ahmad* (2005). Precisely we have the following definitions:

Definition 1.1.

$(i)X$ is said to be new is better than renewal used ($NBRU$) if

$$\widetilde{X}_t \leq X, i.e, \int_{x+t}^{\infty} \overline{F}(u)du \leq \int_{x}^{\infty} \overline{F}(u)du\overline{F}(x)$$

(ii) X is said to be renewal new is better than used ($RNBU$) if

$$X_t \overset{st}{\leq} \widetilde{X}\ , i.e., \quad \overline{F_t}(x) \leq \ \overline{W}_F(x), x \geq 0.$$

(iii) X is said to be renewal new is better than used in expectation ($RNBUE$) if

$$E(X_t) \leq E(\widetilde{X}), \quad i.e, \ 2\mu \int_{x}^{\infty} \overline{F}(u)du \leq \mu_{(2)}\overline{F}(x),$$

where μ is the mean life and $\mu_{(2)}$ is the second moment,both assumed finite.

Definition 1.2.

A random variable X is said to be

(i) renewal new is better than renewal used ($RNBRU$) if

$$\begin{aligned} \widetilde{X}_t \ \overset{st}{\leq} \widetilde{X}, i.e., \overline{W}(x+t) \ &\leq \ \overline{W}_F(x)\overline{W}_F(t), x, t \geq 0, \\ i.e., \mu \int_{x+t}^{\infty} \overline{F}(u)du \ &\leq \ \int_{x}^{\infty} \overline{F}(u)du \int_{t}^{\infty} \overline{F}(u)du, x, t \geq 0. \end{aligned}$$

(ii) renewal new is better than renewal used in expectation ($RNBRUE$) if

$$E(\widetilde{X}_t) \ \overset{st}{\leq} E(\widetilde{X}), i.e., 2\mu \int_{x}^{\infty} \int_{u}^{\infty} \overline{F}(w)dwdu \leq \mu_{(2)} \int_{x}^{\infty} \overline{F}(u)du.$$

The purpose of this paper is to give a moment inequality for the $RNBRUE$ class. The main results are given in *Section* 2. Our proposed tests and their asymptotic normality are shown in *Section* 3. In that section, we obtained *Monte Carlo* null distribution critical values for sample sizes n =40(5)1. In *Section* 4, the PAE values of our tests are calculated. Furthermore, their Pitman asymptotic efficiency (PAE) values relative to the other tests are presented. Finally, in *Section* 5, we apply the proposed test to real practical data in medical science given in *Abouammah et al.* (1994).

2 A moment inequality

In this section we present our main results. The following theorem gives the moment inequality for the $RNBRUE$ life distributions class.

Theorem 2.1.

If F is $RNBRUE$, then

$$\frac{\mu\mu_{(r+3)}}{r+3} \leq \frac{1}{2}\mu_{(2)}\mu_{(r+2)}, \quad r \geq 1, \tag{2.1}$$

where

$$\mu_{(r)} = E(X^r) = r\int_0^\infty x^{r-1}\, \overline{F}\ (u)du.$$

Proof.

Since F is said to be renewal new is better than renewal used in expectation ($RNBRUE$), then

$$2\mu\int_x^\infty \overline{V}(u)du \leq \mu_{(2)}\overline{V}(x), \tag{2.2}$$

where

$$\overline{V}(u) = \int_u^\infty \overline{F}\ (w)dw \text{ and } \overline{V}(x) = \int_x^\infty \overline{F}(u)du.$$

Multiplying both sides in (2.2) by x^r, $r \geq 1$, and integrating over (0,∞) with respect to x , we get

$$2\mu\int_0^\infty \int_x^\infty x^r\ \overline{V}(u)du \leq \mu_{(2)}\int_0^\infty x^r\ \overline{V}(x)dx. \tag{2.3}$$

Now

$$\begin{aligned}\int_0^\infty x^r\ \overline{V}(x)dx &= \int_0^\infty \int_x^\infty x^r\ \overline{F}(u)du\ dx \\ &= \int_0^\infty \overline{F}(x) \int_0^x u^r du\ dx \\ &= \frac{\mu_{(r+2)}}{(r+1)(r+2)}.\end{aligned} \tag{2.4}$$

Also

$$\begin{aligned}\int_0^\infty \int_x^\infty x^r\ \overline{V}(u)du\ dx &= \int_0^\infty \overline{V}\ (x)\frac{x^{r+1}}{r+1}dx \\ &= \frac{1}{r+1}E\ \left[\int_0^\infty x^{r+1}\left(X-x\right)I\ \ (X>x)\right]dx \\ &= \frac{1}{r+1}E\ \left[X\int_0^X x^{r+1}dx - \int_0^X x^{r+2}dx\right] \\ &= \frac{1}{r+1}E\left[\frac{X^{r+3}}{r+2}-\frac{X^{r+3}}{r+3}\right] \\ &= \frac{\mu_{(r+3)}}{(r+1)(r+2)(r+3)}\end{aligned} \tag{2.5}$$

Thus from (2.4) and (2.5),the proof of the theorem is completed.

