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Uncertain Discrete Delay Singular Systems: Delay-Dependent Stability and Stabilization

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Abstract

The problems of delay-dependent robust stability and stabilization of a class of uncertain, linear discrete-time singular systems with state-delay are examined. The parametric uncertainties are assumed to be time-invariant and norm-bounded appearing in the state and delay matrices. A new system representation is developed to derive new delay-dependent stability criteria without relying on overbounding. A solution to delay-dependent state-feedback stabilization is obtained. Seeking computational convenience, all the developed results are cast in the format of linear matrix inequalities (LMIs) and a numerical example is presented.

Keywords: Discrete Singular systems, Time-delay systems, Robust stability, Robust stabilization, LMIs

1 Introduction

Singular systems appear frequently in several applications including large-scale systems, power systems, economic systems, to name a few [9, 10]. The designations singular systems, descriptor systems, implicit systems [1], generalized state-space systems [7], differential-algebraic systems [5] or semistate systems are interchangeably used in the research studies. In recent years, robust stability and robust stabilization problems of singular systems have been under investigation [6, 14, 15, 16].

From these results, it becomes clear that the robust stability problem for singular systems is more involved than the counterpart in state-space systems. Unlike ordinary state-space systems, singular systems require, in addition stability robustness, consideration of regularity and absence of impulses (case of continuous systems) or causality (case of discrete systems) simultaneously [6, 8].

On another research front, it becomes quite evident that delays occur in physical and man-made systems due to various reasons including finite capabilities of information processing among different parts of the system, inherent phenomena like mass transport flow and recycling and/or by product of computational delays [2, 12]. Considerable discussions on delays and their stabilization/destabilization effects in control systems have attracted the interests of numerous investigators in recent years, see [12] and their references. Recent related results on discrete delay systems are presented in [3, 11, 13]

The class of discrete-time singular has been examined for robust stabilization in [15, 16]. From the literature, it seems that the stabilization problem for discrete-time singular and state-delay and bounded-but-unknown parametric uncertainties is not fully investigated and most of the existing results are established under special conditions. In this paper, we focus on the stabilization problem using state-feedback controller. A new expanded state-space representation is developed which converts the singular time-delay system into an equivalent singular system in which all the original system matrices are grouped into the new system matrix and the original delay system becomes easier to handle. The benefit gained is that we do not require overbounding of the quantities involved. These advantages simplify the derivation of new delay-dependent stability and state-feedback stabilization results. All the results are formulated as linear matrix inequalities. A numerical example is worked out to illustrate the theoretical developments.

Notations: In the sequel, the Euclidean norm is used for vectors. We use W^t, W^{-1}, $\lambda(W)$ and $||W||$ to denote, respectively, the transpose, the inverse, the eigenvalues and the induced norm of any square matrix W and $W > 0$ ($W < 0$) stands for a symmetrical and positive- (negative-) definite matrix W. The n-dimensional Euclidean space and the space of bounded sequences are denoted by $\mathbb{R}^{n \times n}$ and ℓ_2, respectively. The symbol $\bullet$ will be used in some matrix expressions to induce a symmetric structure, that is if given matrices $L = L^t$ and $R = R^t$ of appropriate dimensions, then

$$\begin{bmatrix} L & \bullet \\ N & R \end{bmatrix} = \begin{bmatrix} L & N^t \\ N & R \end{bmatrix}$$

Sometimes, the arguments of a function will be omitted when no confusion can arise.

Fact 1: Given a scalar $\epsilon > 0$ and matrices Σ_1, Σ_2 and Φ such that $\Phi^t \Phi \leq I$, then

$$\Sigma_1 \Phi \Sigma_2 + \Sigma_2^t \Phi^t \Sigma_1^t \leq \epsilon^{-1} \Sigma_1 \Sigma_1^t + \epsilon \Sigma_2^t \Sigma_2$$

2 Problem Statement and Definitions

We consider the following class of discrete-time singular systems with state-delay and parametric uncertainties:

$$E\, x_{k+1} = A_{\Delta o} x_k + A_{\Delta d} x_{k-d} + B_o u_k, \quad x_0 = \psi_0 \tag{2.1}$$

where $x_k \in \mathbb{R}^n$ is the state vector; $u_k \in \mathbb{R}^p$ is the control input and $\underline{d} \leq d \leq \bar{d}$ is an unknown integer representing the delay and $\underline{d}, \bar{d}$ are known bounds. The matrix $E \in \mathbb{R}^{n \times n}$ may be singular; we assume that rank $E = r \leq n$. The matrices $A_{\Delta o} \in \mathbb{R}^{n \times n}$ and $A_{\Delta d} \in \mathbb{R}^{n \times n}$ are represented by

$$[A_{\Delta o} \quad A_{\Delta d}] = [A_o \quad A_d] + M \Delta_k [N_a \quad N_d] \tag{2.2}$$

where $A_o \in \mathbb{R}^{n \times n}$, $B_o \in \mathbb{R}^{n \times p}$, $A_d \in \mathbb{R}^{n \times n}$, $M \in \mathbb{R}^{n \times n_m}$, $\Gamma \in \mathbb{R}^{n \times r}$, $N_a \in \mathbb{R}^{n_n \times n}$ and $N_d \in \mathbb{R}^{n_n \times n}$ are real and known constant matrices with Δ_k is a bounded matrix of uncertainties satisfying $\Delta_k^t \Delta_k < I$. The uncertainties that satisfy (2.2) are referred to as admissible uncertainties.

For the time being we set $\Delta_k \equiv 0$, $u_k \equiv 0$, $A_{\Delta d} \equiv 0$, $x_{k-d} \equiv 0$ to yield the free nominal singular system

$$E\, x_{k+1} = A_o x_k \tag{2.3}$$

For system (2.3), we recall the following definitions and results:

Definition 2.1 *[9, 14, 16]:*

1. *System (2.3) is said to be regular if* $det(zE - A_o)$ *is not identically zero.*
2. *System (2.3) is said to be causal if it is regular and* $deg(det(zE - A_o)) = rank(E)$*.*

3. *System (2.3) is said to be stable if all the roots of* $det(zE - A_o)$ *lies inside the unit disk with center at the origin.*

4. *System (2.3) is said to be admissible if it is regular, causal and stable.*

Next we consider the free nominal singular delay system

$$E\, x_{k+1} = A_o x_k + A_d x_{k-d} \tag{2.4}$$

Extending on Definition (2.1), we provide the following

Definition 2.2 *System (2.4) is said to be regular and causal if the pair* (E, A_o) *is regular and causal. System (2.4) is said to be admissible if it is regular, causal and asymptotically stable.*

The objective of this paper is to develop delay-dependent methodologies for robust stability and stabilization for the class of uncertain, discrete-time singular delay systems of the type (2.1). This will be accomplished in Section 3 (delay-dependent stability) and Section 4 (delay-dependent stabilization) through the establishment of a new expanded state-space representation in which converts the singular time-delay system into an equivalent singular system in which the system matrix contains all the matrices of the original and the delay state has simple, certain and fixed matrix even if the original delay matrix is uncertain.

