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Testing Decreasing (Increasing) Variance Residual Class of Life Distributions Using Kernel Method

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Abstract

The decreasing (increasing) variance residual life DVRL (IVRL) class of life distribuation is well known and extensively studied in the literature. A new test is presented for testing exponentiality against DVRL (IVRL) life distributions based on the highly popular "Kernel of curve fitting". The percentiles of these tests are tabulated for sample size $n = 5(1)40$. The proposed test is simple to calculate , dos not depend on the choice of either the bandwidth or the kernal, asymptotically normal and performs well in terms of power and Pitman asymptotic efficiencies for several alternatives.

Keywords: Kernel method; DVRL; IVRL; Efficiency; Exponentiality; asymptotic normality

1 Introduction

Let X be an absolutly continuous non negative random variable with distribution function F, survival function $\overline{F} = 1 - F$. Studies on F as exponential versus that it belongs to a nonparametric class of life distributions have continued over the past three decades or more. Of the most common and practical are the increasing failure rate (IFR), increasing failure rate average (IFRA), new better than used (NBU), new better than used in expectation (NBUE) and decreasing mean residual life (DMRL). An ordering of live variable that proved useful in producing classes of life distributions is due to Stoyan (1983),

Bhattacharjee (1991) for definations and properties.
The variance residual life (VRL) function is useful in many areas including biometry, actuarial science and reliability. Let X, denote the life time of an equipment with distribution function $F(x)$, survival function $\overline{F} = 1 - F$, mean life $\mu = \int_0^\infty \overline{F}(u)du$ and variance $\sigma^2 = var(X)$ both assumed finite. The mean residual life (MRL) and the variance residual life (VRL) functions are defined as the following:

$$\mu(x) = E\{X - x|X \geq x\} = \frac{\int_x^\infty \overline{F}(u)du}{\overline{F}(x)}, \qquad x \geq 0, \tag{1.1}$$

and

$$\sigma^2(x) = var\{X - x|X \geq x\} = var\{X|X \geq x\}. \tag{1.2}$$

A distribution function F is said to be a decereasing (increasin) variance residual life DVRL (IVRL) if $\sigma^2(t)$ is nonincreasing (nondecreasing) function of t, $t \geq 0$. Consider $E[U^2|x] = -\int_0^\infty u^2 d\overline{F}(u/x)$, integrating by parts one has

$$\sigma^2(x) + \mu^2(x) = \frac{2}{\overline{F}(x)} \int_x^\infty \int_y^\infty \overline{F}(t)dtdy,$$

let $\nu(y) = \int_y^\infty \overline{F}(t)dt$, and $r(x) = \frac{f(x)}{\overline{F}(x)}$ then,

$$\sigma^2(x) + \mu^2(x) = \frac{2}{\overline{F}(x)} \int_x^\infty \nu(y)dy, \tag{1.3}$$

$$\mu(x) = \frac{\nu(x)}{\overline{F}(x)}, \tag{1.4}$$

and

$$\frac{d\mu(x)}{dx} = -1 + r(x)\mu(x). \tag{1.5}$$

Differentiating (1.3) with respect to x, we have

$$\frac{d\sigma^2(x)}{dx} = \frac{2f(x)}{\overline{F}^2(x)} \int_x^\infty \nu(y)dy - \frac{2\nu(x)}{\overline{F}(x)} - 2\mu(x)\frac{d\mu(x)}{dx}. \tag{1.6}$$

Using (1.4),(1.5) in (1.6) we obtain

$$\frac{d\sigma^2(x)}{dx} = r(x)(\frac{2}{\overline{F}(x)} \int_x^\infty \nu(y)dy) - 2\mu(x) - 2\mu(x)(-1 + r(x)\mu(x)). \tag{1.7}$$

Using (1.3) in (1.7), we obtain

$$\frac{d\sigma^2(x)}{dx} = r(x)[\sigma^2(x) - \mu^2(x)],$$

Since $\overline{F}(x)$ is DVRL (IVRL) then, $\sigma^2(x) \leq (\geq)\mu^2(x)$ imlies $\sigma^2(x) + \mu^2(x) \leq (\geq)2\mu^2(x)$, hence

$$\frac{2}{\overline{F}(x)}\int_x^\infty \nu(y)dy \leq (\geq)2\mu^2(x).$$

Now, we have the following definition:

Definition (1.1): A life distribution F, with $F(0) = 0$ and its survival function $\overline{F}$ is said to have DVRL (IVRL) if

$$\frac{1}{\overline{F}(x)}\int_x^\infty \nu(y)dy \leq (\geq)\mu^2(x), \tag{1.8}$$

or

$$\overline{F}(x)\int_x^\infty \nu(y)dy \leq (\geq)\nu^2(x). \tag{1.9}$$

Launcer (1984), Gupta (1987)and Gupta et al (1987) studied characterization of this class and used it to find better bounds on moments and survival function. Gupta and Kirmani (2000) charactrized the distribution to the univariate and the bivariate cases. Testing exponentiality versus (IFR, IFRA, NBU, NBUE, DMRL) classes have got a good deal of attention in the literature. For this literature we refer the reader to the surveys by Doksum and Yandell (1984), Hendi and Abouammoh(2001) and Abu-Youssef (2002) among others. All of these approaches are based on devising a measure of departure from H_o in favor of H_1, then estimating this measure empircically. The resulting statistics are all versions of the well known U-statistics class. As with all procedure based on the emprical distribution functon, the procedures mentioned above have little robustness and may be deficient. Thus one may be intersted in a different approach that enjoys more robustness and may be more efficient. One such approach that proved viable in several testing problems is to use nonparametric density estimation . The " Kernel method " is used in some general goodness of fit problems successfully, cf Ahmad and Li (1997a,b), Fan and Li (1995), Hong and White (1995) and Ahmad et al.(1999) among many others. In this paper, we use this approach by defining a measure of departure from H_o that depend on the pdf $f(x)$. Thus the emprical version of these measure require estimating $f(x)$ and thus one may use the celebrated "Kernel method". For a background material on this method, we refer to the books by Scott (1992) and Jones and Wand (1995). Using Kernel methods in reliability appears in early work of Watson and Leadbetter (1964) and Ahmad (1976) among others. While using kernel method for testing NBUC, NBUE and HNBUE are given by Ahmad, et al.(1999). The exponential distribution is the only distribution when the equality is obtained in (1.7) . Hence we test $H_0 : F$ is exponential (μ) against $H_1 : F$ is DVRL (IVRL) and not exponential. In order to test H_0 against H_1 we may use the following measure of departure from H_0:

$$\delta_{KV} = \int_0^\infty f(x)\{\nu^2(x) - \overline{F}(x)\int_x^\infty \nu(y)dy\}dF(x). \tag{1.10}$$

We have

$$\int_x^\infty \nu(y)dy = -x\nu(x) - \frac{x^2}{2}\overline{F}(x) + \frac{1}{2}\int_x^\infty y^2 dF(y). \tag{1.11}$$

From (2.2) and (2.3), we obtain

$$\delta_{KV} = \int_0^\infty f(x)[\nu^2(x) + x\nu(x)\overline{F}(x) + \frac{1}{2}x^2(\overline{F}(x))^2 - \frac{1}{2}\overline{F}(x)\int_0^\infty y^2 I(y > x)dF(y)]dF(x), \tag{1.12}$$

where,

$$I(y > x) = \begin{cases} 1, & y > x \\ 0, & O.W. \end{cases}$$

Note that under $H_0 : \delta_{KV} = 0$, while under $H_1 : \delta_{KV} > (<)0$. To estimate δ_{KV}, let $X_1, X_2, \ldots, X_n$ be a random sample from F, let $\overline{F}_n(x) = \frac{1}{n}\sum_{j=1}^n I(X_j > x)$ denote the empirical distribution of the survival function $\overline{F}(x), dF_n(x) = \frac{1}{n}$, $\nu(x)$ is estimated by $\hat{\nu}_n(x) = \frac{1}{n}\sum_{k=1}^n (X_k - x)I(X_k > x)$, μ is estimated by sample mean $\overline{X}$ and pdf $f(x)$ is estimated by $\hat{f}_n(x) = \frac{1}{na_n}\sum_{l=1}^n k(\frac{x-X_l}{a_n})$, where $k(.)$ be a known pdf, symmetric and bounded with 0 mean and variance $\sigma_k^2 > 0$. Symmetric uniform, normal, double exponential are examples of such pdf. Let $\{a_n\}$ be a sequence of reals such that $a_n \to 0$ and $na_n \to \infty$ as $n \to \infty$. Other conditions on k and a_n will be stated when needed. We propose to estimate δ_{KV} by

