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A Split-Radix DCT Base Structure for Linear Phase Paraunitary Filter Banks

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Abstract

In this study, base on traditional lapped transform's properties, inherit from some of the special characteristic properties of discrete cosine transform, trigonometric identities and the structure of linear phase filter banks. We present some theoretic properties in the generalized structure of linear phase paraunitary filter bank and build some examples corresponding to these properties. These properties not only improve the complexity in the coding scheme of a signal before quantized but also because of the recursive property and adopt the Split-Radix Algorithm of the discrete cosine transform, improve an effective and simplify the lapped structure of linear phase paraunitary filter banks.

Keywords: discrete cosine transform, lapped transform, split radix algorithm, latticed factorization.

INTRODUCTION

There have been a tremendous researches in the field of filter banks (FBs) and multirate systems in the last 30 years [1, 2, 3, 6, 8, 9, 10]. These systems provide new and effective ways to represent signals for processing, understanding, and compression purposes. A system, that there is no information loss at the processing stage, such that the output $\hat{x}[n]$ is a purely delayed version of the input $x[n]$, i.e., $\hat{x}[n] = x[n-l]$, is called perfect reconstruction (PR) filter banks. Linear phase (LP) systems allow us to use simple symmetric extension methods to accurately handle the boundaries of finite-length signals. Furthermore, the LP property can be exploited, leading to faster and more efficient FB implementation. From this point on, all of the FBs in discussion here are LP perfect reconstruction filter banks (LPPRFB).

There has been a great amount of research on pre and post processing algorithms for image and video compression systems. Both classes of algorithms share one common goal: to eliminate or reduce the severity of coding artifacts in the reconstructed signal. The design of linear-phase filter bank has been increasing interest and several methods have been developed so far. Recently, in [8], Soman *et al*. first developed a complete factorization for even-channel LPPUFBs. Tran *et al.*

[10] proposed the design of M-channel linear phase perfect reconstruction filter bank (LPPRFB) through lattice structure, which factorizes the polyphase matrix into a cascade of elementary building blocks. Besides fast implementation structure with minimal number of delay elements, the factorization introduced in [10] also structurally enforces both linear phase and perfect reconstruction properties. Moreover, based on the lattice factorization, Tran *et al.* [10] presented several design examples of LPPRFB, which are quite efficient in image compression, particularly for highly textured images. Thus, further investigation in LPPRFB is motivated. Lu G. and K.K. Ma, [1] present a simplified version of lattice factorization for LPPRFB of [10]. The new lattice factorization substantially reduces the number of free parameters while covering the same class of LPPRFB as shown in [10]. Consequently, the new lattice structure can reduce the search space in nonlinear optimization for the design of filter bank and lower the computation cost in hardware implementation. In [10], Lu G. and K.K. Ma propose a new structure for pairwise mirror image (PMI) property to LPPUFBs, with even-channel, which is a simplified version of the lattice in [9]. The new structure spans the same class of PMI LPPUFBs as the original lattice, while the numbers of free parameters are significantly reduced. Through this way, better results with faster convergence in the optimization can be achieved.

The Type-II discrete cosine transform (DCT-II), Rao [7], has found wide applications in image and video compression, due to its excellent energy compaction capability and to the existence of numerous fast implementations. However, coding schemes exhibit annoying blocking artifacts at low bit rates. The lapped orthogonal transform (LOT) [9] provides a solution to this problem via post-processing of the DCT coefficients. The basis functions of the LOT cover two data blocks. Further suppression of blocking artifacts can be achieved by employing multistage post-processing, as in the generalized LOT (GenLOT) [6]. In [4], Jie presents a general structure of LPPUFB with both pre and post-processing modules added to the DCT.

Notation-wise, vectors and matrices are denoted by boldfaced characters, special matrices $\mathbf{I}, \mathbf{J}, \mathbf{0}, \mathbf{D}$ represents, respectively, the identity, the reversal identity, the zero and the diagonal matrix with entry $+1$ when the corresponding filter is symmetric, and -1 while the corresponding filter is antisymmetric. Capital letters $\boldsymbol{M}, \boldsymbol{L}, \boldsymbol{K}$ denote respectively the number of the channels, the filter length, and the overlapping factor.

II. LAPPED STRUCTURE OF LPPUFB

For an $\boldsymbol{M}$-channel $\boldsymbol{KM}$-tap general structure of LPPUFB system is developed in 1993, [8], by which the analysis polyphase matrix $\mathbf{E}(z)$ is shown in Figure 1 (a), and can be factorized as

$$\mathbf{E}(z) = \prod_{\boldsymbol{i}=\boldsymbol{K}-1}^{1} \mathbf{G}_{\boldsymbol{i}}(z)\mathbf{G}_0 \quad (1)$$

where matrices $\mathbf{G}_0$ and $\mathbf{G}_{\boldsymbol{i}}(z)$, $\boldsymbol{i} = 1, \cdots, \boldsymbol{K}\text{-}1$ are called the initial and $\boldsymbol{i}$th propagate stage of the system, are defined as

$$\mathbf{G}_0 = \mathrm{diag}(\mathbf{U}_0, \mathbf{V}_0)\hat{\mathbf{W}} \tag{2}$$

$$\mathbf{G}_i(z) = \mathrm{diag}(\mathbf{U}_i, \mathbf{V}_i)\mathbf{W}\boldsymbol{\Lambda}_{\boldsymbol{a}}(z)\mathbf{W} \tag{3}$$

$$\boldsymbol{\Lambda}_{\boldsymbol{a}}(\mathrm{z}) = \mathrm{diag}(z\mathbf{I}, \mathbf{I}) \tag{4}$$

matrices $\mathbf{U}_i$ and $\mathbf{V}_i$ in (3) and (4) are arbitrary $(\boldsymbol{M}/2)\times(\boldsymbol{M}/2)$ orthogonal matrices, matrices $\mathbf{W}$ and $\hat{\mathbf{W}}$ in the (3) and (4) represent two kinds of $\boldsymbol{M}\times\boldsymbol{M}$ butterfly matrix, given by

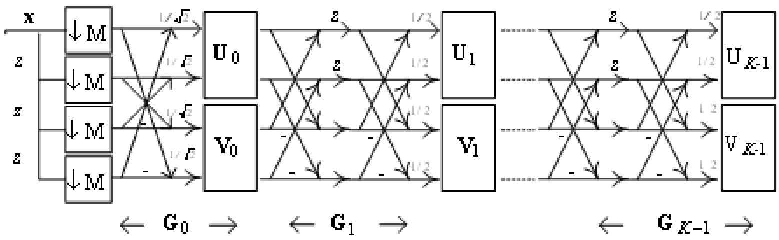
$$\mathbf{W} = \frac{1}{\sqrt{2}}\begin{bmatrix} \mathbf{I} & \mathbf{I} \\ \mathbf{I} & -\mathbf{I} \end{bmatrix}, \qquad \hat{\mathbf{W}} = \frac{1}{\sqrt{2}}\begin{bmatrix} \mathbf{I} & \mathbf{J} \\ \mathbf{J} & -\mathbf{I} \end{bmatrix} \tag{5}$$

In which, $\mathbf{I}$ and $\mathbf{J}$ are $(\boldsymbol{M}/2)\times(\boldsymbol{M}/2)$ identity and reversal identity matrix, respectively.