3 Testing the RNBRUE class

3.1 Test procedure:

Let $X_{1,}X_{2,}...X_{n,}$ be a random sample from a population with distribution function F. We test the null hypothesis $H_0 : \overline{F}$ is exponential with mean μ against H_1: $\overline{F}$ is $RNBRUE$ and not exponential. Using theorem (2.1), we can use the following quantity as a measure of departure from H_0 in favor of H_1 :

$$\delta_{_{RN}}(r) = \frac{1}{2}\mu_{(2)}\mu_{(r+2)} - \frac{\mu\mu_{(r+3)}}{r+3} \tag{3.1}$$

Not that under $H_0 : \delta_{RN}(r) = 0$,and it ispositive under H_1.To make the test scale invariant under H_0,we use

$$\triangle_{RN}(r) = \frac{\delta_{RN}(r)}{\mu^{r+4}}$$

It could be estimated based on a random sample $X_1, X_2, ...X_n$, from F by

$$\begin{aligned} \hat{\triangle}_{RN}(r) &= \frac{\hat{\delta}_{RN}(r)}{\overline{X}^{r+4}} \\ &= \frac{1}{\overline{X}^{r+4}} \left[\frac{1}{n(n-1)} \sum\sum_{i \neq j} \left(\frac{X_i^2 X_j^{r+2}}{2} - \frac{X_i X_j^{r+3}}{r+3} \right) \right] \end{aligned} \tag{3.2}$$

where $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ is the sample mean,and μ is estimated by $\overline{X}$. Setting

$$\phi(X_1 , X_2) = \frac{1}{2} X_1^2 X_2^{r+2} - \frac{1}{r+3} X_1 X_2^{r+3}$$

Again $\hat{\triangle}_{RN}(r)$ and $\frac{\hat{\delta}_{RN}(r)}{\overline{X}^{r+4}}$ have the same limiting distribution. But since $\hat{\triangle}_{RN}(r)$ is the usual U-statistics theory, cf. *Koroljuk* and *Broovskich* (1994), it is asymptotically normal and all we need to evaluate $Var(\frac{\hat{\delta}_{RN}(r)}{\mu^{r+4}})$. The following theorem summarized the large sample properties of $\hat{\triangle}_{RN}(r)$ or U-statistic.

Theorem 3.1.

As $n \to \infty$,$\sqrt{n}(\hat{\triangle}_{RN}(r) - \triangle_{RN}(r))$ is asymptotically normal with mean zero and variance

$$\sigma_{(r)}^2 = \mu^{-2(r+4)} Var \left[\frac{X_1^2 \mu_{(r+2)} + \mu_{(2)} X_1^{r+2}}{2} - \frac{X_1 \mu_{(r+3)} + \mu X_1^{r+3}}{r+3} \right], \tag{3.3}$$

under H_0 this value redused to

$$\sigma_0^2 = (2r+4)! - 2(r+2) \left[(r+2)! \right]^2 \tag{3.4}$$

Proof:

Since $\hat{\triangle}_{RN}(r)$ and $\frac{\hat{\delta}_{RN}(r)}{\overline{X}^{r+4}}$ have the same limiting distribution,we concentrate on $\sqrt{n}\,(\hat{\triangle}_{RN}(r) - \triangle_{RN}(r))$. Now this is asymptotic normal with mean zero and variance $\sigma^2 = Var\left[\phi(X_1\ )\right]$,where

$$\phi(X_1) = E\left[\phi(X_1 , X_2)|X_1\right] + E\left[\phi(X_2 , X_1)|X_1\right]. \tag{3.5}$$

But

$$\phi(X_1) = \frac{X_1^2 \mu_{(r+2)} + \mu_{(2)} X_1^{r+2}}{2} - \frac{X_1 \mu_{(r+3)} + \mu X_1^{r+3}}{r+3}. \tag{3.6}$$

