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Application of Opening by Reconstruction to Characterize the Size Distribution of Catchments Extracted from Digital Elevation Models

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Abstract

In this paper, concepts of mathematical morphology are employed to characterize the size distributions of catchments extracted from digital elevation models (DEMs). First, opening by reconstruction is performed iteratively on the binarised catchment image. The number of extracted catchments remaining and removed, and the total remaining catchment area after each iteration is computed. The well marked peak in the plot of the kernel size and the number of catchments removed indicated the dominant size of the extracted catchments. A power law relationship was derived between the total area of remaining catchments and the number of remaining catchments. This power law arises as a consequence of the fractal properties of the size distribution of the extracted catchments.

1 Introduction

A catchment is the topographic area from which all water runoffs finally reach one single given point. It is from this topographic region from which a stream receives runoff, throughflow, and groundwater flow. From the highest point of land down to the stream bottom, all the surface land is considered part of a stream or river's drainage. The principal components of a drainage basin are its topographic form and the topologic structure of its drainage network [6]. Catchments are important geomorphological features which play an important role in hydrological modelling. Many hydrogeological processes, such as soil erosion, mass movements, sediment transport and land cover changes, are strongly linked to this spatial reference unit.

Traditionally, catchments were derived manually from topographic maps, which was a labour intensive activity. In recent times, extraction techniques have evolved from manual through computer assisted to automated methods; with digital elevation models (DEMs) as the input data. DEMs are a popular source for hydrological modeling and watershed characterization because of their simple data structure, widespread availability and they lend themselves too many GIS processes and operations. In seeking the efficient extraction of watersheds from DEMs, various algorithms have been proposed. These algorithms can be divided into two categories; in a digital topographic context (summarized in Soille and Ansoult [9]), and in an image processing context (summarized in Vincent and Soille [11]).

In this paper, the size distribution of catchments extracted from DEMs is studied using concepts of mathematical morphology. In Section 2, the fundamental mathematical morphological operators are introduced. In Section 3, the characterization of the size distribution of catchments extracted from DEMs is performed. Concluding remarks regarding the scope of the study are provided in the final chapter.

2 Mathematical Morphology

Mathematical morphology is a branch of image processing that deals with the extraction of image components that are useful in representation and description of region shape, such as boundaries, skeletons and the convex hull [3]. Mathematical morphology is well suited to the processing of elevation data because in morphology, any image is viewed as a topographic surface, the grey level of a pixel standing for its elevation [11].

Hence, mathematical morphological operators are extremely useful and important in DEM analysis. The fundamental morphological operators are discussed in Matheron [4], Serra [7], Soille [8]. Morphological operators generally require two inputs; the input image *A*, which can be in binary or grayscale form, and the kernel *B*, which is used to determine the precise effect of the operator [7].

Dilation sets the pixel values within the kernel to the maximum value of the pixel neighbourhood. Binary dilation fills the small holes inside particles and gulfs on the boundary of objects, enlarges the size of the particles and may connect neighbouring particles [1]. The dilation operation is expressed as:

$$A \oplus B = \{a+b: a \in A, b \in B\} \quad (1)$$

Erosion sets the pixels values within the kernel to the minimum value of the kernel. Binary erosion removes isolated points and small particles, shrinks other particles, discards peaks on the boundaries of objects, and disconnects some particles [1]. Erosion is the dual operator of dilation:

$$A \ominus B = (A^c \oplus B)^c \quad (2)$$

where A^c denotes the complement of *A*, and *B* is symmetric with respect to reflection about the origin.

An opening is defined as an erosion followed by a dilation using the same kernel for both operations. Opening tends to remove some of the foreground pixels from the edges of regions of foreground pixels. It preserves the foreground regions that have a similar shape to kernel, or that can completely contain the kernel, while discarding all other regions of foreground pixels [2]. The opening operation is expressed as:

$$A \circ B = (A \ominus B) \oplus B \quad (3)$$

Morphological reconstruction allows for the isolation of certain features within an image based on the manipulation of a mask image *X* and a marker image *Y*. It is founded on the concept of geodesic transformations, where dilations or erosion of a marker image are performed until stability is achieved (represented by a mask image) [10].