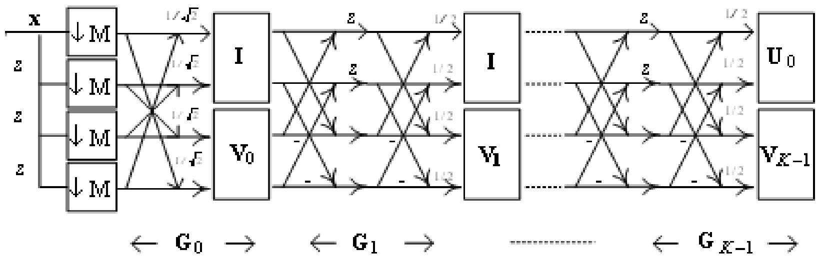
Recently, the reduction of the complexity had been improved significantly, after Gan [2] shows that either all $\mathbf{U}_i$ or $\mathbf{V}_i$ in $\mathbf{G}_i(z)$ can be eliminated, and the completeness of the structure is still guaranteed, that is, $\mathbf{G}_i(z)$, $\boldsymbol{i} = 1,\cdots,\boldsymbol{K}\text{-}1$ in (4) can be reduced to

$$\mathbf{G}_i(z) = \mathrm{diag}(\mathbf{I}, \mathbf{V}_i)\mathbf{W}\Lambda_{\boldsymbol{a}}(z)\mathbf{W} \tag{6}$$

we show this improvement in Figure 1(b).



(a)



(b)

Figure 1: An M -channel KM -tap analysis system: (a) The initial structure. (b) The improved structure.

By the orthogonal property of $\mathbf{U}_0$ and $\mathbf{V}_i$, and with the commutative property between the diagonal matrix $\mathrm{diag}(\mathbf{U}_0,\mathbf{U}_0)$ and the butterfly matrix $\mathbf{W}$ and the advance chain $\mathbf{\Lambda}_a(\mathrm{z})$, the matrix $\mathbf{U}_0$ in $\mathbf{G}_0$ can actually be moved to propagate stage $\mathbf{G}_N(z)$ for $1 \le N \le K-1$ rather than $\mathbf{G}_0$. We propose it as the following proposition.

Proposition 1: The analysis polyphase matrix $\mathbf{E}(z)$ of any $M \times KM$ LPLUFB can be factorized as $\mathbf{E}(z) = \prod_{i=K-1}^{1} \mathbf{G}_i(z)\mathbf{G}_0$, where

$$\mathbf{G}_{K-1}(z) = \mathrm{diag}(\mathbf{U}_0, \mathbf{V}_{K\text{-}1})\mathbf{W}\Lambda_a(z)\mathbf{W} \tag{7}$$

$$\mathbf{G}_0 = \mathrm{diag}(\mathbf{I}, \mathbf{V}_0)\hat{\mathbf{W}} \tag{8}$$

and

$$\mathbf{G}_i(z) = \mathrm{diag}(\mathbf{I}, \mathbf{V}_i)\mathbf{W}\Lambda_a(z)\mathbf{W} \text{ for } 1 \le i \le N-1 \tag{9}$$

In advance, to simplify the structure, turn all $\mathbf{W}$ in $\mathbf{G}_i(z)$ by $\hat{\mathbf{W}}$ through the relation $\mathbf{W} = \mathrm{diag}(\mathbf{I},\mathbf{J})\hat{\mathbf{W}}\mathrm{diag}(\mathbf{I},\mathbf{J})$, this makes each propagate stage $\mathbf{G}_i(z)$, $1 \le i \le K-2$ with the same the structure form, and hence, a new expression of $\mathbf{E}(z)$ is shown in Figure 2, and states in the following proposition:

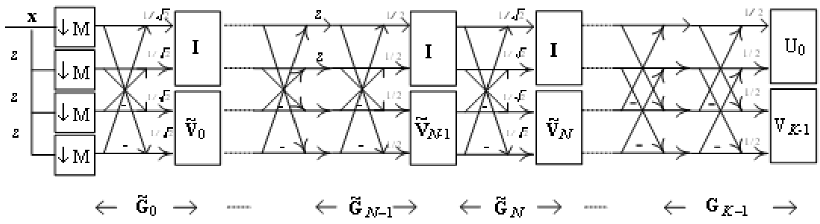


Figure 2: The simplified lapped transform: all $\mathbf{W}$ in $\mathbf{G}_i(z)$, $i = 1,\cdots,N-1$ is replaced by $\hat{\mathbf{W}}$.

Proposition 2: The analysis polyphase matrix of any $M \times KM$ LPLUFB $\mathbf{E}(z)$ can be factorized as

$$\mathbf{E}(z) = \prod_{i=K-1}^{N+1}\mathbf{G}_i(z)\tilde{\mathbf{G}}_N \prod_{j=N-1}^{0}\tilde{\mathbf{G}}_j(z), \tag{10}$$

where $\mathbf{G}_i(z)$ for $i = N+1, N+2, \cdots, K-1$ is the same as (3), and the modified propagate stages

$$\tilde{\mathbf{G}}_N = \mathrm{diag}(\mathbf{U}_0, \mathbf{V}_N\mathbf{J})\hat{\mathbf{W}}; \tag{11}$$

$$\tilde{\mathbf{G}}_0 = \Lambda_a(z)\hat{\mathbf{W}}\mathrm{diag}(\mathbf{I}, \mathbf{J}\mathbf{V}_0)\hat{\mathbf{W}}; \tag{12}$$

$$\tilde{\mathbf{G}}_j = \Lambda_a(z)\hat{\mathbf{W}}\mathrm{diag}(\mathbf{I}, \mathbf{J}\mathbf{V}_i\mathbf{J})\hat{\mathbf{W}} \text{ for } j = 1,2,\cdots,N-1. \tag{13}$$

Proof: After modified $\mathbf{W}$ by $\hat{\mathbf{W}}$, each stage $\mathbf{G}_j(z)$, $j=1,2,\cdots,N$ is of the form

$$\mathbf{G}_i(z) = \operatorname{diag}(\mathbf{I},\mathbf{V}_i\mathbf{J})\hat{\mathbf{W}}\boldsymbol{\Lambda}_a(z)\hat{\mathbf{W}}\operatorname{diag}(\mathbf{I},\mathbf{J}),\ i=1,2,\cdots,N-1 \tag{14}$$

Start from $i=N$, moving $\operatorname{diag}(\mathbf{I},\mathbf{J})$ of $\mathbf{G}_N(z)$ to the next stage $\mathbf{G}_{N-1}(z)$ and product with the first block $\operatorname{diag}(\mathbf{I},\mathbf{V}_{N-1}\mathbf{J})$ of $\mathbf{G}_{N-1}(z)$, this makes

$$\mathbf{G}_{N-1}(z) = \operatorname{diag}(\mathbf{I},\mathbf{J}\mathbf{V}_{N-1}\mathbf{J})\hat{\mathbf{W}}\boldsymbol{\Lambda}_a(z)\hat{\mathbf{W}}\operatorname{diag}(\mathbf{I},\mathbf{J}) \tag{15}$$

The proof of this Proposition is completed after similarly the way for index $i = N\text{-}1, N\text{-}2,\cdots,0$ term by term.