Hence(3.3) follows. Under H_0

$$\phi(X_1\ ) = \frac{(r+2)!X^2 + 2X^{r+2}}{2} - \frac{(r+3)!X + X^{r+3}}{r+3}. \tag{3.7}$$

Thus it is easy to get σ_0^2 as it is defined in (3.4). When $r = 1$,

$$\delta_{RN}(1) = \frac{1}{2}\mu_{(2)}\mu_{(3)} - \frac{1}{4}\mu\mu_{(4)} \tag{3.8}$$

In this case $\sigma_0 = 22.4$ and the test statistic is

$$\hat{\delta}_{RN}(1) = \frac{1}{n(n-1)} \sum\sum_{i \neq j} \left(\frac{X_i^2 X_j^3}{2} - \frac{X_i X_j^4}{4} \right), \tag{3.9}$$

and

$$\hat{\triangle}_{RN}(1) = \frac{\hat{\delta}_{RN}(1)}{\overline{X}^5}, \tag{3.10}$$

which is quite simple statistics. One can use the proposed test to calculate $\frac{\sqrt{n}\hat{\triangle}_{RN}}{\sigma_0}$ and reject H_0 if $\frac{\sqrt{n}\hat{\triangle}_{RN}}{\sigma_0} \geq Z_\alpha$,where Z_α is the $\alpha-$quantile of the standerd normal distribution.

3.2 Monte Carlo null distribution critical values

In practice,simulated percentiles for small samples are commonly used by applied statisticians and reliabilty analyst.We have simulated the upper percentile values for 95%, 98% and 99%. Table(3.1) presented these percentile values of the statistics$\hat{\triangle}_{RN}(1)$ and the calculations are based on 5000 simulated samples of sizes n =5(1)40. It is clear that the percentile values decrease slowly as sample size increses.

Table (3.1) Critical Values of $\hat{\Delta}_{RN}$

n	95%	98%	99%
5	0.9168	0.9258	0.9412
6	0.9240	0.9388	2.2226
7	0.9432	0.9530	0.9593
8	0.9250	0.9380	0.9416
9	0.9277	0.9363	0.9393
10	0.9348	0.9451	0.9509
11	0.9174	0.9263	0.9310
12	0.9118	0.9219	0.9266
13	0.8992	0.9117	0.9168
14	0.9064	0.9135	0.9174
15	0.8675	0.8808	0.8867
16	0.8788	0.8878	0.8920
17	0.8394	0.8519	0.8573
18	0.8376	0.8464	0.8489
19	0.8341	0.8448	0.8465
20	0.8260	0.8334	0.8375
21	0.8119	0.8255	0.8280
22	0.8049	0.8161	0.8194
23	0.7685	0.7776	0.7832
24	0.7583	0.7713	0.7749
25	0.7651	0.7752	0.7778
26	0.7318	0.7442	0.7467
27	0.7227	0.7344	0.7391
28	0.7223	0.7352	0.7396
29	0.6959	0.7084	0.7148
30	0.6874	0.7005	0.7056
31	0.6836	0.6972	0.7009
32	0.6732	0.6855	0.6901
33	0.6661	0.6785	0.6849
34	0.6572	0.6731	0.6771
35	0.6330	0.6455	0.6490
36	0.6001	0.6182	0.6279
39	0.5905	0.6085	0.6140
40	0.5967	0.6176	0.6218

4 Asymptotic Efficiency

In order to asses how good our proposed family of tests are relative to others in the literature we employ the concept of "Pitman's Asymptotic Relative Efficiency" (PARE) of proposed test. To do this,we need to evaluate the "Pitman's

Asymptotic Efficiency" (PAE) for our tests and then compare this (via taking ratios) to the PAEs of other tests to get the (PARE). Let us first evaluate the (PAE) for our proposed family of tests $\overset{\curlywedge}{\triangle}_{RN}$ which is defined in (3.10). It is known that Pitman's Asymptotic Efficiency(PAE) which is defined as Pitman (1979) is given by

$$PAE(\triangle_r(\theta)) = \frac{\left[\frac{d}{d\theta}\triangle_r(\theta)\right]_{\theta\to\theta_0}}{\sigma_0}. \tag{4.1}$$

Hence, In our case,

$$\triangle'_{RN}(1)|_{\theta\to\theta_0} = \frac{1}{2}(r+2)!\mu'_{(2)}(\theta_0) + \mu'_{(r+2)}(\theta_0) - \frac{\mu'_{(r+3)}(\theta_0)}{r+3} - (r+2)!\mu'(\theta_0). \tag{4.2}$$