$$\begin{aligned}\hat{\delta}_{KV_n} &= \int_0^\infty \hat{f}_n(x)[\hat{\nu}_n^2(x) + x\hat{\nu}_n(x)\overline{F}_n(x) + \frac{1}{2}x^2\overline{F}_n^2(x) \\ &\quad - \frac{1}{2}\overline{F}_n(x)\int_0^\infty y^2 I(y > x)dF_n(y)]dF_n(x). \end{aligned}\tag{1.13}$$

or

$$\begin{aligned}\hat{\delta}_{KV_n} &= \frac{1}{n^4 a_n}\sum_{i=1}^n\sum_{j=1}^n\sum_{k=1}^n\sum_{l=1}^n k(\frac{X_i - X_l}{a_n})[(X_j - X_i)(X_k - X_i) \\ &\quad + X_i(X_k - X_i) + \frac{1}{2}X_i^2 - \frac{1}{2}X_k^2]I(X_j > X_i)I(X_k > X_i) \end{aligned}\tag{1.14}$$

i.e.

$$\hat{\delta}_{KV_n} = \frac{1}{n^4 a_n} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} k(\frac{X_i - X_l}{a_n})[X_j X_k. \\ . + \frac{1}{2}X_i^2 - X_j X_i - \frac{1}{2}X_k^2] I(X_j > X_i) I(X_k > X_i). \quad (1.15)$$

let us rewrite(1.15) as

$$\hat{\delta}_{KV_n} = \frac{1}{n(n-1)(n-2)(n-3)} \sum \sum_{i \neq j \neq k \neq l} \sum \sum \phi_n(X_i, X_j, X_k, X_l). \quad (1.16)$$

To make the test scale invariant, we take

$$\hat{\Delta}_{KV_n} = \frac{\hat{\delta}_{KV_n}}{\overline{X}^2}, \quad (1.17)$$

with measure of departure $\Delta_{KV} = \frac{\delta_{KV}}{\mu^2}$. Set

$$\phi_n(X_1, X_2, X_3, X_4) = \frac{1}{a} k(\frac{X_1 - X_4}{a}) \left[X_2 X_3 + \frac{1}{2}X_1^2 - X_2 X_1 - \frac{1}{2}X_3^2 \right] \\ I(X_2 > X_1) I(X_3 > X_1), \quad (1.18)$$

and define the symmetric kernel

$$\xi(X_1, X_2, X_3, X_4) = \frac{1}{4!} \sum_R \phi_n(X_{i1}, X_{i2}, X_{i3}, X_{i4}),$$

where the sum over all arrangements of $(X_1, X_2, X_3$ and $X_4)$. Then $\hat{\delta}_{KV_n}$ is equivalent to the U-statistic. In section 2, condition under which $\sqrt{n}(\hat{\Delta}_{KV_n} - \Delta_{KV})$ is asymptotically normal are given , the null and nonnull variance are obtained. The test basecd on $\hat{\Delta}_{KV_n}$ is shown to be consistent and its relative efficiencies to other test and its power estimate for 95% percentile are given for some well known alternatives. Finally small samles Monte Carlo critical values are also given.

2 Testing against DVRL alternatives

2.1 The test procedure

The "kernel method" was used for testing exponentiality against some classes of life distributions cf. Ahmad et al. (1999). In this section, we derive a kernel-test for $H_0 : F$ is exponential (μ) against $H_1 : F$ is DVRL and not exponential. First, we prove the following

Theorem 2.1. If $na_n^4 \to 0$ as $n \to \infty$, if f has bounded second derivative and if $V(\psi_n(X_1)) < \infty$, where $\psi_n(X_1)$ is as given (2.7), then $\sqrt{n}(\Delta_{KV_n} - \Delta_{KV})$ is asymptotically normal with mean 0 and variance $\lim_n V(\psi_n(X_1)$. Under H_0, the variance $= 0.0714$

The following simple lemma is needed in the proof of theorem 2.1.

Lemma 2.1. Let $\theta_n = E[\hat{\Delta}_{KV_n}]$, then

$$\theta_n = \int_0^\infty E[\hat{f}_n(x)][\nu^2(x) + x\nu(x)\overline{F}(x) + \frac{1}{2}x^2(\overline{F}(x))^2 - \frac{1}{2}\overline{F}(x)\int_0^\infty y^2 I(y > x)dF(y)]dF(x), \quad (2.1)$$