In (12), $\mathbf{V}_N\mathbf{J}$ is the rotation of $\mathbf{V}_N$ with axis the first column of $\mathbf{V}_N$; while $\mathbf{J}\mathbf{V}_0$ is the rotation of $\mathbf{V}_0$ with axis the first row in (13); and in (14), $\mathbf{J}\mathbf{V}_i\mathbf{J}$ is the rotation of $\mathbf{V}_i$ with axis the antidiagonal.

Since all these matrices $\mathbf{U}_i$, $\mathbf{V}_i$ and $\mathbf{J}$ are arbitrary $(M/2)\times(M/2)$ orthogonal matrices, for simplicity, we replace $\mathbf{V}_N\mathbf{J}$, $\mathbf{J}\mathbf{V}_0$ and $\mathbf{J}\mathbf{V}_i\mathbf{J}$ in (12), (13) and (14), by $\tilde{\mathbf{V}}_N$, $\tilde{\mathbf{V}}_0$ and $\tilde{\mathbf{V}}_i$, respectively.

Take some special cases of $\tilde{\mathbf{V}}_i$, which result in a great improvement of simplifying process before coding.

Proposition 3: As state in the Proposition 2, take $\mathbf{E}_{N\text{-}1}(z) = \prod_{j=N-1}^{0} \tilde{\mathbf{G}}_j(z)$, under condition $\tilde{\mathbf{V}}_i = -\mathbf{I}$ for all $i=0,1,2,\cdots,N-1$, then $\tilde{\boldsymbol{G}}_i(z) = \begin{bmatrix} \mathbf{0} & z\mathbf{J} \\ \mathbf{J} & \mathbf{0} \end{bmatrix}$ and hence the cascade product of two cascade stages $\tilde{\boldsymbol{G}}_i(z)\tilde{\boldsymbol{G}}_{i+1}(z) = z\mathbf{I}$, for $i=0,1,2,\cdots,N-2$. And hence, the analysis polyphase matrix

$$\mathbf{E}(z) = \begin{cases} z^k \prod_{i=K-1}^{N+1} \mathbf{G}_i(z) \ \ \tilde{\mathbf{G}}_N & \text{if } N = 2k, \\ z^k \prod_{i=K-1}^{N+1} \mathbf{G}_i(z) \ \ \tilde{\mathbf{G}}_N \begin{bmatrix} \mathbf{0} & z\mathbf{J} \\ \mathbf{J} & \mathbf{0} \end{bmatrix} & \text{f } N = 2k+1. \end{cases} \tag{16}$$

Theorem 1: Given any orthogonal matrix $\mathbf{E}_0$, the polyphase matrix $\mathbf{E}(z)$ of any $\boldsymbol{M} \times \boldsymbol{KM}$ ($\boldsymbol{M}$ even, $\boldsymbol{K}$ integer) linear phase paraunitary filter bank satisfying (1) can be factorized as

$$\mathbf{E}(z) = \mathbf{E}_0 \mathbf{G}'_{\boldsymbol{K}-1} \prod_{\boldsymbol{i}=\boldsymbol{K}-2}^{1} \mathbf{G}_{\boldsymbol{i}}(z) \mathbf{G}_0 \tag{17}$$

or

$$\mathbf{E}(z) = \prod_{i=K-1}^{1} \mathbf{G}_i(z) \mathbf{G}'_0 \mathbf{E}_0 \tag{18}$$

where $\mathbf{G}'_{\boldsymbol{K}-1}(z) = \mathbf{E}_0^{\boldsymbol{T}} \mathbf{G}_{\boldsymbol{K}-1}(z)$ and $\mathbf{G}'_0 = \mathbf{G}'_0 \mathbf{E}_0^{\boldsymbol{T}}$.

Proof: It is simply using the orthogonality of $\mathbf{E}_0$. To (17), since the final stage $\mathbf{G}_{\boldsymbol{K}-1}(z) = \mathbf{E}_0 \mathbf{E}_0^{\boldsymbol{T}} \mathbf{G}_{\boldsymbol{K}-1}(z)$, while for the (18) initial stage $\mathbf{G}_0 = \mathbf{G}_0 \mathbf{E}_0^{\boldsymbol{T}} \mathbf{E}_0$.

Though it states only on the final and initial stage, in fact, we can embed any orthogonal operation $\mathbf{E}_0$ at any stage $\boldsymbol{N}$ between stage 0 and $\boldsymbol{K}-1$.

III. SPLIT RADIX ALGORITHM OF DCT

The popular DCT in JPEG and MPEG is the eight-point type-II DCT. Because of its practical value, numerous fast DCT-II algorithms have been proposed [7], the most effective are ones based on sparse matrix even partly recursive factorizations, i.e., an even M-point DCT-II can be implemented via an $M/2$-point DCT-II and an $M/2$- point DCT-IV it is a close neighbor of the optimal solution in the search space.

All lapped transforms can be viewed as post- and pre-processing of the DCT coefficients with the quantizer $\mathbf{Q}$ in between, as shown in Fig. 4(a). A more intuitive viewpoint is depicted in Fig. 4(b), where the pre- and post-filter are outside the existing framework. This way, we have a chance at improving coding performance while achieving standard-compliance with minimal software/ hardware modifications.

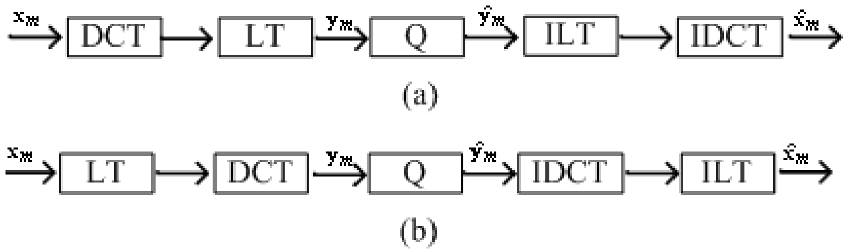


Figure 4: The pre/post processing structure. (a) DCT as a preprocessing , and (b) as a postprocessing of a lapped transform.