The geodesic dilation, δ^G used in the reconstruction process is performed through iteration of elementary geodesic dilations, $\delta_{(1)}$, until stability is achieved.

$$\delta^G(Y) = \delta_{(1)}(Y) \ o \ \delta_{(1)}(Y) \ o \ \delta_{(1)}(Y) \ldots \text{until stability} \quad (4)$$

The elementary dilation process is performed using a standard dilation of size one followed by an intersection.

$$\delta_{(1)}(Y) = Y \oplus B \cap X \quad (5)$$

The operation in equation 5 is used for elementary dilation in binary reconstruction. In greyscale reconstruction, the intersection in the equation is replaced with a pointwise minimum [10].

Morphological reconstruction is a useful filtering tool. Figure 1(a) shows an image with circles of various sizes. In order to filter the smaller sized circles, first opening is performed using a square kernel of size 30. The circles that are unable to completely contain the kernel are removed, while the shape of remaining circles is altered (Figure 1(b)). Morphological reconstruction is implemented with Figure 1(a) being the mask and Figure 1(b) being the marker. This restores the original shape of the remaining circles (Figure 1(c)). This process is known as opening by reconstruction.

3 Characterization of The Size Distribution of Catchments Extracted From DEMs

The DEM in Figure 2 shows the area of Great Basin, Nevada, USA. The area is bounded by latitude 38° 15’ to 42° N and longtitude 118° 30’ to 115° 30’W. The DEM is a Global Digital Elevation Model (GTOPO30 DEM) and was downloaded from the USGS GTOPO30 website (http://edcwww.cr.usgs.gov/landdaac/gtopo30/ gtopo30.html). GTOPO30 DEMs are available at a global scale, providing a digital representation of the Earth’s surface at a 30 arc-seconds sampling interval. The land data used to derive GTOPO30 DEMs are obtained from digital terrain elevation data (DTED), the 1-degree

DEM for USA and the digital chart of the world (DCW). The accuracy of GTOPO30 DEMs varies by location according to the source data. The DTED and the 1-degree dataset have a vertical accuracy of $\pm$ 30m while the absolute accuracy of the DCW vector dataset is $\pm$2000m horizontal error and $\pm$650 vertical error [5].

The catchments of the DEM are extracted using the immersion simulation algorithm proposed in Vincent and Soille [11]. This algorithm is based on a progressive flooding of an image, is applicable to n-dimensional images. The pixels are first sorted in increasing order of their grey levels. The successive grey levels are processed in order to simulate the flooding propagation. A distributive sorting technique combined with breadth-first scannings of each grey level allow for an extremely fast computation of catchments. A total of 167 catchments are extracted from the DEM (Figure 3).

The size distribution of the extracted catchments is characterized by performing opening by reconstruction iteratively on the binarised catchment image (Figure 4). In each iteration, opening removes the catchments that are unable to completely contain the kernel. The shape of the remaining catchments are modified. The reconstruction process restores the original shape of the remaining catchments. The number of extracted catchments remaining and removed, and the total remaining catchment area after each iteration of opening by reconstruction is computed (Table 1).

A plot of the kernel size r and number of catchments removed n is drawn (Figure 5). The plot exhibits a well marked peak at kernel size 13. This can be interpreted as the dominant size of the extracted catchments.

