Some Doubts on the Validity of the Foreground Galactic Contribution Subtraction from Microwave Anisotropies

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Abstract. The Galactic foreground contamination in CMBR anisotropies, especially from the dust component, is not easily separable from the cosmological or extragalactic component. In this paper, some doubts will be raised concerning the validity of the methods used until now to remove Galactic dust emission and will show that none of them achieves its goal.

First, I review the recent bibliography on the topic and discuss critically the methods of foreground subtraction: the cross-correlation with templates, analysis assuming the spectral shape of the Galactic components, the "maximum entropy method", "internal linear combination", and "wavelet-based high resolution fitting of internal templates". Second, I analyse the Galactic latitude dependence from WMAP data. The frequency dependence is discussed with data in the available literature. The result is that all methods of subtracting the Galactic contamination are inaccurate. The Galactic latitude dependence analysis or the frequency dependence of the anisotropies in the range 50–250 GHz put a constraint on the maximum Galactic contribution in the power spectrum to be less than $\sim 10\%$ (68% C. L.) for an ~ 1 degree scale, and possibly higher for larger scales.

The origin of most of the signals in the CMBR anisotropies is not Galactic. In any case, the subtraction of the galaxy is not accurate enough to allow a "precision Cosmology"; other sources of contamination (extragalactic, solar system) are also present.

Key words. Cosmic microwave background—ISM: clouds—dust, extinction—ISM: structure.

1. Introduction

The cosmic microwave background radiation (CMBR) has been interpreted as the relict radiation of an early stage of the Universe. Its black-body spectrum (Mather *et al.* 1994) of 2.75 K reveals a very small dependence on sky position. Measurements of the anisotropies, carried out by several teams of researchers over the last two decades, have claimed to provide information on the structural formation of the Universe, inflation in its early stages, quantum gravity, topological defects (strings, etc.), dark matter type

and abundance, the determination of cosmological parameters (H_0, Ω_m, Ω_Λ, etc.), the geometry and dynamics of the Universe, the thermal history of the Universe at the recombination epoch, etc. (e.g., Bennett *et al.* 2003a). There are even some researchers who claim that the golden era of cosmology, the age of the precision cosmology, has arrived with new experiments to measure these anisotropies and other optimistic interpretations of observational cosmology. However, all that glitters is not gold and what is claimed as a source of cosmological information is often something else. One should be very careful to ensure that there are no contaminants along the light path of this microwave radiation, Galactic contamination being one important factor. If one wants to do "precision cosmology", then even greater care must be taken. Many authors (see the brief review in section 2.) have studied the different components of the Galactic microwave foreground radiation, but in my view these efforts are still insufficient to separate the components appropriately. The foreground emission is small; there is no "*Galactic foreground contamination which is 1000 times more intense than the desired signal*" (as claimed by Robitaille 2007) in off-plane regions; this is completely wrong, but neither is the foreground contamination entirely negligible. At least, some doubts on the validity of the foreground Galactic subtraction from microwave anisotropies can be expressed, and this is precisely the topic of this paper, together with some analysis to constrain Galactic emission. In section 2., I review the recent bibliography on the topic and discuss critically the methods of foreground subtraction. I analyse the Galactic latitude dependence (section 3.) from WMAP first year data. The frequency dependence is discussed (section 4.) with data in the available literature. Other possible sources of contamination are also discussed in section 5..

2. Some comments on the present status of CMBR foreground analysis

The consideration of the Galactic foregrounds has been present for a long time now. However, an analysis of the papers analysing the problem reveals that it was sometimes underestimated, and that the problem of galaxy subtraction has become more complex in recent years.

For instance, Gutiérrez de la Cruz *et al.* (1995); Davies *et al.* (1996) thought that Tenerife data at 15 GHz (see also Gutiérrez *et al.* 2000) were likely to be dominated by cosmological fluctuations, and that 33 GHz data were almost unaffected by the Galaxy ($\sim 4\,\mu$K in scales of 5–15 degrees). More recent analyses (WMAP, Bennett *et al.* 2003b, Fig. 10), however, have claimed that for comparable angular scales the galaxy is dominant in the anisotropies at 22.8 GHz (and the Galactic contamination at 15 GHz is not much smaller than at 22.8 GHz), and at 33 GHz the Galactic anisotropies are of the same order as those from the CMBR. Also, at small scales, less than a degree, the foreground Galactic emission dominates (Leitch *et al.* 2000). Halverson *et al.* (2002) and Fernández-Cerezo *et al.* (2006) claim that the cosmological signal is still dominant at $|b| > 40°$ around 15 GHz and 31 GHz respectively for intermediate and small angular scales (around a degree and down to ≈ 12 arcminutes respectively), but their analysis is based on correlations with templates, which, as we will see below, might not be appropriate. An explanation for this is that it is mainly due to the existence of a new kind of emission correlated with the dust that was not discovered previously; the synchrotron and or free–free emission might also have been underestimated. Positive correlations between the microwave anisotropies, including the region around 15 GHz, and far-infrared maps, which trace Galactic dust, were found (Kogut *et al.*

