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On T_1 Separation Axioms in I-Fuzzy Topological Spaces

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Abstract

In this note, we introduce the degree to which an I-fuzzy topological space (X, τ) is Sub T_1 (in short, ST_1), which we denote by $ST_1(X, \tau)$ and proved that $KT_1(X, \tau)$, defined by Yue and Fang and $ST_1(X, \tau)$ are equal.

Keywords: I- fuzzy topology, I- fuzzy quasi-coincident neighborhood system, T_1 axiom, KT_1 axiom

1 Introduction

After the introduction of fuzzy sets by Zadeh [13] in 1965, various mathematicians generalized the notion of a fuzzy set. Initially Chang [1] introduced the concept of an I- topology on a set X by replacing 'subsets'by 'fuzzy sets', in the usual definition of a topology on X. Later on Kubiak[4] and Šostak [9] generalized this concept by introducing an I-fuzzy topology on a set X. Pu and Liu [5] established the theory of quasi-coincident neighborhood system in I-topology. Fang [2] extended this concept and defined I-fuzzy quasi-coincident neighborhood system in I-fuzzy topological spaces.

Separation is a crucial branch of fuzzy topology, many mathematicians did a lot of work in this frame. In this note, we are concerned with some separation axioms in an I-fuzzy topological space. Rodabaugh [6, 7] defined RT_0 and Kubiak [4] defined KT_1 axioms in an L- topological space. Yue and Fang [12] defined the degree to which an I-fuzzy topological space (X, τ) is ST_0, RT_0 and KT_1, denoted by $ST_0(X, \tau)$, $RT_0(X, \tau)$ and $KT_1(X, \tau)$ respectively and pointed out that $ST_0(X, \tau) \leq RT_0(X, \tau)$. Later on Shi and Li [8] proved a stronger result that $ST_0(X, \tau)$ and $RT_0(X, \tau)$ are equal.

In [12] Yue and Fang proved that $T_1(X, \tau) \leq KT_1(X, \tau)$. In this note, we introduce $ST_1(X, \tau)$ where $T_1(X, \tau) \leq ST_1(X, \tau)$ and proved that $ST_1(X, \tau)$ and $KT_1(X, \tau)$ are equal.

2 Preliminaries

Definition 2.1 *(C. K. Wong [11]). A fuzzy point x_r in X is a fuzzy set in X taking value $r \in (0,1)$ at x and zero elsewhere. A fuzzy singleton (Zadeh [14]) x_r in X is a fuzzy set in X taking value $r \in (0,1]$. x and r are respectively called the support and value of x_r. Two fuzzy points/fuzzy singletons are said to be distinct if their supports are distinct. A fuzzy point x_r is said to belong to a fuzzy set A if $r < A(x)$.*

It can be easily seen that $x_r \in \vee_{i\in\Lambda} A_i \Leftrightarrow x_r \in A_i$ for some $i \in \Lambda$.

Definition 2.2 *(Pu and Liu [5]). Let x_r be a fuzzy point in X and $A \in I^X$. Then x_r is said to be quasi-coincident with A (notation: $x_r qA$) if $A(x)+r > 1$. Two fuzzy sets A, B in X are said to be quasi-coincident (notation: AqB) if $A(x)+B(x) > 1$ for some $x \in X$.The relation (is not quasi-coincident with) is denoted by $\neg q$. A Q-neighborhood (in short, Q-nbd) of a fuzzy singleton x_r in an I-topology (X,τ) is a fuzzy set $N \in I^X$ such that $\exists\ U \in \tau$ with $x_r qU \subseteq N$.*

Definition 2.3 *(Šostak [9], Kubiak [4]). An I- fuzzy topology on a set X is a map $\tau : I^X \longrightarrow I$ such that*

(i) $\tau(\underline{1}) = \tau(\underline{0}) = 1$;

(ii) $\tau(U \cap V) \geq \tau(U) \wedge \tau(V)$, $\forall U, V \in I^X$;

(iii) $\tau(\cup_{j\in J} U_j) \geq \wedge_{j\in J} \tau(U_j)$, $\forall U_j \in I^X$, $j \in J$.

The pair (X,τ) is called an I- fuzzy topological space (in short, I-fts).

Definition 2.4 *(Yue and Fang [12]). Let (X,τ) be an I-fts and x_λ be a fuzzy singleton in X, Define $Q_{x_\lambda} : I^X \to I$ as follows:*

$$Q_{x_\lambda}(U) = \begin{cases} \bigvee_{x_\lambda qV \leq U} \tau(V), & if \quad x_\lambda qU \\ 0 & \text{otherwise} \end{cases}$$

$Q_{x_\lambda}(U)$ is called the degree to which U is quasi-coincident neighborhood of x_λ. The set Q=$\{Q_{x_\lambda} \mid x_\lambda$ is a fuzzy singleton in X $\}$ is called the fuzzy quasi-coincident neighborhood system of τ.