Proof. Note that $E\hat{f}_n(x) = \frac{1}{a}\int K(\frac{x-y}{a})f(y)dy$. Set $g_n(x) = E\hat{f}_n(x)$, thus

$$E\hat{\Delta}_{KV_n} = \theta_n = E[\phi_n(X_1, X_2, X_3, X_4). \quad (2.2)$$

where

$$\phi_n(X_1, X_2, X_3, X_4) = (1/a)K(\frac{X_1 - X_4}{a})\left[X_2X_3 + \frac{1}{2}X_1^2 - X_1X_2 - \frac{1}{2}X_3^2\right] I(X_2 > X_1)I(X_3 > X_1).$$

Hence

$$\begin{aligned}\theta_n &= Eg_n(X_1)[\nu^2(X_1) + X_1\nu(X_1)\overline{F}(X_1) + \frac{1}{2}X_1^2(\overline{F}(X_1))^2 - \\ &\quad \frac{1}{2}\overline{F}(X_1)\int_0^\infty y^2 I(y > x)dF(y)] \\ &= \int_0^\infty g_n(x)[\nu^2(x) + x\nu(x)\overline{F}(x) + \frac{1}{2}x^2(\overline{F}(x))^2 \\ &\quad -\frac{1}{2}\overline{F}(x)\int_0^\infty y^2 I(y > x)dF(y)]dF(x). \quad (2.3)\end{aligned}$$

Proof theorem 2.1. Note that

$$\sqrt{n}(\hat{\Delta}_{KV_n} - \Delta_{KV}) = \sqrt{n}(\hat{\Delta}_{KV_n} - \theta_n) + \sqrt{n}(\theta_n - \Delta_{KV}) \quad (2.4)$$

But

$$\begin{aligned}E\hat{f}_n(x) &= \frac{1}{a}\int k(\frac{x-y}{a})f(y)dy = \int k(w)f(x - aw)dw \\ &\simeq f(x) + \frac{a^2}{2}f''(x)\sigma_k^2,\end{aligned}$$

under the condition assumed on k. Hence

$$\begin{aligned}\theta_n^{(1)} &\simeq \delta_F^{(1)} + \frac{a^2}{2}\sigma_k^2 \left\{ \int_0^\infty f''(x)[\nu^2(x) + x\nu(x)\overline{F}(x) + \frac{1}{2}x^2(\overline{F}(x))^2 \right. \\ &\left. -\frac{1}{2}\overline{F}(x)\int_0^\infty y^2 I(y > x)dF(y)]dF(x) \right\}. \qquad (2.5)\end{aligned}$$

Thus $\sqrt{n}(\theta_n^{(1)} - \Delta_F^{(1)}) = O(a^2\sqrt{n}) = 0(1)$, by assumptions. Note also $\hat{\Delta}_{KV_n}$ is unbiased estimate of $\theta_n = E\hat{\Delta}_{KV}$ and is asymptotically unbiased estimate of Δ_{KV_n}. Next, write

$$\begin{aligned}\sqrt{n}(\hat{\Delta}_{KV_n} - \theta_n) &= \sqrt{n}\{(\frac{1}{n}\sum_{i=1}^n \psi_n(X_i)) + (n(n-1)(n-2)(n-3))^{-1} \\ &\sum\sum\sum_{i\neq j\neq l\neq k} \xi_n(X_i, X_j, X_k, X_l,)\}. \qquad (2.6)\end{aligned}$$

where

$$\begin{aligned}\psi_n(X_1) &= E[\phi_n(X_1, X_2, X_3, X_4)|X_1] + E[\phi_n(X_2, X_1, X_3, X_4)|X_1] \\ &+E[\phi_n(X_2, X_3, X_1, X_4)|X_1] \\ &+E[\phi_n(X_2, X_3, X_4, X_1)|X_1] - 4\theta_n, \qquad (2.7)\end{aligned}$$

and

$$\xi_n(X_1, X_2, X_3, X_4) = \phi_n(X_1, X_2, X_3, X_4) - \psi_n(X_1) - 3\theta_n. \qquad (2.8)$$

Now, by Layaponouff's central theorem, the first term in the right hand side of (2.6) is asymptotically normal if $L_n = \frac{E[\psi_n(X_1)]^{2+\delta}}{\sqrt{n}}[V(\psi_n(X_1))]^{1+\delta/2} \to 0$ as $n \to \infty$. Now using (2.4) it is easy to see for large n

$$\begin{aligned}E[\phi_n(X_1, X_2, X_3, X_4)|X_1] &= f(X_1)\{(\int_{X_1}^\infty udF(u))^2 + \frac{1}{2}X_1^2\overline{F}^2(X_1) \\ &-X_1\overline{F}(X_1)\int_{X_1}^\infty udF(u) \\ &-\frac{1}{2}\overline{F}(X_1)\int_{X_1}^\infty u^2dF(u)\}, \qquad (2.9)\end{aligned}$$