3 Delay-Dependent Stability

In the sequel, we employ the difference operator $\mathcal{D}_k \triangleq x_{k+1} - x_k$ along with $x_{k-d} = x_k - \sum_{j=k-d}^{k-1} \mathcal{D}_j$ to rewrite system (2.1):

$$\begin{aligned} E\, x_{k+1} &= A_{\Delta o} x_k + A_{\Delta d} x_{k-d} + B_o u_k \\ &= (A_{\Delta o} + A_{\Delta d}) x_k - A_{\Delta d} \sum_{j=k-d}^{k-1} \mathcal{D}_j + B_o u_k \end{aligned}$$

Together with the definition of $\mathcal{D}_k$, we get

$$0 = (A_{\Delta o} + A_{\Delta d} - E) x_k - E\, \mathcal{D}_k - A_{\Delta d} \sum_{j=k-d}^{k-1} \mathcal{D}_j + B_o u_k \tag{3.1}$$

Define $\sigma_k = \sum_{j=k-d}^{k-1} \mathcal{D}_j$, then it follows that

$$\sigma_{k+1} = \sigma_k + \mathcal{D}_k - \mathcal{D}_{k-d}$$

Introducing

$$\xi_k = [x_k^t \quad \mathcal{D}_k^t \quad \sigma_k^t]^t, \quad A_{\Delta od} \ = \ A_{\Delta o} + A_{\Delta d}$$

we readily obtain the new expanded state-space system

$$(\Sigma_2) : \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I \end{bmatrix} \xi_{k+1}$$

$$= \begin{bmatrix} I & I & 0 \\ A_{\Delta od} - E & -E & -A_{\Delta d} \\ 0 & I & I \end{bmatrix} \xi_k \ + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -I & 0 \end{bmatrix} \xi_{k-d} + \begin{bmatrix} 0 \\ B_o \\ 0 \end{bmatrix} u_k$$

$$U \ \xi_{k+1} \quad = \quad \bar{A}_{\Delta\xi} \ \xi_k \ + \ \bar{A}_{\xi d} \ \xi_{k-d} \ + \ \bar{B}_o u_k \tag{3.2}$$

where the initial conditions are characterized by

$$\xi_0 = \begin{bmatrix} x_0 \\ \mathcal{D}_0 \\ \sigma_0 \end{bmatrix} = \begin{bmatrix} \psi_0 \\ (A_o - E)\psi_0 - A_d \psi_{-d_0} \\ \sum_{j=-\bar{d}}^{-1} \mathcal{D}_{x_j} \end{bmatrix} \tag{3.3}$$

Remark 3.1 *In short, if x_k is a solution of uncertain delay system (2.3) with $\Delta_k \equiv 0$ and $u_k \equiv 0$, then ξ_k is a solution of the new expanded state-space system (3.2) subject to (3.3) and the reverse is true. This is the essence of descriptor transformation. It is significant to observe that in system (3.2) the delay matrix has a simple, certain and fixed matrix even although the original delay matrix $A_{\Delta d}$ is uncertain. In addition, all the matrices of the original singular system are grouped into the new system matrices and henceforth we call it the "Compact Form (CF)".*

We rewrite the CF matrix

$$\bar{\mathcal{A}}_{\Delta\xi} \quad = \quad \bar{\mathcal{A}}_{\xi o} \ + \ \bar{M} \ \Delta_k \ \bar{N} \tag{3.4}$$

with

$$\bar{A}_{\xi o} \quad = \quad \begin{bmatrix} I & I & 0 \\ A_{od} - E & -E & -A_d \\ 0 & I & I \end{bmatrix}, \quad \bar{M} = \begin{bmatrix} 0 \\ M \\ 0 \end{bmatrix},$$

$$\bar{N} \quad = \quad [N_{ad} \quad 0 \quad - N_d], \quad N_{ad} = N_a + N_d, \quad A_{od} = A_o + A_d$$

Now to derive tractable conditions for stability, we introduce the following Lyapunov functional

$$V(\xi_k) \quad = \quad V_a(\xi_k) \ + \ V_b(\xi_k) \ + \ V_c(\xi_k) \ + \ V_d(\xi_k) \tag{3.5}$$

with

$$\begin{aligned}
V_a(\xi_k) &= \xi_k^t U^t \mathcal{P} U \xi_k, \quad 0 < \mathcal{P}^t = \mathcal{P} \in \mathrm{IR}^{3n\times 3n} \\
V_b(\xi_k) &= \sum_{j=k-d}^{k-1} \xi_j^t \bar{I}^t \mathcal{W} \bar{I} \xi_j, \quad 0 < \mathcal{W}^t = \mathcal{W} \in \mathrm{IR}^{n\times n}, \quad \bar{I} = [I \;\; 0 \;\; 0] \\
V_c(\xi_k) &= \sum_{p=-\bar{d}+2}^{-\underline{d}+1} \sum_{j=k+p-1}^{k-1} \xi_j^t \tilde{I}^t \mathcal{Q} \tilde{I} \xi_j, \quad 0 < \mathcal{Q}^t = \mathcal{Q} \in \mathrm{IR}^{n\times n}, \quad \tilde{I} = [0 \;\; I \;\; 0] \\
V_d(\xi_k) &= \sum_{p=-\bar{d}+1}^{-\underline{d}} \sum_{j=k+p}^{k-1} [(j-p-k+1)\xi_j^t \tilde{I}^t \mathcal{Q} \tilde{I} \xi_j]
\end{aligned}$$

where

$$\mathcal{P} \triangleq \begin{bmatrix} \mathcal{P}_x & \mathcal{P}_f & 0 \\ \bullet & \mathcal{P}_d & 0 \\ \bullet & \bullet & \mathcal{P}_s \end{bmatrix}, \; 0 < \mathcal{P}_x = \mathcal{P}_x^t \in \mathrm{IR}^{n\times n}, \; 0 < \mathcal{P}_d = \mathcal{P}_d^t \in \mathrm{IR}^{n\times n}, \; 0 < \mathcal{P}_s = \mathcal{P}_s^t \in \mathrm{IR}^{n\times n}$$

We consider V_k and evaluate the first difference of the functionals V_a, V_b, V_c and V_d. For V_a, we have

$$\begin{aligned}
V_a(\xi_{k+1}) - V_a(\xi_k) &= \xi_{k+1}^t U^t \mathcal{P} U \xi_{k+1}^t - \xi_k^t U^t \mathcal{P} U \xi_k^t \\
&= [\bar{A}_{\Delta\xi}\xi_k + \bar{A}_{\xi d}\xi_{k-d}]^t \mathcal{P} [\bar{A}_{\Delta\xi}\xi_k + \bar{A}_{\xi d}\xi_{k-d}] - \xi_k^t U^t \mathcal{P} U \xi_k^t \\
&= \xi_k^t [\bar{A}_{\Delta\xi}^t \mathcal{P} \bar{A}_{\Delta\xi} - U^t \mathcal{P} U] \xi_k^t + \xi_{k-d}^t \bar{A}_{\xi d}^t \mathcal{P} \bar{A}_{\xi d} \xi_{k-d}^t + 2\xi_k^t \bar{A}_{\Delta\xi}^t \mathcal{P} \bar{A}_{\xi d} \xi_{k-d}
\end{aligned} \tag{3.6}$$