For even integer $n \ge 4$, consider a finite length signal $\mathbf{x} = (x_0, x_1, \cdots, x_{n-1})^T$, define the even-odd permutation matrix $\mathbf{P}_n$ on $\mathbf{x}$ by

$$\mathbf{P}_n \mathbf{x} \equiv (x_0, x_2, \cdots, x_{n-2}, x_1, x_3, \cdots, x_{n-1})^T \tag{19}$$

Proposition 4: (First order factorization) Let $M \ge 4$ be an even integer, the matrix $\mathbf{C}_M^{II}$ can be factorized in the form

$$\mathbf{C}_{\boldsymbol{M}}^{\boldsymbol{II}} = \mathbf{P}_{\boldsymbol{n}}^{\boldsymbol{T}}\, \boldsymbol{diag}\left(\mathbf{C}_{\boldsymbol{M}/2}^{\boldsymbol{II}} \quad \mathbf{C}_{\boldsymbol{M}/2}^{\boldsymbol{IV}}\right) \boldsymbol{diag}\left(\mathbf{I}_{\boldsymbol{M}/2} \quad \mathbf{J}_{\boldsymbol{M}/2}\right)\hat{\mathbf{W}} \tag{20}$$

Proof: First, assume $\boldsymbol{M} = 2\boldsymbol{m}$, for some integer $\boldsymbol{m} \geq 2$. We permute the row of $\mathbf{C}_{\boldsymbol{M}}^{\boldsymbol{II}}$ by multiplying with $\mathbf{P}_{\boldsymbol{M}}$ and write the result as a block matrix

$$\mathbf{P}_{\boldsymbol{M}}\mathbf{C}_{\boldsymbol{M}}^{\boldsymbol{II}} = \begin{bmatrix} \mathbf{A}_{00} & \mathbf{A}_{01} \\ \mathbf{A}_{10} & \mathbf{A}_{11} \end{bmatrix} \tag{21}$$

where the four matrices are defined as

$$\mathbf{A}_{00} = \left\{ \sqrt{\frac{1}{\boldsymbol{M}/2}}\, \varepsilon(2\boldsymbol{i})\, \boldsymbol{cos}\frac{(2\boldsymbol{i})\cdot(2\boldsymbol{j}+1)}{2\boldsymbol{M}}\pi \right\}_{\boldsymbol{i},\boldsymbol{j}=0}^{\boldsymbol{m}-1} \tag{22}$$

$$\mathbf{A}_{01} = \left\{ \sqrt{\frac{1}{\boldsymbol{M}/2}}\, \varepsilon(2\boldsymbol{i})\, \boldsymbol{cos}\frac{(2\boldsymbol{i})\cdot(2\boldsymbol{j}+1)}{2\boldsymbol{M}}\pi \right\}_{\boldsymbol{i}=0,\boldsymbol{j}=\boldsymbol{m}}^{\boldsymbol{i}=\boldsymbol{m}-1,\boldsymbol{j}=2\boldsymbol{m}-1} \tag{23}$$

$$\mathbf{A}_{10} = \left\{ \sqrt{\frac{1}{\boldsymbol{M}/2}}\, \varepsilon(2\boldsymbol{i}+1)\, \boldsymbol{cos}\frac{(2\boldsymbol{i}+1)(2\boldsymbol{j}+1)}{2\boldsymbol{M}}\pi \right\}_{\boldsymbol{i},\boldsymbol{j}=0}^{\boldsymbol{m}-1} \tag{24}$$

$$\mathbf{A}_{11} = \left\{ \sqrt{\frac{1}{\boldsymbol{M}/2}}\, \varepsilon(2\boldsymbol{i}+1)\, \boldsymbol{cos}\frac{(2\boldsymbol{i}+1)\cdot(2\boldsymbol{j}+1)}{2\boldsymbol{M}}\pi \right\}_{\boldsymbol{i}=0,\boldsymbol{j}=\boldsymbol{m}}^{\boldsymbol{m}-1,\boldsymbol{j}=2\boldsymbol{m}-1} \tag{25}$$

In which, $\varepsilon_{\boldsymbol{M}}(\boldsymbol{i}) = 1/\sqrt{2}$ fo $\boldsymbol{i} = 0, \boldsymbol{M}$; and 1 for $\boldsymbol{i} = 1, \cdots, \boldsymbol{M}-1$. It can be found out that $\mathbf{A}_{00} = \mathbf{C}_{\boldsymbol{n}}^{\boldsymbol{II}}$ and $\mathbf{A}_{10} = \mathbf{C}_{\boldsymbol{n}}^{\boldsymbol{IV}}$. To the two blocks $\mathbf{A}_{01}$ and $\mathbf{A}_{11}$, by changing index $\boldsymbol{j}' = \boldsymbol{j}\text{-}\boldsymbol{n}$, and using identities $\boldsymbol{cos}(2\boldsymbol{k}\pi - \theta) = \boldsymbol{cos}\,\theta$ and $\boldsymbol{cos}[(2\boldsymbol{k}+1)\pi - \theta] = -\boldsymbol{cos}\,\theta$ for integer $\boldsymbol{k}$. We have

$$\mathbf{A}_{01} = \mathbf{C}_{\boldsymbol{m}}^{\boldsymbol{II}}\ \mathbf{J} \tag{26}$$

$$\mathbf{A}_{11} = -\mathbf{C}_{\boldsymbol{n}}^{\boldsymbol{IV}}\ \mathbf{J} \tag{27}$$

And hence,

$$\begin{aligned}\mathbf{P}_M \mathbf{C}_M^{II} &= \frac{1}{\sqrt{2}}\begin{bmatrix} \mathbf{C}_{M/2}^{II} & \mathbf{C}_{M/2}^{II} & \mathbf{J} \\ \mathbf{C}_{M/2}^{IV} & -\mathbf{C}_{M/2}^{IV} & \mathbf{J} \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{C}_{M/2}^{II} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_{M/2}^{IV} \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} \mathbf{I} & \mathbf{J} \\ \mathbf{I} & -\mathbf{J} \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{C}_{M/2}^{II} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_{M/2}^{IV} \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{J} \end{bmatrix} \hat{\mathbf{W}} \end{aligned} \tag{28}$$

Since $\mathbf{P}_M^{-1} = \mathbf{P}_M^T$, finally we complete the proof

$$\mathbf{C}_M^{II} = \mathbf{P}_n^T \, diag\left(\mathbf{C}_{M/2}^{II} \quad \mathbf{C}_{M/2}^{IV}\right) diag\left(\mathbf{I}_{M/2} \quad \mathbf{J}_{M/2}\right)\hat{\mathbf{W}} \, .$$

Example 1: For $n = 4$, we have the factorization of $\mathbf{C}_4^{II}$ as

$$\mathbf{C}_4^{II} = \mathbf{P}_4^T \, diag\left(\mathbf{C}_2^{II} \quad \mathbf{C}_2^{IV}\right) diag\left(\mathbf{I}_2 \quad \mathbf{J}_2\right)\hat{\mathbf{W}}$$