A logarithmic plot of the number of remaining catchments N and the total remaining area S is drawn (Figure 6). A slope and y-intercept of the plot is computed. A power law relationship is observed between the two parameters:

$$Log\, N = 1.49*Log\, S + 3.22 \quad (6)$$

$$N = 3.22*S^{1.49} \quad (7)$$

This power law arises as a consequence of the fractal properties of the size distribution of the extracted catchments. In Equation 7, 3.22 is a constant of proportionality, while 1.49 is the fractal dimension of the size distribution of the extracted catchments, which indicates the variance of the size distribution of the extracted catchments; a higher value of D indicates a more varied distribution, while a lower value of D indicates a more even distribution.

4 Conclusion

The size distribution of the catchments extracted from DEMs was characterized using iterative opening by reconstruction. The well marked peak in the plot of the kernel size and the number of catchments removed indicated the dominant size of the extracted catchments. A power law relationship was derived between the total area of remaining catchments and the number of remaining catchments. This power law arises as a consequence of the fractal properties of the size distribution of the extracted catchments. More experiments are being carried out to further quantify the fractal properties of catchments extracted from DEMs.

References

1) Duchene, P. and Lewis, D., Visilog 5 Documentation. Quebec, Noesis Vision Inc., Canada (1996).
2) Fisher, B., Perkins, S., Walker, A. and Wolfart, E., Hypermedia image processing reference, Wiley, New York (1994).
3) Gonzalez, R.C. and Woods, R.E., Digital image processing. Addison-Wesley Pubslishing Company, New York. (1993).
4) Matheron, G., Random Sets and Integral Geometry. Wiley, New York (1975).
5) Miliaresis, G.C., and Argialas, D.P., Quantitative representation of mountain objects extracted from the Global Digital Elevation Model (GTOPO30). International Journal of Remote Sensing, 23 (5) (2002), 949-964.
6) Monkhouse, F.J., Principles of physical geography. University of London Press Ltd., New York (1965).
7) Serra, J., Image Analysis and Mathematical Morphology. Academic Press, London (1982).

8) Soille, P., Morphological image analysis: principles and applications. Springer Verlag, Berlin (2003).
9) Soille, P. and Ansoult, M.M., Automated basin delineation from digital elevation models using mathematical morphology. Signal Processing, 20 (1990), 171-182.
10) Vincent, L., Morphological reconstruction in image analysis: applications and efficient algorithms. IEEE Transactions on Image Processing, 2 (2) (1993), 176-201.
11) Vincent, L. and Soille, P., Watersheds in digital spaces: An efficient algorithm based on immersion simulations. IEEE Transactions on Pattern Analysis and Machine Intelligence, 13 (6) (2003), 583-598.

Tables

Table 1: Statistics of the catchments after each iteration of opening by reconstruction

Kernel size	**Number of catchments removed**	**Number of catchments remaining**	**Total remaining catchment area (pixels)**
1	0	167	69371
2	0	167	69371
3	0	167	69371
4	0	167	69371
5	1	166	69264
6	3	163	69076
7	8	155	68326
8	8	147	67339
9	11	136	65718
10	11	125	63617
11	9	116	61589
12	14	102	57927
13	16	86	53098
14	11	75	49182
15	13	62	44158
16	9	53	39631
17	6	47	35938
18	14	33	27248
19	3	30	25553
20	12	18	17636
21	2	16	15853
22	3	13	13504
23	2	11	11725
24	4	7	7713
25	4	3	3524
26	1	2	2395
27	1	1	1246
28	1	0	0
29	0	0	0
30	0	0	0

Figures

Figure 1: Application of opening by reconstruction to perform filtering. (a) The original image. (b) The opened image. (c) The reconstructed image.