For V_b, we have

$$\begin{aligned}
V_b(\xi_{k+1}) \; - \; V_b(\xi_k) &= \sum_{j=k+1-d}^{k} \xi_j^t \bar{I}^t \mathcal{W} \bar{I} \xi_j \; - \; \sum_{j=k-d}^{k-1} \xi_j^t \bar{I}^t \mathcal{W} \bar{I} \xi_j \\
&= \xi_k^t \bar{I}^t \mathcal{W} \bar{I} \xi_k \; - \; \xi_{k-d}^t \bar{I}^t \mathcal{W} \bar{I} \xi_{k-d}
\end{aligned} \tag{3.7}$$

For V_c, we have

$$\begin{aligned}
V_c(\xi_{k+1}) \; - \; V_c(\xi_k) &= \sum_{p=-\bar{d}+2}^{-\underline{d}+1} \sum_{j=k+p}^{k} \xi_j^t \tilde{I}^t \mathcal{Q} \tilde{I} \xi_j \; - \; \sum_{p=-\bar{d}+2}^{-\underline{d}+1} \sum_{j=k+p-1}^{k-1} \xi_j^t \tilde{I}^t \mathcal{Q} \tilde{I} \xi_j \\
&= \sum_{p=-\bar{d}+2}^{-\underline{d}+1} \Bigg[\xi_k^t \tilde{I}^t \mathcal{Q} \tilde{I} \xi_k + \sum_{j=k+p}^{k-1} \xi_j^t \tilde{I}^t \mathcal{Q} \tilde{I} \xi_j - \xi_{k+p-1}^t \tilde{I}^t \mathcal{Q} \tilde{I} \xi_{k+p-1} \\
&- \sum_{j=k+p-1}^{k-1} \xi_j^t \tilde{I}^t \mathcal{Q} \tilde{I} \xi_j \Bigg]
\end{aligned}$$

$$
\begin{aligned}
&= \sum_{p=-\bar{d}+2}^{-\underline{d}+1} \xi_k^t \tilde{I}^t \mathcal{Q} \tilde{I} \xi_k - \sum_{p=-\bar{d}+2}^{-\underline{d}+1} \xi_{k+p-1}^t \tilde{I}^t \mathcal{Q} \tilde{I} \xi_{k+p-1} \\
&= (\bar{d} - \underline{d})\, \xi_k^t \tilde{I}^t \mathcal{Q} \tilde{I} \xi_k - \sum_{j=k-\bar{d}+1}^{k-\underline{d}} \xi_j^t \tilde{I}^t \mathcal{Q} \tilde{I} \xi_j
\end{aligned} \tag{3.8}
$$

For V_d, we have

$$
\begin{aligned}
V_d(\xi_{k+1}) - V_d(\xi_k) &= \sum_{p=-\bar{d}+1}^{-\underline{d}} \sum_{j=k+p+1}^{k} [(j-p-k)\xi_j^t \tilde{I}^t \mathcal{Q} \tilde{I} \xi_j] \\
&- \sum_{p=-\bar{d}+1}^{-\underline{d}} \sum_{j=k+p}^{k-1} [(j-p-k+1)\xi_j^t \tilde{I}^t \mathcal{Q} \tilde{I} \xi_j] \\
&= -\sum_{p=-\bar{d}+1}^{-\underline{d}} p\, \xi_k^t \tilde{I}^t \mathcal{Q} \tilde{I} \xi_k - \sum_{p=-\bar{d}+1}^{-\underline{d}} \xi_{k+p}^t \tilde{I}^t \mathcal{Q} \tilde{I} \xi_{k+p} \\
&= -\sum_{j=k-\bar{d}+1}^{k-\underline{d}} \xi_j^t \tilde{I}^t \mathcal{Q} \tilde{I} \xi_j + \frac{1}{2}(\bar{d}+\underline{d})\,(\underline{d}-\bar{d}+1)\, \xi_k^t \tilde{I}^t \mathcal{Q} \tilde{I} \xi_k
\end{aligned} \tag{3.9}
$$

It follows from (3.5) and (3.6)-(3.9) that

$$
\begin{aligned}
V_{k+1} - V_k &= \xi_k^t [\bar{A}_{\Delta\xi}^t \mathcal{P} \bar{A}_{\Delta\xi} - U^t \mathcal{P} U] \xi_k^t + \xi_{k-d}^t \bar{A}_{\xi d}^t \mathcal{P} \bar{A}_{\xi d} \xi_{k-d}^t + 2\xi_k^t \bar{A}_{\Delta\xi}^t \mathcal{P} \bar{A}_{\xi d} \xi_{k-d} \\
&+ \xi_k^t \bar{I}^t \mathcal{W} \bar{I} \xi_k - \xi_{k-d}^t \bar{I}^t \mathcal{W} \bar{I} \xi_{k-d} \\
&+ (\bar{d}-\underline{d})\, \xi_k^t \tilde{I}^t \mathcal{Q} \tilde{I} \xi_k + \sum_{j=k-\bar{d}+1}^{k-\underline{d}} \xi_j^t \tilde{I}^t \mathcal{Q} \tilde{I} \xi_j \\
&- \sum_{j=k-\bar{d}+1}^{k-\underline{d}} \xi_j^t \tilde{I}^t \mathcal{Q} \tilde{I} \xi_j + \frac{1}{2}(\bar{d}+\underline{d})\,(\underline{d}-\bar{d}+1)\, \xi_k^t \tilde{I}^t \mathcal{Q} \tilde{I} \xi_k \\
&= \xi_k^t [\bar{A}_{\Delta\xi}^t \mathcal{P} \bar{A}_{\Delta\xi} - U^t \mathcal{P} U] \xi_k^t + \xi_{k-d}^t \bar{A}_{\xi d}^t \mathcal{P} \bar{A}_{\xi d} \xi_{k-d}^t + 2\xi_k^t \bar{A}_{\Delta\xi}^t \mathcal{P} \bar{A}_{\xi d} \xi_{k-d} \\
&+ \xi_k^t \bar{I}^t \mathcal{W} \bar{I} \xi_k - \xi_{k-d}^t \bar{I}^t \mathcal{W} \bar{I} \xi_{k-d} \\
&+ [\bar{d} + \frac{1}{2}(\bar{d}-\underline{d})\,(\bar{d}+\underline{d}-1)]\, \xi_k^t \tilde{I}^t \mathcal{Q} \tilde{I} \xi_k \\
&= \begin{bmatrix} \xi_k \\ \xi_{k-d} \end{bmatrix}^t \begin{bmatrix} \begin{array}{c} \bar{A}_{\Delta\xi}^t \mathcal{P} \bar{A}_{\Delta\xi} - U^t \mathcal{P} U + \\ \bar{I}^t \mathcal{W} \bar{I} + d^+\, \tilde{I}^t \mathcal{Q} \tilde{I} \end{array} & \bar{A}_{\Delta\xi}^t \mathcal{P} \bar{A}_{\xi d} \\ \bullet & -\bar{I}^t \mathcal{W} \bar{I} + \bar{A}_{\xi d}^t \mathcal{P} \bar{A}_{\xi d} \end{bmatrix} \begin{bmatrix} \xi_k \\ \xi_{k-d} \end{bmatrix} \\
&= \begin{bmatrix} \xi_k \\ \xi_{k-d} \end{bmatrix}^t \Upsilon(d^+) \begin{bmatrix} \xi_k \\ \xi_{k-d} \end{bmatrix}
\end{aligned} \tag{3.10}
$$