1996; Leitch *et al.* 1997, 2000; de Oliveira-Costa *et al.* 1997, 1998; Finkbeiner *et al.* 1999, 2004; Fernández-Cerezo *et al.* 2006). Casassus *et al.* (2004) report that in the Helix region the emission at 31 GHz and 100 μm are well correlated; they estimate that the 100 μm-correlated radio emission, presumably due to dust, accounts for at least 20% of the 31 GHz emission in the Helix (so the total dust emission is higher than 20% because there is a non-correlated component too). Watson *et al.* (2005) show that the anomalous emission at around 23 GHz of the Perseus molecular cloud (Temp $\sim$ 1 mK) is an order of magnitude larger than the emission expected from synchrotron + free–free + thermal dust. There is also some correlation of 10, 15 GHz maps with H_α maps, but very low, so the free-free emission is detected at levels far lower than the dust correlation (de Oliveira-Costa *et al.* 1999, 2002). The most likely current explanation for this emission correlated with dust around 15–50 GHz is spinning dust grain emission (Draine & Lazarian 1998) and/or magnetic dipole emission from ferromagnetic grains (Draine & Lazarian 1999); although some other authors (e.g., Mukherjee *et al.* 2001) do not agree. Whatever it is, it is now clear that the 15–40 GHz range is dominated by the galaxy.

A question might arise as to whether the contamination in the remaining frequencies (40–200 GHz) is being correctly accounted for. Typical calculations of dust contamination claim that it should not be predominant, and that its contribution can be subtracted accurately, but how sure can we be of this statement? How accurate is the subtraction of the dust foreground signal?

2.1 *Dust foreground removal with templates*

The first difficulty in the subtraction of the dust component is to know exactly how much emission there is in each line of sight. First approximations came with extrapolations from the IRAS far-infrared and DIRBE data (with the zodiacal light subtracted, as well as the cosmic infrared background) used to model the dust thermal emission in microwaves. Templates were taken from these infrared maps and extrapolated in amplitude by a common factor for all pixels. The problem is that, as said by Finkbeiner *et al.* (1999), *'a template approach is often carelessly used to compare observations with expected contaminants, with the correlation amplitude indicating the level of contamination. (...) These templates ignore well-measured variation in dust temperature and variations in dust/gas ratio.'* According to López-Corredoira (1999), the growing contrast of colder clouds in the background of the diffuse interstellar medium will produce much higher microwave anisotropies than the product of the template extrapolation. Neither is it a good strategy to subtract a scaled IRAS template to remove the spinning dust in multifrequency data since it produces large residual differences (Leitch *et al.* 2000).

A better approximation to this dust emission in the microwave region came with the adoption of an extrapolation with colour corrections in each pixel. This method assigns a different temperature to each pixel. This colour correction, together with a ν^2 emission emissivity (Schlegel *et al.* 1998), gave a much tighter agreement with FIRAS data. However, it was still inconsistent with the FIRAS data below 800 GHz in amplitude (Finkbeiner *et al.* 1999): a 14% error in the mean amplitude at 500 GHz. Indeed, no power-law emissivity function fits the FIRAS data in the 200–2100 GHz region (Finkbeiner *et al.* 1999). Furthermore, laboratory measurements suggest that the universality of ν^2 emissivity is an oversimplification, with different species of grains

having different emissivity laws (Finkbeiner *et al.* 1999). A better approximation is an extrapolation with two components ($\nu^{1.7}$, $\langle T \rangle = 9.5$ K and $\nu^{2.7}$, $\langle T \rangle = 16$ K); each pixel has an assignation of two temperatures (Finkbeiner *et al.* 1999) with correction factors that are a function of these temperatures. This still fails in the predictions of FIRAS from the IRAS–DIRBE extrapolation by 15% in zones dominated by atomic gas (Finkbeiner *et al.* 1999).

Finkbeiner *et al.*'s (1999) approach, although much better than a direct extrapolation of the template, is insufficient. The problem is difficult to solve because we are trying to do an extrapolation by a factor ~ 15–60 in frequency (from 1250 GHz [240 μm] or 3000 GHz [100 μm]) to 50–90 GHz). This is equivalent, for instance, to the attempt to derive a map of stellar emission in 12 μm as an extrapolation of the emission in optical B-filter. Frankly, when I see IRAS-12 μm map of point sources and the Palomar plates in blue filters, I observe huge differences, and I do not know how we can extrapolate the second map to obtain the first one. Each star has a different colour, and stars which are very bright in blue may be very faint at mid-infrared and *vice versa*. The same thing happens with the diffuse + cloud emission: there are hot regions, cold regions, different kinds of emitters (molecular gas, atomic gas) and we have to integrate all this into each line of sight. The assumption that with only two temperatures we can extrapolate the average flux, and that a colour term can correct the pixel-to-pixel differences is comparable to the assumption that a model with only two kinds of stars and the knowledge of $(B - V)$ for each star we can extrapolate the star counts from optical to mid-infrared for the whole galaxy.