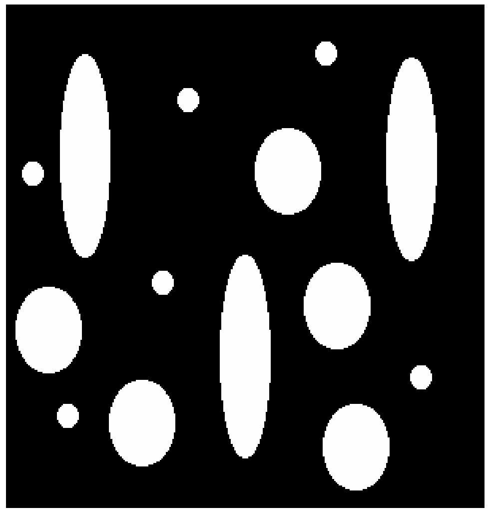
Figure 2: The GTOPO30 DEM of Great Basin. The elevation values of the terrain (minimum 1005 meters and maximum 3651 meters) are rescaled to the interval of 0 to 255 (the brightest pixel has the highest elevation).

Figure 3: The catchments extracted from the DEM of Great Basin

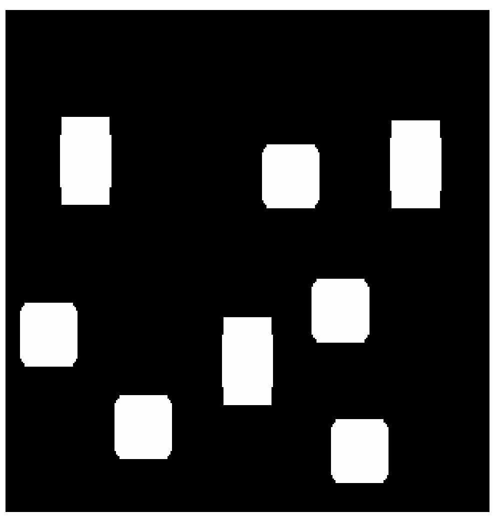
Figure 4: The binarised catchment image

Figure 5: The plot of the kernel size and the number of catchments removed

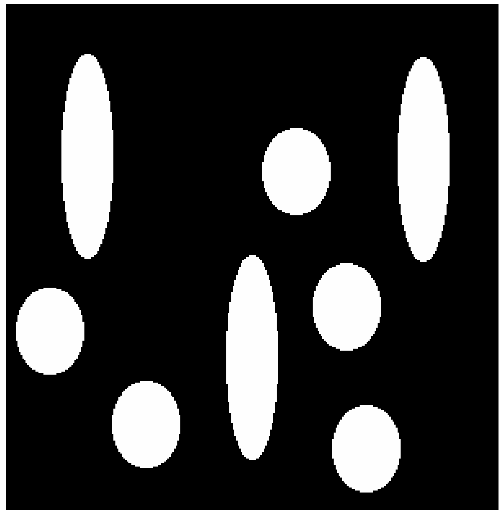
Figure 6: The logarithmic plot of the total area of the remaining catchments and the number of remaining catchments



(a)



(b)



(c)