Indeed,

$$\mathbf{C}_4^{II} = \frac{1}{2}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & \sqrt{2}\, cos\frac{\pi}{8} & \sqrt{2}\, sin\frac{\pi}{8} \\ 0 & 0 & \sqrt{2}\, sin\frac{\pi}{8} & -\sqrt{2}\, cos\frac{\pi}{8} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \end{bmatrix} \tag{29}$$

To factorize DCT-IV, we need to introduce some matrices using in this paper. First denote the modified identity matrices $\mathbf{I}'_M$, $\mathbf{I}''_M$ and the forward shift matrix $\mathbf{V}_M$ by

$$\mathbf{I}'_M \equiv \text{diag}\left(\varepsilon_M(i)^{-1}\right)_{i=0}^{M-1}, \qquad \mathbf{I}''_M \equiv \text{diag}\left(\varepsilon_M(i+1)^{-1}\right)_{i=0}^{M-1} \tag{30}$$

$$\mathbf{D}_M \equiv \text{diag}\left((i)^{-1}\right)_{i=0}^{M-1}, \qquad \mathbf{V}_M \equiv \left[\delta(i-j-1)\right]_{i,j=0}^{M-1} \tag{31}$$

the modified addition matrices:

$$\mathbf{A}_M(0) \equiv \mathbf{I}_M \qquad \mathbf{A}_M(1) \equiv \frac{1}{\sqrt{2}}\begin{bmatrix} \mathbf{I}'_{M_1} & \mathbf{V}_{M_1}\mathbf{D}_{M_1} \\ \mathbf{V}^T_{M_1} & -\mathbf{I}''_{M_1}\mathbf{D}_{M_1} \end{bmatrix} \boldsymbol{diag}\left(\mathbf{I}_{M_1} \ \ \mathbf{J}_{M_1}\right) \quad (32)$$

Furthermore, we define cosine and sine vectors by

$$\mathbf{C}_M \equiv \left[\boldsymbol{cos}\frac{(2\boldsymbol{k}+1)\pi}{8\boldsymbol{M}} \right]_{\boldsymbol{i}=0}^{\boldsymbol{M}-1}, \qquad \mathbf{S}_M \equiv \left[\boldsymbol{sin}\frac{(2\boldsymbol{k}+1)\pi}{8\boldsymbol{M}} \right]_{\boldsymbol{i}=0}^{\boldsymbol{M}-1} \quad (33)$$

And the cross-shaped twiddle matrices:

$$\mathbf{T}_M(0) \equiv \frac{1}{\sqrt{2}}\begin{bmatrix} \mathbf{I}_{M_1} & \mathbf{J}_{M_1} \\ \mathbf{I}_{M_1} & \text{-}\mathbf{J}_{M_1} \end{bmatrix} \qquad \mathbf{T}_M(1) \equiv diag\left(\mathbf{I}_{M_1} \ \ \mathbf{D}_{M_1}\right)\begin{bmatrix} diag\ \ \mathbf{C}_{M_1} & \mathbf{J}_{M_1}\ diag\ \ \mathbf{J}_{M_1}\mathbf{S}_{M_1} \\ -\mathbf{J}_{M_1}\ diag\ \ \mathbf{S}_{M_1} & diag\ \ \mathbf{J}_{M_1}\mathbf{C}_{M_1} \end{bmatrix} \quad (34)$$

We present the factorization of DCT-IV $\mathbf{C}_M^{IV}$ for even integer $M \geq 4$ in the following subsection.

Proposition 5: For even integer $M = 2m \geq 4$, the matrix $\mathbf{C}_M^{IV}$ can be factorized in the form:

$$\mathbf{C}_M^{IV} = \mathbf{P}_M^T \mathbf{A}_M(1)\boldsymbol{diag}\left(\mathbf{C}_m^{II} \quad \mathbf{C}_m^{II}\right)\mathbf{T}_M(1) \quad (35)$$

In which, $\mathbf{A}_M(1)$ is the modified addition matrix

$$\mathbf{A}_M(1) \equiv \frac{1}{\sqrt{2}}\begin{bmatrix} \mathbf{I}'_m & \mathbf{V}_m\mathbf{D}_m \\ \mathbf{V}^T_m & -\mathbf{I}''_m\mathbf{D}_m \end{bmatrix}\boldsymbol{diag}\left(\mathbf{I}_m \quad \mathbf{J}_m\right) \quad (36)$$

and the cross-shaped twiddle matrix is

$$\mathbf{T}_M(1) \equiv diag\left(\mathbf{I}_m \quad \mathbf{D}_m\right)\begin{bmatrix} diag\ \ \mathbf{C}_m & \mathbf{J}_m\ diag\ \ \mathbf{J}_m\mathbf{S}_m \\ -\mathbf{J}_m\ diag\ \ \mathbf{S}_m & diag\ \ \mathbf{J}_m\mathbf{C}_m \end{bmatrix} \quad (37)$$

Proof: First, we permute the rows of $\mathbf{C}_M^{IV}$ by multiplying with $\mathbf{P}_M$ and write the results as block matrix

$$\mathbf{P}_M\mathbf{C}_M^{IV} = \begin{bmatrix} \mathbf{A}_{00} & \mathbf{A}_{01} \\ \mathbf{A}_{10} & \mathbf{A}_{11} \end{bmatrix} \quad (38)$$

where these four blocks are defined as

$$\mathbf{A}_{00} = \frac{1}{\sqrt{\boldsymbol{m}}}\left\{\boldsymbol{cos}\frac{(4\boldsymbol{i}+1)(2\boldsymbol{j}+1)\pi}{4\boldsymbol{M}}\right\}_{\boldsymbol{i},\boldsymbol{j}=0}^{\boldsymbol{m}-1} \tag{39}$$

$$\mathbf{A}_{01} = \frac{1}{\sqrt{\boldsymbol{m}}}\left\{\boldsymbol{cos}\frac{(4\boldsymbol{i}+1)(\boldsymbol{M}+2\boldsymbol{j}+1)\pi}{4\boldsymbol{M}}\right\}_{\boldsymbol{i},\boldsymbol{j}=0}^{\boldsymbol{m}-1} \tag{40}$$

$$\mathbf{A}_{10} = \frac{1}{\sqrt{\boldsymbol{m}}}\left\{\boldsymbol{cos}\frac{(4\boldsymbol{i}+3)(2\boldsymbol{j}+1)\pi}{4\boldsymbol{M}}\right\}_{\boldsymbol{i},\boldsymbol{j}=0}^{\boldsymbol{m}-1} \tag{41}$$

$$\mathbf{A}_{11} = \frac{1}{\sqrt{\boldsymbol{m}}}\left\{\boldsymbol{cos}\frac{(4\boldsymbol{i}+3)(\boldsymbol{M}+2\boldsymbol{j}+1)\pi}{4\boldsymbol{M}}\right\}_{\boldsymbol{i},\boldsymbol{j}=0}^{\boldsymbol{m}-1} \tag{42}$$