Definition 2.5 *(Kubiak [4]). Let (X,τ) be an I-fts, The degree to which two distinguished crisp points $x, y \in X$ are KT_1, is defined as follows:*

$$KT_1(x,y) = \bigvee_{U(x)>U(y)} \tau(U) \wedge \bigvee_{V(y)>V(x)} \tau(V)$$

.

Definition 2.6 *(Yue and Fang [12]). Let (X,τ) be an I-fts. The degree to which two distinct fuzzy singletons x_λ and y_μ are T_1 is defined as,*

$$T_1(x_\lambda, y_\mu) = \Big(\bigvee_{x_\lambda \neg q U} Q_{y_\mu}(U) \Big) \wedge \Big(\bigvee_{y_\mu \neg q V} Q_{x_\lambda}(V) \Big).$$

The degree to which (X,τ) is T_1, is defined by

$$T_1(X,\tau) = \wedge \{ T_1(x_\lambda, y_\mu) \mid x_\lambda, y_\mu \text{are distinct fuzzy singletons } \}.$$

3 Main Result

Definition 3.1 *Let (X,τ) be an I-fts. The degree to which (X,τ) is ST_1, is defined as follows:*

$$ST_1(X,\tau) = \bigwedge \{ \bigvee_{\lambda>0} T_1(x_\lambda, y_\lambda) \mid x \neq y \}.$$

Theorem 3.1 *Let (X,τ) be an I-fts. Then $KT_1(X,\tau) = ST_1(X,\tau)$.*

Proof: In order to prove that $KT_1(X,\tau) = ST_1(X,\tau)$, we will show that for any $x, y \in X$, $KT_1(x,y) = \bigvee_{\lambda>0} T_1(x_\lambda, y_\lambda)$.
For any $x, y \in X$, we have,

$$\begin{aligned}
KT_1(x,y) &= \bigvee_{U(x)>U(y)} \tau(U) \wedge \bigvee_{V(y)>V(x)} \tau(V) \\
&= \big(\bigvee \{ \tau(U) \mid U(x) > U(y) \} \big) \wedge \big(\bigvee \{ \tau(V) \mid V(y) > V(x) \} \big) \\
&= \big(\bigvee \{ \tau(U) \mid U^{'}(y) > U^{'}(x) \} \big) \wedge \big(\bigvee \{ \tau(V) \mid V^{'}(x) > V^{'}(y) \} \big) \\
&= \Big(\bigvee_{\lambda>0} \bigvee \{ \tau(U) \mid U^{'}(y) \geq \lambda > U^{'}(x) \} \Big) \wedge \Big(\bigvee_{\lambda>0} \bigvee \{ \tau(V) \mid V^{'}(x) \geq \lambda > V^{'}(y) \} \Big) \\
&= \Big(\bigvee_{\lambda>0} \bigvee \{ \tau(U) \mid y_\lambda \neg q U, x_\lambda q U \} \Big) \wedge \Big(\bigvee_{\lambda>0} \bigvee \{ \tau(V) \mid x_\lambda \neg q V, y_\lambda q V \} \Big)
\end{aligned}$$

$$= \left(\bigvee_{\lambda>0} \bigvee_{y_\lambda \neg q U} \bigvee_{x_\lambda q A \leq U} \tau(A) \right) \wedge \left(\bigvee_{\lambda>0} \bigvee_{x_\lambda \neg q V} \bigvee_{y_\lambda q B \leq V} \tau(B) \right)$$
$$= \left(\bigvee_{\lambda>0} \bigvee_{y_\lambda \neg q U} Q_{x_\lambda}(U) \right) \wedge \left(\bigvee_{\lambda>0} \bigvee_{x_\lambda \neg q V} Q_{y_\lambda}(V) \right)$$
$$= \bigvee_{\lambda>0} \left((\bigvee_{y_\lambda \neg q U} Q_{x_\lambda}(U)) \wedge (\bigvee_{x_\lambda \neg q V} Q_{y_\lambda}(V)) \right)$$
$$= \bigvee_{\lambda>0} T_1(x_\lambda, y_\lambda)$$

$\Rightarrow \wedge\{KT_1(x,y) \mid x \neq y\} = \wedge\{\vee_{\lambda>0} T_1(x_\lambda, y_\lambda)\}$
$\Rightarrow KT_1(X,\tau) = ST_1(X,\tau)$.
Hence proved.

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