Figure 1

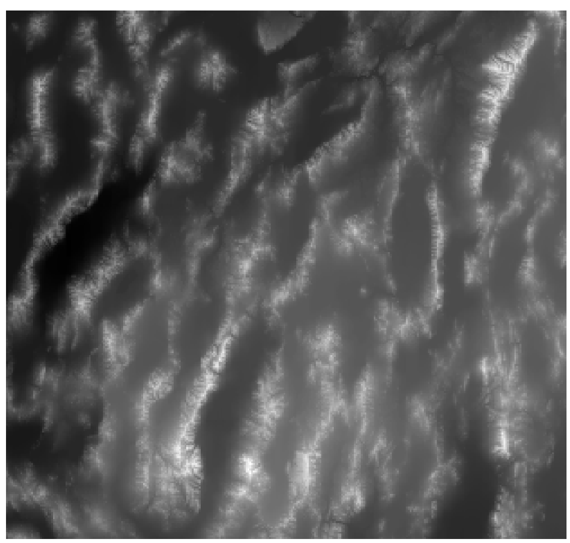


Figure 2

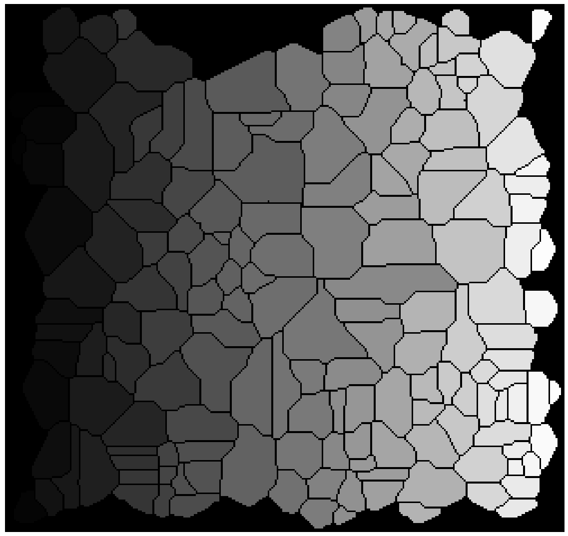


Figure 3

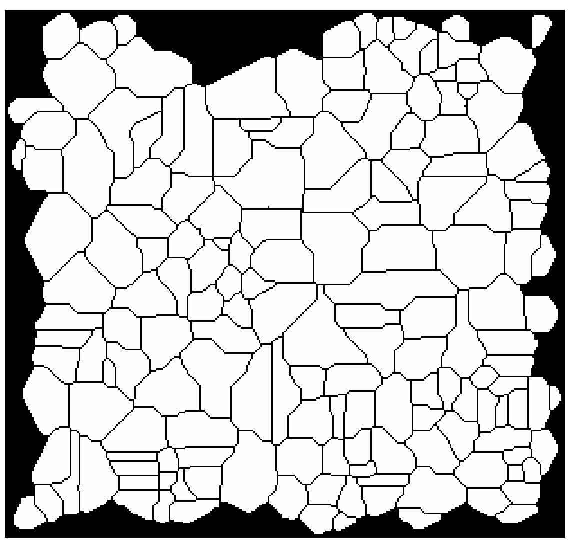


Figure 4

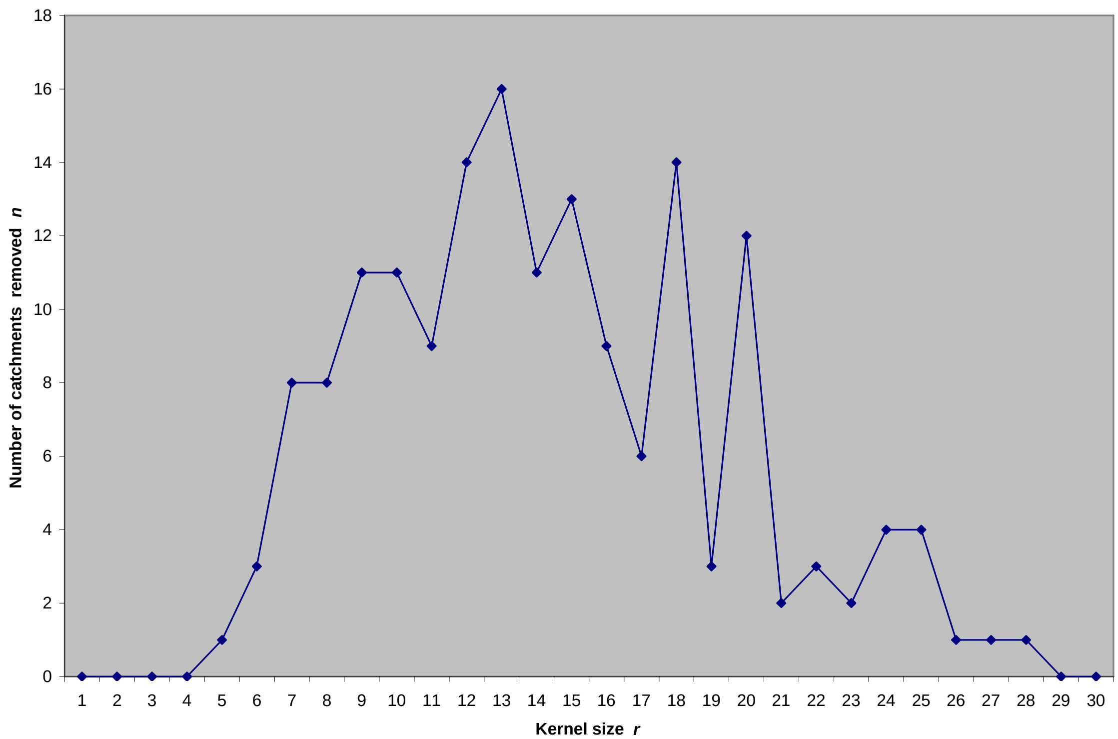