It can be proved easily that

$$\mathbf{A}_{00} = \frac{1}{\sqrt{2}}\left(\mathbf{I}'_{\boldsymbol{m}}\ \mathbf{C}^{\boldsymbol{II}}_{\boldsymbol{m}}\,\boldsymbol{diag}\,\boldsymbol{C}_{\boldsymbol{m}} - \mathbf{V}_{\boldsymbol{m}}\mathbf{D}_{\boldsymbol{m}}\ \mathbf{S}^{\boldsymbol{II}}_{\boldsymbol{m}}\mathbf{J}_{\boldsymbol{m}}\ \boldsymbol{diag}\,\boldsymbol{S}_{\boldsymbol{m}}\right) \tag{43}$$

and

$$\mathbf{A}_{10} = \frac{1}{\sqrt{2}}\left(\mathbf{V}^{\boldsymbol{T}}_{\boldsymbol{m}}\ \mathbf{C}^{\boldsymbol{II}}_{\boldsymbol{m}}\ \boldsymbol{diag}\,\boldsymbol{C}_{\boldsymbol{m}} + \mathbf{I}''_{\boldsymbol{m}}\ \mathbf{D}_{\boldsymbol{m}}\ \mathbf{S}^{\boldsymbol{II}}_{\boldsymbol{m}}\ \mathbf{J}_{\boldsymbol{m}}\ \boldsymbol{diag}\,\boldsymbol{S}_{\boldsymbol{m}}\right) \tag{44}$$

To the other two matrices $\mathbf{A}_{01}$ and $\mathbf{A}_{11}$, it is more complex than the two matrices above, and needs more trigonometric identities, such as use $\cos(\alpha+(\pi/2)) = -\boldsymbol{sin}\,\alpha$ to expand these two matrices, these transpose the order of column vector $\mathbf{C}_{\boldsymbol{M}}$ and $\mathbf{S}_{\boldsymbol{M}}$.

$$\begin{aligned}
\mathbf{A}_{01} &= \frac{1}{\sqrt{\boldsymbol{m}}}\left\{\boldsymbol{cos}\frac{(4\boldsymbol{i}+1)(\boldsymbol{M}+2\boldsymbol{j}+1)\boldsymbol{\pi}}{4\boldsymbol{M}}\right\}_{\boldsymbol{i},\boldsymbol{j}=0}^{\boldsymbol{m}-1} \\
&= \frac{1}{\sqrt{\boldsymbol{m}}}\left\{\boldsymbol{cos}\left(\left[\left(\frac{\boldsymbol{i}(\boldsymbol{M}+2\boldsymbol{j}+1)}{\boldsymbol{M}}\right)+\frac{\boldsymbol{M}+2\boldsymbol{j}+1}{4\boldsymbol{M}}-\frac{\pi}{2}+\frac{\pi}{2}\right]\boldsymbol{\pi}\right)\right\}_{\boldsymbol{i},\boldsymbol{j}=0}^{\boldsymbol{m}-1} \\
&= \frac{1}{\sqrt{\boldsymbol{m}}}\left\{\boldsymbol{cos}\left(\left[\left(\boldsymbol{i}+\frac{\boldsymbol{i}(2\boldsymbol{j}+1)}{\boldsymbol{M}}\right)+\frac{\boldsymbol{M}+2\boldsymbol{j}+1}{4\boldsymbol{M}}\right]\boldsymbol{\pi}\right)\right\}_{\boldsymbol{i},\boldsymbol{j}=0}^{\boldsymbol{m}-1} \\
&= \frac{(-1)^{\boldsymbol{i}}}{\sqrt{\boldsymbol{m}}}\left\{\boldsymbol{cos}\left(\left[\left(\boldsymbol{i}+\frac{\boldsymbol{i}(2\boldsymbol{j}+1)}{\boldsymbol{M}}\right)+\frac{\boldsymbol{M}+2\boldsymbol{j}+1}{4\boldsymbol{M}}\right]\boldsymbol{\pi}\right)\right\}_{\boldsymbol{i},\boldsymbol{j}=0}^{\boldsymbol{m}-1}
\end{aligned}$$

$$=\frac{1}{\sqrt{m}}\left\{(-1)^{i}\cos i\left(\frac{2j+1}{M}\right)\pi\cos\left(\frac{M+2j+1}{4M}\right)\pi-(-1)^{i}\sin i\left(\frac{2j+1}{M}\right)\pi\sin\left(\frac{M+2j+1}{4M}\right)\pi\right\}_{i,j=0}^{m-1}$$

$$=\frac{1}{\sqrt{m}}\left\{(-1)^{i}\cos i\left(\frac{2j+1}{M}\right)\pi\sin\left(\frac{M-2j-1}{4M}\right)\pi-(-1)^{i}\sin i\left(\frac{2j+1}{M}\right)\pi\cos\left(\frac{M-2j-1}{4M}\right)\pi\right\}_{i,j=0}^{m-1}$$

$$=\frac{1}{\sqrt{2}}\left\{\mathbf{D}_m\,\mathbf{I}'_m\,\mathbf{C}_m^{II}\,diag(\mathbf{J}_m S_m)-\mathbf{D}_m\,\mathbf{V}_m\,\mathbf{S}_m^{II}\,diag(\mathbf{J}_m C_m)\right\} \quad (45)$$

Analogous the way, we get block

$$\mathbf{A}_{11}=\frac{1}{\sqrt{2}}\left\{\mathbf{V}_m^{T}\,\mathbf{C}_m^{II}\,\mathbf{J}_m\,diag(\mathbf{J}_m S_m)-\mathbf{I}''_m\,\mathbf{D}_m\,\mathbf{S}_m^{II}\,diag(\mathbf{J}_m C_m)\right\} \quad (46)$$

And hence, the block form of $\mathbf{P}_M\mathbf{C}_M^{IV}$ become

$$\mathbf{P}_M\mathbf{C}_M^{IV}=\frac{1}{\sqrt{2}}\begin{bmatrix}\mathbf{I}'_m & \mathbf{V}_m\,\mathbf{D}_m\\ \mathbf{V}_m^{T} & -\mathbf{I}''_m\,\mathbf{D}_m\end{bmatrix}\begin{bmatrix}\mathbf{C}_m^{II} & \mathbf{0}\\ \mathbf{0} & \mathbf{S}_m^{II}\end{bmatrix}\begin{bmatrix}diag\,C_m & \mathbf{J}_m\,diag\mathbf{J}_m S_m\\ -\mathbf{J}_m\,diag\,S_m & diag\,C_m\,\mathbf{J}_m\end{bmatrix} \quad (47)$$

Finally, we complete the proof by moving $\mathbf{P}_M$ from the left to the right.