By Laypunov theory, asymptotic stability ($V_{k+1} - V_k < 0, \forall \xi_k \neq 0$) implies that $\Upsilon(d^+) < 0$ which by Schur complement is equivalent to

$$\begin{bmatrix} -U^t \mathcal{P} U + \bar{I}^t \mathcal{W} \bar{I} + d^+ \tilde{I}^t \mathcal{Q} \tilde{I} & 0 & \bar{A}^t_{\Delta\xi} \mathcal{P} \\ \bullet & -\bar{I}^t \mathcal{W} \bar{I} & \bar{A}^t_{\xi d} \mathcal{P} \\ \bullet & \bullet & -\mathcal{P} \end{bmatrix} < 0 \tag{3.11}$$

Define

$$\mathcal{X} \triangleq \mathcal{P}^{-1} = \begin{bmatrix} \mathcal{X}_x & \mathcal{X}_f & 0 \\ \bullet & \mathcal{X}_d & 0 \\ \bullet & \bullet & \mathcal{X}_s \end{bmatrix} \in \mathbb{R}^{3n \times 3n}, \quad 0 < \mathcal{X}_x = \mathcal{X}_x^t \in \mathbb{R}^{n \times n},$$
$$0 < \mathcal{X}_d = \mathcal{X}_d^t \in \mathbb{R}^{n \times n}, \quad \mathcal{X}_f \in \mathbb{R}^{n \times n} \tag{3.12}$$

and

$$d^+ = \bar{d} + \frac{1}{2}(\bar{d} - \underline{d})\ (\bar{d} + \underline{d} - 1)$$

Using the congruence transformation $diag[\mathcal{X} \quad I \quad \mathcal{X}]$ and invoking the linearizations

$$\mathcal{X} \bar{I}^t \mathcal{W} \bar{I} \mathcal{X} \triangleq \mathcal{M} = d^{\dagger} \begin{bmatrix} \mathcal{M}_x & \mathcal{M}_f & 0 \\ \bullet & \mathcal{M}_d & 0 \\ \bullet & \bullet & 0 \end{bmatrix},$$
$$0 < \mathcal{M}_x^t = \mathcal{M}_x \in \mathbb{R}^{n \times n},\ \mathcal{M}_f \in \mathbb{R}^{n \times n},\ 0 < \mathcal{M}_d^t = \mathcal{M}_d \in \mathbb{R}^{n \times n},$$

$$d^+\ \mathcal{X} \tilde{I}^t \mathcal{Q} \tilde{I} \mathcal{X} \triangleq d^+\ \mathcal{N} = d^+ \begin{bmatrix} \mathcal{N}_x & \mathcal{N}_f & 0 \\ \bullet & \mathcal{N}_d & 0 \\ \bullet & \bullet & 0 \end{bmatrix},$$
$$0 < \mathcal{N}_x^t = \mathcal{N}_x \in \mathbb{R}^{n \times n},\ \mathcal{N}_f \in \mathbb{R}^{n \times n},\ 0 < \mathcal{N}_d^t = \mathcal{N}_d \in \mathbb{R}^{n \times n},$$

$$\mathcal{Z} = \mathcal{X} U^t \mathcal{P} U \mathcal{X} \triangleq \begin{bmatrix} \mathcal{Z}_x & \mathcal{Z}_f & 0 \\ \bullet & \mathcal{Z}_d & 0 \\ \bullet & \bullet & \mathcal{Z}_s \end{bmatrix},$$
$$0 < \mathcal{Z}_x^t = \mathcal{Z}_x \in \mathbb{R}^{n \times n},\ \mathcal{Z}_f \in \mathbb{R}^{n \times n},\ 0 < \mathcal{Z}_d^t = \mathcal{Z}_d \in \mathbb{R}^{n \times n},\ 0 < \mathcal{Z}_s^t = \mathcal{Z}_s \in \mathbb{R}^{n \times n},$$

inequality (3.11) becomes

$$\begin{bmatrix} -\mathcal{Z} + \mathcal{M} + d^+ \mathcal{N} & 0 & \mathcal{X} \bar{A}^t_{\Delta\xi} \\ \bullet & -\bar{I}^t \mathcal{W} \bar{I} & \bar{A}^t_{\xi d} \\ \bullet & \bullet & -\mathcal{X} \end{bmatrix} < 0 \tag{3.13}$$

The following theorem establishes LMI-based sufficient conditions for dealy-dependent robust stability of system (Σ_2).

Theorem 3.1 *System* (Σ_2) *with* $u_k \equiv 0$ *is delay-dependent robustly stable if there exist matrices* $0 < \mathcal{X}_x = \mathcal{X}_x^t \in I\!R^{n\times n}$, $0 < \mathcal{X}_d = \mathcal{X}_d^t \in I\!R^{n\times n}$, $0 < \mathcal{X}_s = \mathcal{X}_s^t \in I\!R^{n\times n}$, $\mathcal{X}_f \in I\!R^{n\times n}$, $0 < \mathcal{Z}_x = \mathcal{Z}_x^t \in I\!R^{n\times n}$, $0 < \mathcal{Z}_d = \mathcal{Z}_d^t \in I\!R^{n\times n}$, $\mathcal{Z}_f \in I\!R^{n\times n}$, $0 < \mathcal{Z}_s = \mathcal{Z}_s^t \in I\!R^{n\times n}$, $0 < \mathcal{M}_x = \mathcal{M}_x^t \in I\!R^{n\times n}$, $0 < \mathcal{M}_d = \mathcal{M}_d^t \in I\!R^{n\times n}$, $0 < \mathcal{N}_x = \mathcal{N}_x^t \in I\!R^{n\times n}$, $0 < \mathcal{N}_d = \mathcal{N}_d^t \in I\!R^{n\times n}$, $\mathcal{M}_f \in I\!R^{n\times n}$, $\mathcal{N}_f \in I\!R^{n\times n}$, $0 < \mathcal{W} = \mathcal{W}^t \in I\!R^{n\times n}$ *and scalars* $\delta > 0$, $\alpha > 0$ *such that the following inequality holds for all admissible uncertainties*