$$\begin{aligned}E[\phi_n(X_2, X_1, X_3, X_4)|X_1] &= \int_0^{X_1} f^2(y)[X_1)\int_y^\infty udF(u) + \frac{1}{2}y^2\overline{F}(y) \\ &-X_1y\overline{F}(y) - \frac{1}{2}\int_y^\infty u^2dF(u)]dy, \qquad (2.10)\end{aligned}$$

$$\begin{aligned}E[\phi_n(X_2, X_3, X_1, X_4)|X_1] &= \int_0^{X_1} f^2(y)[X\int_y^\infty udF(u) + \frac{1}{2}y^2\overline{F}(y) \\ &-y\int_y^\infty udF(u) - \frac{1}{2}X_1^2\overline{F}(y)]dy \qquad (2.11)\end{aligned}$$

observe that $E[\phi_n(X_2, X_3, X_4, X_1)|X_1]$ has the same representation as (2.9). Set $\eta(X_1)$ to be the sum of twice of right hand side of (2.9) plus that of (2.10) and (2.11). Thus

$$\psi_n(X_1) = \eta(X_1) + O_p(a^2) \text{ say}, \tag{2.12}$$

Then, $V(\psi_n(X_1)) = Var(\eta_1(X_1)) + O(a^2)$, and for $p > 2$, $E|\psi_n(X_1)|^p \leq C_p E|\eta(X_1)|^p = O(1)$. Hence, $L_n \to 0$ as $n \to \infty$ provided that $na^4 \to 0$ as $n \to \infty$. Next, look at

$$\begin{aligned} & E\left[\frac{\sqrt{n}}{n(n-1)(n-2)(n-3)} \sum\sum_{i\neq j\neq k\neq l}\sum\sum \xi_n(X_i, X_j, X_k, X_l)\right] \\ = & \frac{1}{n(n-1)^2(n-2)^2(n-3)^2} \sum\sum_{i\neq j\neq k\neq l}\sum\sum \\ & E[\xi_n(X_i, X_j, X_k, X_l) \times \xi_n(X_i, X_j, X_k, X_l)] \\ = & \frac{1}{(n-1)} E\xi_n^2(X_1, X_2, X_3, X_4) = O(na)^{-1} = O(1). \end{aligned} \tag{2.13}$$

Under H_0, $\overline{F}(x) = e^{-x}$and $\eta(X_1) = \frac{-16}{27} + \frac{7}{9}X - \frac{X^2}{6} + \frac{16}{27}e^{-3X}$. Thus $E_0[\eta_(X_1)] = 0$ and $\sigma_0^2 = Var(\eta(X_1) = 0.0714$ by direct calculation. The theorem is proved.

2.2 Asymptotic Relative Efficiency

To asses how good this procedure is relative others, we use the concept of "Pitman's asymptotic relative efficiency" (PARE). To do this we need to evaluate the "Pitman's asymptotic efficiency"(PAE) of our test $\hat{\Delta}_{KV_n}$ in (1.15) and compare this (by taking ratios) with PAE of other tests to get PARE. For the proposed test the PAE is given by

$$\begin{aligned} \frac{1}{\sigma_0}\left\{\frac{d}{d\theta}\Delta_{Kv}(\theta)\right\}_{\theta\to\theta_0} &= \frac{1}{\sigma_0}\left\{\frac{d}{d\theta}\int_0^\infty f_\theta(x)[\nu_\theta^2(x) + x\nu_\theta(x)\overline{F}_\theta(x) + \frac{1}{2}x^2\overline{F}_\theta(x)\right. \\ & \left. -\frac{1}{2}\overline{F}_\theta(x)\int_x^\infty y^2 dF_\theta(y)]dF_\theta(x)\right\}_{\theta\to\theta_0}. \end{aligned} \tag{2.1}$$

We compare our test statistic $\hat{\Delta}_{KV_n}$ to the test statistics V^* and $\hat{\Delta}_n$ presented by Hollander and Proschan (1975) and Abu-Youssef (2002) respectively. Two of the most commonly used alternatives [see Hollander and Proschan, 1975]

(i) Linear failure rate	: $\bar{F}_\theta = e^{-x-\frac{\theta x^2}{2}}$,	$x > 0, \theta > 0$
(ii) Weibull	: $\bar{F}_\theta = e^{-x^\theta}$,	$x > 0, \theta > 0$.