But we easly see that

$$\mu_{\theta,(r)} = r\int_0^\infty x^{r-1}\overline{F}_\theta\ (x)dx, \tag{4.3}$$

givining that

$$\mu'_r(\theta) = r\int_0^\infty x^{r-1}\overline{F}'_\theta(x)dx.$$

Hence

$$\begin{aligned}\triangle'_{RN}(1)|_{\theta\to\theta_0} &= (r+2)!\int_0^\infty x\overline{F}'_{\theta_0}(x)dx + (r+2)\int_0^\infty x^{r+1}\overline{F}'_{\theta_0}(x)dx \\ &\quad - \int_0^\infty x^{r+2}\overline{F}'_{\theta_0}(x)dx - (r+2)!\int_0^\infty \overline{F}'_{\theta_0}(x)dx.\end{aligned} \tag{4.4}$$

Three of the most commonly used alternatives with this area:

(i) The linear Failure Rate Family:

$$\overline{F}_\theta(x) = e^{-x-\frac{\theta}{2}x^2}, \quad x \geq 0, \theta \geq 0$$

(ii) The MakehamFamily:

$$\overline{F}_\theta(x) = e^{-x-\theta(e^{-x+x-1})} \qquad x \geq 0, \theta \geq 0$$

(iii) The Weibull Family:

$$\overline{F}_\theta(x) = e^{-x^\theta} \quad x \geq 0, \theta \geq 1$$

Directly calculations most of the efficiencies of these families give:

(i) The linear Failure Rate Family

$$PAE(\triangle_r(\theta)) = (r+1)(r+2)!$$

(ii) The MakehamFamily

$$PAE(\triangle_r(\theta)) = (r+2)!(\frac{3}{4} - (\frac{1}{2})^{r+3}$$

(iii) The Weibull Family

$$PAE(\triangle_r(\theta)) = (r+2)! \left[\sum\nolimits_{i=1}^{r+2} \frac{1}{i} - 1\right].$$

As far as, no other tests have as yet been proposed for testing against $RNBRUE$ alternatives.Thus compare it to others that may be usefule for this problem.Here we choose the tests K^* and $\delta_{(3)}$ which represented by *Kanjo* (1993) and Mugdadi,A and *Ahmad* (2005) respectively.

Direct calculations of the tests K^* and $\delta_{(3)}$ are summarized in Table(4.1). Also, in Table (4.2) we give (PAREs) of K^* and $\delta_{(3)}$ tests whose PAE are mentioned in Table(4.1).

Table (4.1)

Distribution	$\hat{\Delta}_{RN}$	K^*	$\delta_{(3)}$
Linear failure rate	0.535	0.433	0.408
Makham	0.184	0.144	0.039
Weibull	0.223	0.132	0.170

Table (4.2)

Distribution	e_{F_i} $(\hat{\Delta}^{(1)}_{RN}, K^*)$	e_{F_i} $(\hat{\Delta}^{(1)}_{RN}, \delta_{(3)})$
Linear failure	1.24	1.31
Makham	1.28	4.72
Weibull	1.69	1.31

It is cleare from Table 4.2 ,we can see that the statistic $\hat{\Delta}_{RN}(1)$ for $RNBRUE$ is more efficiently than both $K^*$and $\delta_{(3)}$ and for all cases and also simpler. Note that:Since $\hat{\Delta}_{RN}$defines a class (with parameter) r of test statistics,we choose r that the maximizes the PAE of that alternatives.If we take $r = 1$ then our test will have more efficiency than others.

5 Numerical Examples for RNBRUE test

Consider the data in *Abouammoh et al.* (1994). These data represent 40 patients suffering from blood cancer from one of the Ministry of Health Hospital in Saudi Arabia and the ordered life times (in day are 115, 181, 255, 418, 441, 461, 516, 739, 743, 789, 807, 865, 924, 983, 1024, 1062, 1063, 1169, 1191, 1222, 1222, 1251, 1277, 1290, 1357, 1369, 1408, 1455, 1478, 1549, 1578, 1578, 1599, 1603, 1604, 1696, 1735, 1799, 1815, 1852. Using equation (3.9), the value of test statistics, based on the above data is $\hat{\Delta}_{RN} = 0.3047$. This value is smaller than the critical value in Table (3.1).Hence H_0 is note rejected at the signficance level $\alpha = 0.95$ This means that the data set has the exponential property.

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