$$\begin{bmatrix} -\mathcal{Z} + \mathcal{M} + d^+\mathcal{N} & 0 & \Pi_a & \Pi_n \\ \bullet & -\bar{I}^t\mathcal{W}\bar{I} - \alpha\mathcal{B} & \bar{A}_{\xi d}^t & 0 \\ \bullet & \bullet & -\mathcal{X} + \delta\bar{M}\bar{M}^t & 0 \\ \bullet & \bullet & \bullet & -\delta I \end{bmatrix} < 0 \qquad (3.14)$$

where

$$\Pi_a = \begin{bmatrix} \mathcal{X}_x + \mathcal{X}_f & \mathcal{X}_x A_{od}^t - \mathcal{X}_x E^t - \mathcal{X}_f E^t & \mathcal{X}_f \\ \mathcal{X}_f^t + \mathcal{X}_d & \mathcal{X}_f^t A_{od}^t - \mathcal{X}_f^t E^t - \mathcal{X}_d E^t & \mathcal{X}_d \\ 0 & -\mathcal{X}_s A_d^t & \mathcal{X}_s \end{bmatrix}, \quad \Pi_n = \begin{bmatrix} \mathcal{X}_x N_{ad}^t \\ \mathcal{X}_f^t N_{ad}^t \\ -\mathcal{X}_s N_d^t \end{bmatrix} \qquad (3.15)$$

Proof: Using (3.4) in (3.13) and manipulating with the aid of **Fact 1**, it yields

$$\begin{bmatrix} -\mathcal{Z} + \mathcal{M} + d^+\mathcal{N} & 0 & \mathcal{X}\bar{A}_{\xi o}^t + \mathcal{X}\bar{N}^t\Delta^t\bar{M} \\ \bullet & -\bar{I}^t\mathcal{W}\bar{I} & \bar{A}_{\xi d}^t \\ \bullet & \bullet & -\mathcal{X} \end{bmatrix}$$

$$= \begin{bmatrix} -\mathcal{Z} + \mathcal{M} + d^+\mathcal{N} & 0 & \mathcal{X}\bar{A}_{\xi o}^t \\ \bullet & -\bar{I}^t\mathcal{W}\bar{I} & \bar{A}_{\xi d}^t \\ \bullet & \bullet & -\mathcal{X} \end{bmatrix} + \begin{bmatrix} \mathcal{X}\bar{N}^t \\ 0 \\ 0 \end{bmatrix} \Delta_k^t \begin{bmatrix} 0 & 0 & \bar{M}^t \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ 0 \\ \bar{M} \end{bmatrix} \Delta_k \begin{bmatrix} \bar{N}\mathcal{X} & 0 & 0 \end{bmatrix}$$

$$\leq \begin{bmatrix} -\mathcal{Z} + \mathcal{M} + d^+\mathcal{N} & 0 & \mathcal{X}\bar{A}_{\xi o}^t \\ \bullet & -\bar{I}^t\mathcal{W}\bar{I} & \bar{A}_{\xi d}^t \\ \bullet & \bullet & -\mathcal{X} \end{bmatrix} + \delta^{-1} \begin{bmatrix} \mathcal{X}\bar{N}^t \\ 0 \\ 0 \end{bmatrix} [\bar{N}\mathcal{X} \quad 0 \quad 0]$$

$$+\delta \begin{bmatrix} 0 \\ 0 \\ \bar{M} \end{bmatrix} [0 \quad 0 \quad \bar{M}^t], \quad \delta > 0$$

$$= \begin{bmatrix} -\mathcal{Z} + \mathcal{M} + d^+\mathcal{N} + \delta^{-1}\mathcal{X}\bar{N}^t\bar{N}\mathcal{X} & 0 & \mathcal{X}\bar{A}_{\xi o}^t \\ \bullet & -\bar{I}^t\mathcal{W}\bar{I} & \bar{A}_{\xi d}^t \\ \bullet & \bullet & -\mathcal{X} + \delta\bar{M}\bar{M}^t \end{bmatrix} \qquad (3.16)$$

By Schur complement, the last inequality in (3.16) becomes

$$\begin{bmatrix} -\mathcal{Z} + \mathcal{M} + d^+\mathcal{N} & 0 & \mathcal{X}\bar{A}_{\xi o}^t & \mathcal{X}\bar{N}^t \\ \bullet & -\bar{I}^t\mathcal{W}\bar{I} & \bar{A}_{\xi d}^t & 0 \\ \bullet & \bullet & -\mathcal{X} + \delta\bar{M}\bar{M}^t & 0 \\ \bullet & \bullet & \bullet & -\delta I \end{bmatrix} < 0, \quad \delta > 0 \qquad (3.17)$$

Let

$$\mathcal{B} \;=\; \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix}, \;\; \mathbf{B} = \begin{bmatrix} 0 & 0 \\ 0 & \mathcal{B} \end{bmatrix} \geq 0 \tag{3.18}$$

It follows from the **S**-procedure [4] that there exists $\alpha > 0$ such that the following inequality holds

$$\begin{bmatrix} -\mathcal{Z} + \mathcal{M} + d^{+}\mathcal{N} & 0 & \mathcal{X}\bar{A}^{t}_{\xi o} & \mathcal{X}\bar{N}^{t} \\ \bullet & -\bar{I}^{t}\mathcal{W}\bar{I} - \alpha\mathcal{B} & \bar{A}^{t}_{\xi d} & 0 \\ \bullet & \bullet & -\mathcal{X} + \delta\bar{M}\bar{M}^{t} & 0 \\ \bullet & \bullet & \bullet & -\delta I \end{bmatrix} < 0, \;\; \delta > 0 \tag{3.19}$$

Algebraic manipulation of LMI (3.19) using (3.4) and (3.12) yields (3.14) as desired. $\nabla\nabla\nabla$

In the absence of uncertainties we get the following corollary

Corollary 3.1 *System* (Σ_2) *with* $u_k \equiv 0,\;\; M \equiv 0,\; N_a \equiv 0$ *and* $N_d \equiv 0$ *is delay-dependent quadratically stable if there exist matrices* $0 < \mathcal{X}_x = \mathcal{X}_x^t \in \mathbb{R}^{n\times n},\; 0 < \mathcal{X}_d = \mathcal{X}_d^t \in \mathbb{R}^{n\times n},\; 0 < \mathcal{X}_s = \mathcal{X}_s^t \in \mathbb{R}^{n\times n},\; \mathcal{X}_f \in \mathbb{R}^{n\times n},\; 0 < \mathcal{Z}_x = \mathcal{Z}_x^t \in \mathbb{R}^{n\times n},\; 0 < \mathcal{Z}_d = \mathcal{Z}_d^t \in \mathbb{R}^{n\times n},\; \mathcal{Z}_f \in \mathbb{R}^{n\times n},\; 0 < \mathcal{Z}_s = \mathcal{Z}_s^t \in \mathbb{R}^{n\times n},\; 0 < \mathcal{M}_x = \mathcal{M}_x^t \in \mathbb{R}^{n\times n},\; 0 < \mathcal{M}_d = \mathcal{M}_d^t \in \mathbb{R}^{n\times n},\; 0 < \mathcal{N}_x = \mathcal{N}_x^t \in \mathbb{R}^{n\times n},\; 0 < \mathcal{N}_d = \mathcal{N}_d^t \in \mathbb{R}^{n\times n},\; \mathcal{M}_f \in \mathbb{R}^{n\times n},\; \mathcal{N}_f \in \mathbb{R}^{n\times n},\; 0 < \mathcal{W} = \mathcal{W}^t \in \mathbb{R}^{n\times n}$ *and a scalar* $\alpha > 0$ *satisfying the following inequality*