The Corollary next, demonstrates the propagate property of DCT-II and DCT–IV [5], which results in the Split Radix Algorithm of DCT-II and DCT–IV.

Corollary (Second order factorization) Let M be an integer $M\geq 8$ with $M\equiv 0\bmod 4$ be given. Then the matrices $\mathbf{C}_M^{II}$, and $\mathbf{C}_M^{IV}$ can be factorized as follows

$$\mathbf{C}_M^{II}=\mathbf{P}_M^{T}\,diag\left(\mathbf{P}_{M/2}^{T},\,\mathbf{P}_{M/2}^{T}\right)diag\left(\mathbf{I}_{M/2},\,\mathbf{A}_{M/2}(1)\right)diag\left(\mathbf{C}_{M/4}^{II},\,\mathbf{C}_{M/4}^{IV},\,\mathbf{C}_{M/4}^{II},\,\mathbf{C}_{M/4}^{II}\right)diag\left(\mathbf{T}_{M/2}(0),\,\mathbf{T}_{M/2}(1)\right)\mathbf{T}_M(0) \quad (48)$$

$$\mathbf{C}_M^{IV}=\mathbf{P}_M^{T}\left[\mathbf{A}_M(1)\,diag\left(\mathbf{P}_{M/2}^{T},\,\mathbf{P}_{M/2}^{T}\right)diag\left(\mathbf{C}_{M/4}^{II},\mathbf{C}_{M/4}^{IV},\mathbf{C}_{M/4}^{II},\mathbf{C}_{M/4}^{IV}\right)diag\left(\mathbf{T}_{M/2}(0),\mathbf{T}_{M/2}(1)\right)\right]\mathbf{T}_M(1) \quad (49)$$

Let $M=2^{m}, m\geq 3$, and let $M_s=2^{m-s}$, $s=0,1,\cdots,m-1$. In the first order factorization, we have $\mathbf{C}_M^{II}$ split into $C_{M_1}^{II}\oplus C_{M_1}^{IV}$ (proposition 4). Recursive application, the second order factorization (proposition 5), $C_{M_1}^{II}\oplus C_{M_1}^{IV}$ is split into $\boldsymbol{C}_{M_2}^{II} \oplus \boldsymbol{C}_{M_2}^{IV} \oplus \boldsymbol{C}_{M_2}^{II} \oplus \boldsymbol{C}_{M_2}^{II}$, to the s order factorization, we illustrate the expand procedure of the split radix DCT-II and DCT-IV in the Figure 5 for the third order factorization.

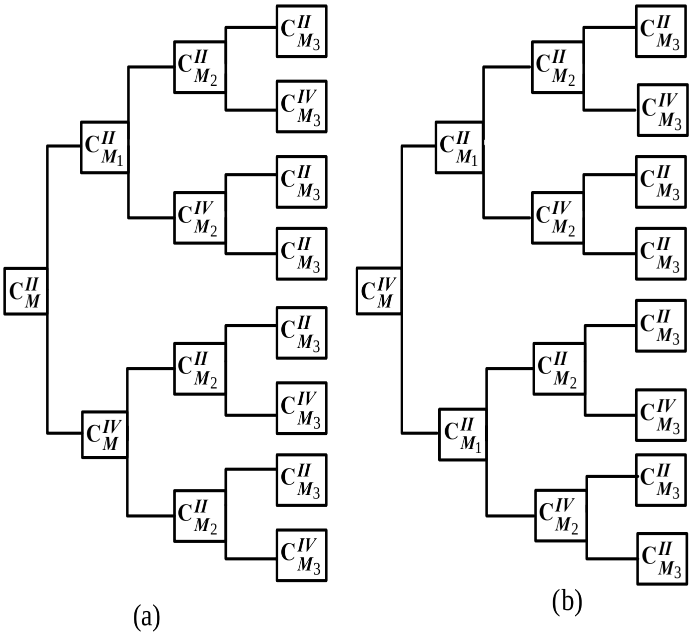


Figure 5. The Split Radix Expansion of DCT-II (a), and DCT–IV (b).

Example 2: The 8-point DCT-II $\mathbf{C}_8^{II}$ has the second order factorization as

$$\mathbf{C}_8^{II} = \frac{1}{2}\begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \cos\frac{\pi}{16} & \cos\frac{3\pi}{16} & \cos\frac{5\pi}{16} & \cos\frac{7\pi}{16} & \cos\frac{9\pi}{16} & \cos\frac{11\pi}{16} & \cos\frac{13\pi}{16} & \cos\frac{15\pi}{16} \\ \cos\frac{2\pi}{16} & \cos\frac{6\pi}{16} & \cos\frac{10\pi}{16} & \cos\frac{14\pi}{16} & \cos\frac{18\pi}{16} & \cos\frac{22\pi}{16} & \cos\frac{26\pi}{16} & \cos\frac{30\pi}{16} \\ \cos\frac{3\pi}{16} & \cos\frac{9\pi}{16} & \cos\frac{15\pi}{16} & \cos\frac{21\pi}{16} & \cos\frac{27\pi}{16} & \cos\frac{33\pi}{16} & \cos\frac{39\pi}{16} & \cos\frac{45\pi}{16} \\ \cos\frac{4\pi}{16} & \cos\frac{12\pi}{16} & \cos\frac{20\pi}{16} & \cos\frac{28\pi}{16} & \cos\frac{36\pi}{16} & \cos\frac{44\pi}{16} & \cos\frac{52\pi}{16} & \cos\frac{60\pi}{16} \\ \cos\frac{5\pi}{16} & \cos\frac{15\pi}{16} & \cos\frac{25\pi}{16} & \cos\frac{35\pi}{16} & \cos\frac{45\pi}{16} & \cos\frac{55\pi}{16} & \cos\frac{65\pi}{16} & \cos\frac{75\pi}{16} \\ \cos\frac{6\pi}{16} & \cos\frac{18\pi}{16} & \cos\frac{30\pi}{16} & \cos\frac{42\pi}{16} & \cos\frac{54\pi}{16} & \cos\frac{66\pi}{16} & \cos\frac{78\pi}{16} & \cos\frac{90\pi}{16} \\ \cos\frac{7\pi}{16} & \cos\frac{21\pi}{16} & \cos\frac{35\pi}{16} & \cos\frac{49\pi}{16} & \cos\frac{63\pi}{16} & \cos\frac{77\pi}{16} & \cos\frac{91\pi}{16} & \cos\frac{105\pi}{16} \end{bmatrix}$$

$$= \mathbf{P}_8^T \begin{bmatrix} \mathbf{P}_4^T & \mathbf{0} \\ \mathbf{0} & \mathbf{P}_4^T \end{bmatrix} \begin{bmatrix} \mathbf{I}_4 & \mathbf{0}_4 \\ \mathbf{0}_4 & \mathbf{A}_4(1) \end{bmatrix} \begin{bmatrix} C_2^{II} & \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 \\ \mathbf{0}_2 & C_2^{IV} & \mathbf{0}_2 & \mathbf{0}_2 \\ \mathbf{0}_2 & \mathbf{0}_2 & C_2^{II} & \mathbf{0}_2 \\ \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 & C_2^{II} \end{bmatrix} \begin{bmatrix} \mathbf{T}_4(0) & \mathbf{0}_4 \\ \mathbf{0}_4 & \mathbf{T}_4(1) \end{bmatrix} \mathbf{T}_8(0)$$

We show the procedure in Figure 6(a), there are four rotation angles and three butterfly matrices in the factorizing procedure.