Direct calculations of the PAE of the tests V^*, $\hat{\Delta}_n$ and $\hat{\Delta}_{v_n}$ are summarized in Table 2.1.

Table 2.1 PAE of $\hat{\Delta}_{v_n}$, V^* and $\hat{\Delta}_n$

Distribution	V^*	$\hat{\Delta}^n$	$\hat{\Delta}_{Kv_n}$
Linear failure rate (F_1)	0.906	0.9192	0.9356
Weibull (F_2)	0.846	0.71	1.8313

In Table 2.2 we give PARE's of $\hat{\Delta}_{v_n}$ with respect to V^* and $\hat{\Delta}_n$ and whose PAE are mentioned in in Table 2.1.

Table 2.2 PARE of $\hat{\Delta}_{v_n}$ with respect toV^* and $\hat{\Delta}_n$

Distribution	$e_{F_i}(\hat{\Delta}_{v_n},V^*)$	$e_{F_i}(\hat{\Delta}_{v_n},\hat{\Delta}_n)$
Linear failure rate (F_1)	0.99	1.007
Weibull (F_2)	2.236	2.665

From Table 2.2 it appears that the test statistic $\hat{\Delta}_{KV_n}$ performs well for $\overline{F}_1$ and $\overline{F}_2$ and it is more efficient than both $\hat{\Delta}_n$ and V^*

2.3 The Power Estimates

The power of the test statistics $\hat{\Delta}_{Kv_n}$ is considerd for 95% percentille in Table 2.3 for three of most commonly used alternatives [see Hollander and Proschan (1975)], they are

(i) Linear failure rate : $\bar{F}_\theta = e^{-x-\frac{\theta x^2}{2}}$, $x > 0, \theta > 0$

(ii) Makeham : $\bar{F}_\theta = e^{-x-\theta(x+e^{-x}-1)}$, $x \geq 0, \theta > 0$

(iii) Weibull : $\bar{F}_\theta = e^{-x^\theta}$, $x > 0, \theta > 0$.

These distributions are reduced to exponential distribution for appropriate values of θ . To conduct the test, calculate $\sqrt{\frac{1000n}{714}}\hat{\delta}_{KV_n}$ and reject H_0 if this value exceeds Z_α, the standard normal variate at level α.

Table 2.3 Power Estimate of $\hat{\Delta}_{KV_n}$

Distribution	θ	Sample Size		
		n=10	n=20	n=30
F_1	2	0.605	0.578	0.546
Linear failure	3	0.802	0.863	0.911
rate	4	0.926	0.981	0.988
F_2	2	0.390	0.299	0.234
Makham	3	0.572	0.591	0.569
	4	0.732	0.792	0.833
F_3	2	0.249	0.157	0.147
Weibull	3	0.491	0.603	0.741
	4	0.545	0.606	0.615

2.4 Monte-Carlo Null Distribution Critical Points

In pratice, simulated percentiles for small samples are commonly used by applied statistications and reliability analyst. We have simulated the upper percentile points for 95%, 98%, and 99%. Table 2.4 gives these percentile points of the statistic $\hat{\Delta}_{KV_n}$ in (1.15) and the calculations are based on 5000 simulated samples of sizes $n = 6(2)40$. The percentile values change slowly as n increases. To conduct the test, calculate $\sqrt{\frac{1000n}{714}}\hat{\delta}_{KV_n}$ and reject H_0 if this value exceeds Z_α, the standard normal variate at level α.
Note that: since the above procedure is independent of choosing a_n and k, we select k to be the standard normal and those a_n by the normal scale rule (cf. Jones and Wand (1995) p.60).

Table (2.4) Critical Values of $\hat{\delta}^{(1)}_{R_n}$

n	95%	98%	99%
6	0.1559	0.2239	0.3070
8	0.1051	0.1624	0.2019
10	0.0557	0.1096	0.1323
12	0.0647	0.0858	0.1155
14	0.0595	0.0737	0.0819
16	0.0562	0.0784	0.0906
18	0.0488	0.0646	0.0768
20	0.0440	0.0590	0.0652
22	0.0408	0.0530	0.0645
24	0.0418	0.0537	0.0623
26	0.0390	0.0489	0.0640
28	0.0366	0.0450	0.0532
30	0.0362	0.0453	0.0518
32	0.0343	0.0444	0.0449
34	0.0326	0.0413	0.0444
36	0.0313	0.0416	0.0454
38	0.0279	0.0341	0.0406
40	0.0281	0.0372	0.0420

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