$$\begin{bmatrix} -\mathcal{Z} + \mathcal{M} + d^{+}\mathcal{N} & 0 & \Pi_a \\ \bullet & -\bar{I}^{t}\mathcal{W}\bar{I} - \alpha\mathcal{B} & \bar{A}^{t}_{\xi d} \\ \bullet & \bullet & -\mathcal{X} \end{bmatrix} < 0 \tag{3.20}$$

4 Delay-Dependent State-Feedback Stabilization

Consider system (3.2) and applying the state-feedback control law

$$u = K_o\, x_k = K_o\, \bar{I}\, \xi_k \tag{4.1}$$

we obtain the following closed-loop system

$$\begin{aligned} (\Sigma_2):\;\; U\,\xi_{k+1} \;&=\; \begin{bmatrix} I & I & 0 \\ A_{\Delta odk} - E & -E & -A_{\Delta d} \\ 0 & I & I \end{bmatrix} \xi_k \;+\; \bar{A}_{\xi d}\,\xi_{k-d} \\ &=\; \bar{A}_{\Delta\xi k}\xi \;+\; \bar{A}_{\xi d}\,\xi_{k-d} \end{aligned} \tag{4.2}$$

with

$$\bar{\mathcal{A}}_{\Delta\xi k} = (\bar{\mathcal{A}}_{\xi ko} + \bar{M}\Delta\bar{N}) \; , \quad \bar{\mathcal{A}}_{\xi ko} = \begin{bmatrix} I & I & 0 \\ A_{od} + B_o K_o - E & -E & -A_d \\ 0 & I & I \end{bmatrix}$$

It follows from inequality (3.19) that system (4.2) is delay-dependent robustly stabilizable if the following LMI is satisfied for all admissible uncertainties:

$$\begin{bmatrix} -\mathcal{Z} + \mathcal{M} + d^+\mathcal{N} & 0 & \mathcal{X}\bar{A}^t_{\xi ko} & \mathcal{X}\bar{N}^t \\ \bullet & -\bar{I}^t\mathcal{W}\bar{I} & \bar{A}^t_{\xi d} & 0 \\ \bullet & \bullet & -\mathcal{X} + \delta\bar{M}\bar{M}^t & 0 \\ \bullet & \bullet & \bullet & -\delta I \end{bmatrix} < 0 \tag{4.3}$$

Following the steps of section 3, we get the stabilization result which is summarized by the following theorem:

Theorem 4.1 *System* (Σ_2) *is delay-dependent robustly stabilizable by the feedback controller (4.1) if there exist matrices* $0 < \mathcal{X}_x = \mathcal{X}_x^t \in I\!\!R^{n\times n}$, $0 < \mathcal{X}_d = \mathcal{X}_d^t \in I\!\!R^{n\times n}$, $0 < \mathcal{X}_s = \mathcal{X}_s^t \in I\!\!R^{n\times n}$, $\mathcal{X}_f \in I\!\!R^{n\times n}$, $0 < \mathcal{Z}_x = \mathcal{Z}_x^t \in I\!\!R^{n\times n}$, $0 < \mathcal{Z}_d = \mathcal{Z}_d^t \in I\!\!R^{n\times n}$, $\mathcal{Z}_f \in I\!\!R^{n\times n}$, $0 < \mathcal{Z}_s = \mathcal{Z}_s^t \in I\!\!R^{n\times n}$, $0 < \mathcal{M}_x = \mathcal{M}_x^t \in I\!\!R^{n\times n}$, $0 < \mathcal{M}_d = \mathcal{M}_d^t \in I\!\!R^{n\times n}$, $0 < \mathcal{N}_x = \mathcal{N}_x^t \in I\!\!R^{n\times n}$, $0 < \mathcal{N}_d = \mathcal{N}_d^t \in I\!\!R^{n\times n}$, $\mathcal{M}_f \in I\!\!R^{n\times n}$, $\mathcal{N}_f \in I\!\!R^{n\times n}$, $\mathcal{Y}_x \in I\!\!R^{m\times n}$, $\mathcal{Y}_f \in I\!\!R^{m\times n}$, $0 < \mathcal{W} = \mathcal{W}^t \in I\!\!R^{n\times n}$ *and scalars* $\delta > 0$, $\alpha > 0$ *such that the following LMI holds for all admissible uncertainties*

$$\begin{bmatrix} -\mathcal{Z} + \mathcal{M} + d^+\mathcal{N} & 0 & \Pi_b & \Pi_n \\ \bullet & -\bar{I}^t\mathcal{W}\bar{I} - \alpha\mathcal{B} & \bar{A}^t_{\xi d} & 0 \\ \bullet & \bullet & -\mathcal{X} + \delta\bar{M}\bar{M}^t & 0 \\ \bullet & \bullet & \bullet & -\delta I \end{bmatrix} < 0 \tag{4.4}$$

where

$$\Pi_b = \begin{bmatrix} \mathcal{X}_x + \mathcal{X}_f & \mathcal{X}_x A^t_{od} + \mathcal{Y}^t_x B^t_o - \mathcal{X}_x E^t - \mathcal{X}_f E^t & \mathcal{X}_f \\ \mathcal{X}^t_f + \mathcal{X}_d & \mathcal{X}^t_f A^t_{od} + \mathcal{Y}^t_f B^t_o - \mathcal{X}^t_f E^t - \mathcal{X}_d E^t & \mathcal{X}_d \\ 0 & -\mathcal{X}_s A^t_d & \mathcal{X}_s \end{bmatrix}, \quad \Pi_n = \begin{bmatrix} \mathcal{X}_x N^t_{ad} \\ \mathcal{X}^t_f N^t_{ad} \\ -\mathcal{X}_s N^t_d \end{bmatrix} \tag{4.5}$$

The feedback gain is given by $K_o = \mathcal{Y}_x \mathcal{X}_x^{-1}$.