Example 3: The second factorization of 8-point DCT-IV $\mathbf{C}_8^{IV}$ is

$$\mathbf{C}_8^{IV} = \frac{1}{\sqrt{2}} \mathbf{P}_8^T \begin{bmatrix} \mathbf{I}'_4 & \mathbf{V}_4\mathbf{D}_4 \\ \mathbf{V}_4^T & \mathbf{I}''_4\ \mathbf{D}_4 \end{bmatrix} \begin{bmatrix} \mathbf{I}_4 & \mathbf{0}_4 \\ \mathbf{0}_4 & \mathbf{J}_4 \end{bmatrix} \begin{bmatrix} \mathbf{P}_4^T & \mathbf{0}_4 \\ \mathbf{0}_4 & \mathbf{P}_4^T \end{bmatrix} \begin{bmatrix} C_2^{II} & \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 \\ \mathbf{0}_2 & C_2^{IV} & \mathbf{0}_2 & \mathbf{0}_2 \\ \mathbf{0}_2 & \mathbf{0}_2 & C_2^{II} & \mathbf{0}_2 \\ \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 & C_2^{II} \end{bmatrix} \begin{bmatrix} \mathbf{T}_4(0) & \mathbf{0}_4 \\ \mathbf{0}_4 & \mathbf{T}_4(1) \end{bmatrix} \begin{bmatrix} \mathbf{I}_4 & \mathbf{0}_4 \\ \mathbf{0}_4 & \mathbf{D}_4 \end{bmatrix} \begin{bmatrix} \mathit{diag}\ \mathbf{C}_4 & \mathbf{J}_4\ \mathit{diag}\ \mathbf{J}_4\mathbf{S}_4 \\ -\mathbf{J}_4\ \mathit{diag}\ \mathbf{S}_4 & \mathit{diag}\ \mathbf{J}_4\mathbf{C}_4 \end{bmatrix}$$

We depict the procedure in Figure 6(b), there are four rotation angles and three butterfly matrices in the factorizing procedure..

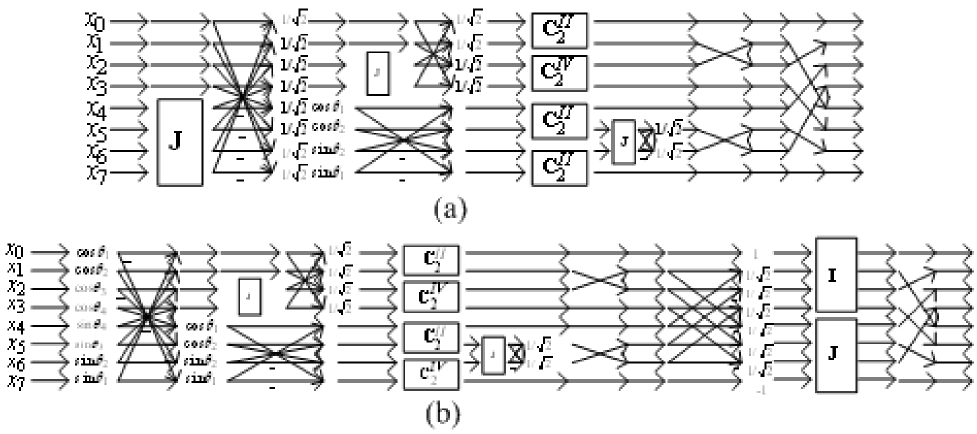


Figure 6. The structure of Split-Radix algorithm: (a) 8-point DCT-II. (b) 8-point DCT-IV.

At the end of this section, follows from the Theorem in section II, we embed DCT $\mathbf{C}$, which is been radix split for purposes, into the N th stage $\tilde{\mathbf{G}}_N$, $0 \le N \le K-1$, such that

$$\tilde{\mathbf{G}}_N = \mathbf{C}\,\mathbf{C}^T \mathrm{diag}\left(\mathbf{U}_0, \tilde{\mathbf{V}}'_N\right)\hat{\mathbf{W}} \tag{50}$$

and hence, the M -channel KM -tap analysis system $\mathbf{E}(z)$ can be factorized as

$$\mathbf{E}(z) = \prod_{i=K-1}^{N+1} \mathbf{G}_i(z)\tilde{\mathbf{G}}_N \prod_{j=N-1}^{0} \hat{\mathbf{G}}'_j(z) \tag{51}$$

we demonstrate the structure of $\mathbf{E}(z)$ in Fig. 7.

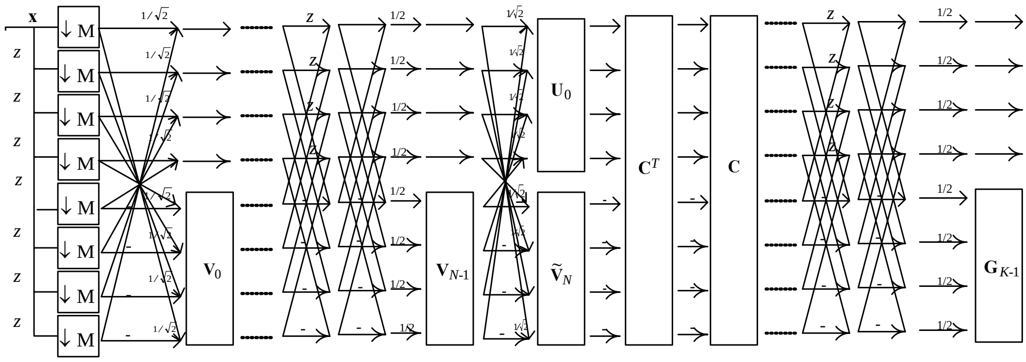


Figure 7. An M -channel KM -tap Split-Radix analysis system.

IV. CONCLUSION

In this paper, we generalize the structure of linear phase paraunitary filter banks through elementary matrices operations. Then, by the special characteristic of discrete cosine transform, we adopt split radix algorithm in the processing. These properties not only improve the complexity of coding algorithm for a finite signal before quantized but also due to the compactness property and the recursive radix sparse matrix factorizations property of the discrete cosine transform, it is indeed support an effective and simplified lapped structure of linear phase paraunitary filter banks.

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