18
16
14
12
10
8
6
4
2
0
Number of catchments removed n
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30
Kernel size r

Figure 5

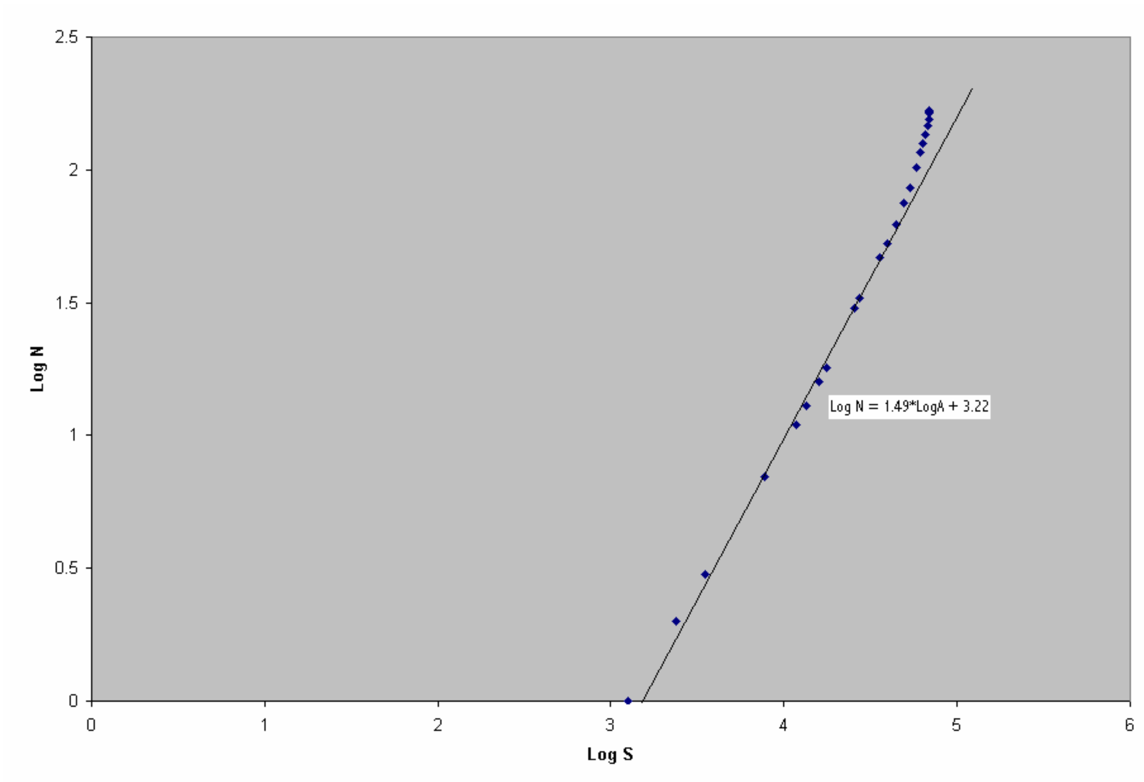


Figure 6