Proof: Follows from **Theorem** 3.1 after taking $\mathcal{Y}_x = K_o\mathcal{X}_x$ and $\mathcal{Y}_f = K_o\mathcal{X}_f$. $\nabla\nabla\nabla$

In the absence of uncertainties we have the following corollary

Corollary 4.1 *System* (Σ_2) *with* $M \equiv 0$, $N_a \equiv 0$ *and* $N_d \equiv 0$ *is delay-dependent quadratically stabilizable by the feedback controller (4.1) law if there exist matrices*

$0 < \mathcal{X}_x = \mathcal{X}_x^t \in I\!R^{n\times n}$, $0 < \mathcal{X}_d = \mathcal{X}_d^t \in I\!R^{n\times n}$, $0 < \mathcal{X}_s = \mathcal{X}_s^t \in I\!R^{n\times n}$, $\mathcal{X}_f \in I\!R^{n\times n}$, $0 < \mathcal{Z}_x = \mathcal{Z}_x^t \in I\!R^{n\times n}$, $0 < \mathcal{Z}_d = \mathcal{Z}_d^t \in I\!R^{n\times n}$, $\mathcal{Z}_f \in I\!R^{n\times n}$, $0 < \mathcal{Z}_s = \mathcal{Z}_s^t \in I\!R^{n\times n}$, $0 < \mathcal{M}_x = \mathcal{M}_x^t \in I\!R^{n\times n}$, $0 < \mathcal{M}_d = \mathcal{M}_d^t \in I\!R^{n\times n}$, $0 < \mathcal{N}_x = \mathcal{N}_x^t \in I\!R^{n\times n}$, $0 < \mathcal{N}_d = \mathcal{N}_d^t \in I\!R^{n\times n}$, $\mathcal{M}_f \in I\!R^{n\times n}$, $\mathcal{N}_f \in I\!R^{n\times n}$, $\mathcal{Y}_x \in I\!R^{m\times n}$, $\mathcal{Y}_f \in I\!R^{m\times n}$, $0 < \mathcal{W} = \mathcal{W}^t \in I\!R^{n\times n}$ *and a scalar* $\alpha > 0$ *satisfying the following LMI*

$$\begin{bmatrix} -\mathcal{Z} + \mathcal{M} + d^{+}\mathcal{N} & 0 & \Pi_b \\ \bullet & -\bar{I}^t\mathcal{W}\bar{I} - \alpha\mathcal{B} & \bar{A}_{\xi d}^t \\ \bullet & \bullet & -\mathcal{X} \end{bmatrix} < 0 \tag{4.6}$$

The feedback gain is given by $K_o = \mathcal{Y}_x\mathcal{X}_x^{-1}$.

4.1 Example

In order to illustrate the validity of our approach, we consider the following discrete-time singular systems with state-delay

$$\begin{aligned} A_o &= \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, \ A_d = \begin{bmatrix} 0.01 & 0.10 \\ 0 & 0.10 \end{bmatrix}, \ B_o = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \\ M &= \begin{bmatrix} 0.01 & 0.02 \end{bmatrix}, \ N_a = \begin{bmatrix} 0.02 & 0.01 \end{bmatrix}, \ N_d = \begin{bmatrix} 0.01 & 0.01 \end{bmatrix}, \\ N_b &= 0.01, \ E = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \ \underline{d} = 0, \ \bar{d} = 5 \end{aligned}$$

We record that the open-lop system is unstable since its eigenvalues are outside the unit disc. Implementation of the LMI (dceq177) yields the feasible solution

$$\begin{aligned} \mathcal{X}_x &= \begin{bmatrix} 1.2416 & 0.1119 \\ 0.1119 & 1.2997 \end{bmatrix}, \ \mathcal{X}_f = \begin{bmatrix} -0.4575 & -0.1602 \\ 0.0570 & -0.2360 \end{bmatrix}, \\ \mathcal{X}_d &= \begin{bmatrix} 0.8779 & -0.0480 \\ -0.0480 & 0.5658 \end{bmatrix}, \ \mathcal{X}_s = \begin{bmatrix} 1.6944 & -0.0079 \\ -0.0079 & 1.4331 \end{bmatrix}, \\ \mathcal{Z}_x &= \begin{bmatrix} 7.2877 & -0.0397 \\ -0.0397 & 7.3704 \end{bmatrix}, \ \mathcal{Z}_f = \begin{bmatrix} -0.1055 & -0.0427 \\ 0.0445 & -0.0829 \end{bmatrix}, \\ \mathcal{Z}_d &= \begin{bmatrix} 7.4585 & 0.0049 \\ 0.0049 & 7.3351 \end{bmatrix}, \ \mathcal{Z}_s = \begin{bmatrix} 3.9645 & -0.0026 \\ -0.0026 & 3.5666 \end{bmatrix}, \\ \mathcal{M}_x &= \begin{bmatrix} 0.2993 & 0.0006 \\ 0.0006 & 0.2980 \end{bmatrix}, \ \mathcal{M}_d = \begin{bmatrix} 0.2971 & 0.0002 \\ 0.0002 & 0.2986 \end{bmatrix}, \\ \mathcal{N}_x &= \begin{bmatrix} 0.4787 & 0.0008 \\ 0.0008 & 0.4770 \end{bmatrix}, \ \mathcal{N}_d = \begin{bmatrix} 0.4753 & -0.0003 \\ -0.0003 & 0.4777 \end{bmatrix}, \ \delta = 2.1820 \end{aligned}$$

These values give the feedback gain as $K_o = [1.9760 \quad 2.7106]$ which renders the closed-loop singular system stable.

5 Robust Results with Uncertain E-Matrix

All the foregoing robust analysis and design results are valied for nominal E-matrix. In the case when it is uncertain, then an effective modification has to be introduced. We limit ourselves to one class of uncertain E-matrix, that is, $E_n = E + M\Delta_t N_e$ where $M \in \mathbb{R}^{n\times n_m}$, $N_e \in \mathbb{R}^{n_n\times n}$ are real and known constant matrices with Δ_t is a bounded matrix of uncertainties satisfying $\Delta_t^t\Delta_t < I$. It is obvious that this class of norm-bounded uncertainties is selected to render the analysis tractable. Accordingly, the CF matrix becoms

$$\breve{\mathcal{A}}_{\Delta\xi} = \bar{\mathcal{A}}_{\xi o} + \bar{M}\,\Delta_t\,\hat{N} \tag{5.1}$$

where

$$\begin{aligned} \bar{A}_{\xi o} &= \begin{bmatrix} I & I & 0 \\ A_{od} - E & -E & -A_d \\ 0 & I & I \end{bmatrix}, \quad \bar{M} = \begin{bmatrix} 0 \\ M \\ 0 \end{bmatrix}, \\ \hat{N} &= [N_{ad} - N_e \quad -N_e \quad -N_d], \;\; N_{ad} = N_a + N_d, \;\; A_{od} = A_o + A_d \end{aligned} \tag{5.2}$$

In the following, we list without proof the main delay-dependent robust stability and stabilization results

Theorem 5.1 *System* (Σ_2) *with uncertain E-matrix and* $u_k \equiv 0$ *is delay-dependent robustly stable if there exist matrices* $0 < \mathcal{X}_x = \mathcal{X}_x^t \in \mathbb{R}^{n\times n}$, $0 < \mathcal{X}_d = \mathcal{X}_d^t \in \mathbb{R}^{n\times n}$, $0 < \mathcal{X}_s = \mathcal{X}_s^t \in \mathbb{R}^{n\times n}$, $0 < \mathcal{X}_f^t = \mathcal{X}_f \in \mathbb{R}^{n\times n}$, $\mathcal{X}_h \in \mathbb{R}^{n\times n}$, $\mathcal{X}_g \in \mathbb{R}^{n\times n}$, $\mathcal{M}_d = \mathcal{M}_d^t \in \mathbb{R}^{n\times n}$, $\mathcal{M}_e \in \mathbb{R}^{n\times n}$, $\mathcal{M}_q \in \mathbb{R}^{n\times n}$, $\mathcal{M}_f \in \mathbb{R}^{n\times n}$, $\mathcal{M}_x = \mathcal{M}_x^t \in \mathbb{R}^{n\times n}$, $0 < \mathcal{M}_s = \mathcal{M}_s^t \in \mathbb{R}^{n\times n}$, $\mathcal{L}_d = \mathcal{L}_d^t \in \mathbb{R}^{n\times n}$, $\mathcal{L}_e \in \mathbb{R}^{n\times n}$, $\mathcal{L}_q \in \mathbb{R}^{n\times n}$, $\mathcal{L}_f \in \mathbb{R}^{n\times n}$, $\mathcal{L}_x = \mathcal{L}_x^t \in \mathbb{R}^{n\times n}$, $0 < \mathcal{L}_s = \mathcal{L}_s^t \in \mathbb{R}^{n\times n}$, $0 < \mathcal{N}_d^t = \mathcal{N}_d \in \mathbb{R}^{n\times n}$, $0 < \mathcal{N}_e^t = \mathcal{N}_e \in \mathbb{R}^{n\times n}$, $0 < \mathcal{N}_q^t = \mathcal{N}_q \in \mathbb{R}^{n\times n}$, $0 < \mathcal{N}_f^t = \mathcal{N}_f \in \mathbb{R}^{n\times n}$, $0 < \mathcal{N}_x = \mathcal{N}_x \in \mathbb{R}^{n\times n}$, $0 < \mathcal{N}_s^t = \mathcal{N}_s \in \mathbb{R}^{n\times n}$ *and scalars* $\delta > 0$, $\sigma > 0$ *such that the following inequalities hold for all admissible uncertainties*

$$\begin{aligned} &\begin{bmatrix} -\mathcal{Z} + \mathcal{M} + d^{+}\mathcal{N} & 0 & \Pi_a & \hat{\Pi}_n \\ \bullet & -\bar{I}^t\mathcal{W}\bar{I} - \alpha\mathcal{B} & \bar{A}_{\xi d}^t & 0 \\ \bullet & \bullet & -\mathcal{X} + \delta\bar{M}\bar{M}^t & 0 \\ \bullet & \bullet & \bullet & -\delta I \end{bmatrix} < 0 \\ &\mathcal{X}^t U^t = U\mathcal{X} \geq 0 \end{aligned} \tag{5.3}$$

where

$$\hat{\Pi}_n = \begin{bmatrix} \mathcal{X}_x(N_{ad}^t - N_e^t) - \mathcal{X}_f^t N_e^t \\ \mathcal{X}_f(N_{ad}^t - N_e^t) - \mathcal{X}_d N_e^t \\ -\mathcal{X}_s N_d^t \end{bmatrix} \tag{5.4}$$

Theorem 5.2 *System* (Σ_2) *with uncertain E-matrix is delay-dependent robustly stablizable by the feedback controller (4.1) if there exist matrices* $0 < \mathcal{X}_x = \mathcal{X}_x^t \in I\!R^{n\times n}$, $0 < \mathcal{X}_d = \mathcal{X}_d^t \in I\!R^{n\times n}$, $0 < \mathcal{X}_s = \mathcal{X}_s^t \in I\!R^{n\times n}$, $0 < \mathcal{X}_f^t = \mathcal{X}_f \in I\!R^{n\times n}$, $\mathcal{X}_h \in I\!R^{n\times n}$, $\mathcal{X}_g \in I\!R^{n\times n}$, $\mathcal{M}_d = \mathcal{M}_d^t \in I\!R^{n\times n}$, $\mathcal{M}_e \in I\!R^{n\times n}$, $\mathcal{M}_q \in I\!R^{n\times n}$, $\mathcal{M}_f \in I\!R^{n\times n}$, $\mathcal{M}_x = \mathcal{M}_x^t \in I\!R^{n\times n}$, $0 < \mathcal{M}_s = \mathcal{M}_s^t \in I\!R^{n\times n}$, $\mathcal{L}_d = \mathcal{L}_d^t \in I\!R^{n\times n}$, $\mathcal{L}_e \in I\!R^{n\times n}$, $\mathcal{L}_q \in I\!R^{n\times n}$, $\mathcal{L}_f \in I\!R^{n\times n}$, $\mathcal{L}_x = \mathcal{L}_x^t \in I\!R^{n\times n}$, $0 < \mathcal{L}_s = \mathcal{L}_s^t \in I\!R^{n\times n}$, $0 < \mathcal{N}_d^t = \mathcal{N}_d \in I\!R^{n\times n}$, $0 < \mathcal{N}_e^t = \mathcal{N}_e \in I\!R^{n\times n}$, $0 < \mathcal{N}_q^t = \mathcal{N}_q \in I\!R^{n\times n}$, $0 < \mathcal{N}_f^t = \mathcal{N}_f \in I\!R^{n\times n}$, $0 < \mathcal{N}_x = \mathcal{N}_x \in I\!R^{n\times n}$, $0 < \mathcal{N}_s^t = \mathcal{N}_s \in I\!R^{n\times n}$, $\mathcal{Y}_s \in I\!R^{m\times n}$, $\mathcal{Y}_f \in I\!R^{m\times n}$ *and scalars* $\delta > 0$, $\sigma > 0$ *such that the following LMIs hold for all admissible uncertainties*

$$\begin{bmatrix} -\mathcal{Z} + \mathcal{M} + d^+\mathcal{N} & 0 & \Pi_a \\ \bullet & -\bar{I}^t\mathcal{W}\bar{I} - \alpha\mathcal{B} & \bar{A}_{\xi d}^t \\ \bullet & \bullet & -\mathcal{X} \end{bmatrix} < 0$$
$$\mathcal{X}^t U^t \;=\; U\mathcal{X} \;\geq\; 0 \tag{5.5}$$

The feedback gain is given by $K_o = \mathcal{Y}_s\mathcal{X}_x^{-1}$.

6 Conclusions

The problems of Delay-dependent robust stability and stabilization of uncertain, linear discrete-time singular systems with state-delay have been examined. A new system representation has been developed to derive new delay-dependent stability criteria without relying on overbounding. A solution to delay-dependent state-feedback stabilization has thus been obtained. A numerical example has been presented.

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