The cold clouds, invisible or very faint in far-infrared surveys, are potential elements to produce a very significant emission at 40–200 GHz at the amplitude level of the observed anisotropies. Bernard *et al.* (1999) have shown that some cirrus features with a high value of $I_{100\,\mu\mathrm{m}}/I_{60\,\mu\mathrm{m}}$ of 29.5 (the average is 3.2) are cold dust regions ($T \approx 13$ K instead of the average of 17.5 K). Colder clouds may also exist. Lawrence (2001) claims that part of the SCUBA-850 μm sources might be local sources at 7 K. And who knows whether there are even colder clouds that might emit significantly only in the range of microwaves? Paladini *et al.* (2007) show that the microwave emission in the outer Galactic plane ($R > 8.9$ kpc) is much higher than expected, suggesting the possible presence of very cold dust.

It is also very common (e.g., Masi *et al.* 2001; Fernández-Cerezo *et al.* 2006) to calculate the Galactic dust contribution in some microwave data by just making a cross-correlation between these data and some far-infrared map of the sky. This is simply wrong and not even valid for ascertaining the order of magnitude of such contamination. Since the templates in infrared cannot be used as maps of the microwave Galactic emission, there will be non-correlated emission coming from the dust too. That is, there will be many clouds that emit substantially in microwaves, and that are not detected in far-infrared maps.

Therefore, any method of foreground subtraction which uses templates will have serious credibility problems with regard to the goodness of the subtraction. This applies not only to those methods that use templates directly with coupling coefficients derived from cross-correlation but also to MEM (maximum entropy method; Bennett *et al.* 2003b), which uses in the initial stage templates for the dominant foreground components and also establishes some *a priori* conditions of their spectral behaviour. Moreover, any calculation of the limits of such contamination based on cross-correlations will not be totally accurate.

2.2 *Dust foreground removal without templates*

If removal of foreground contamination is applied without the use of templates, we might need a knowledge of all of the physical components of this foreground emission and their spectral behaviour.

With ISO-90,180 μm, a power law down to scale of $3'$ is found in the power spectrum (Herbstmeier *et al.* 1998), with similar shape as that derived from the 21 cm line for Galactic cirrus. At 410 GHz, the dust emission is also roughly a power law in off-plane regions (Masi *et al.* 2001), although I see in the plot for $b = -17°$ from Fig. 2 of Masi *et al.* (2001) that there would be a better fit if we admitted that there was a maximum in the dust emission power spectrum around $l = 200$. In any case, even if a power law is present in the far infrared or at 410 GHz, the extrapolation to 50–90 GHz is not direct and, moreover, the spectral index of this power law is highly variable. The presence of different dust-emitting components causes the spectral indices of the foregrounds to vary with position. Kiss *et al.* (2003) have measured spectral index changes from -5.1 to -2.1, depending on the region, and also changes with wavelength; they point out the existence of dust at different temperatures, in particular of a cold, extended component. Dupac *et al.* (2004) also show with the PRONAOS balloon-borne experiment that the submillimetre/millimetre spectral index is found to vary between roughly 1 and 2.5 on small scales ($3.5'$ resolution). Even small spectral index variations as small as $\Delta\alpha \sim 0.1$ can have a substantial impact on how channels should be combined and on the attainable accuracy (Tegmark 1998), so the errors of the subtraction assuming certain power spectrum for the dust are serious.

Another technique that does not use templates is (Bennett *et al.* 2003b): ILC (internal linear combination). It assumes nothing about the particular frequency dependencies or morphologies of the foregrounds and tries to minimize the variance in twelve different regions of the sky with the combination of the five available WMAP frequencies. There is degeneracy of solutions, an infinite number of maps can be generated from the five basis sets, there is no way to test whether the maximum likelihood solution is the correct one (Robitaille 2007), and it is not enough to divide the sky into twelve regions. Bennett *et al.* (2003b) themselves warn against its use for cosmological analysis, and Eriksen *et al.* (2004b) show that it is not effective to remove all residual foregrounds. The ILC method performs quite badly, especially for dust (Eriksen *et al.* 2004b), in part because of the variability of the spectral indices. Indeed, a tremendous coefficient variability in the twelve sections was obtained (Robitaille 2007). Therefore, ILC maps are not clean enough to allow cosmological conclusions to be arrived at (Eriksen *et al.* 2004b): '*[The] ILC map, which by eye looks almost free of foreground residuals, has been extensively used for scientific purposes—despite the fact that there are strong (and difficult to quantify) residual foregrounds present in the map. (. . .) the ILC map is indeed highly contaminated by residual foregrounds, and in particular, that the low-l components, which have received the most attention so far, are highly unstable under the ILC cleaning operation*' (Eriksen *et al.* 2005).

The WI-FIT method ('Wavelet based hIgh resolution Fitting of Internal Templates', Hansen *et al.* 2006) does not require *a priori* templates, but takes the information about the foregrounds by taking differences of temperature maps at different frequencies. However, for the application in presently available maps, it requires the assumption that the spectral indices are constant in space, which, as said, is a very inaccurate approximation. Their assumption that the Galactic emission in each pixel is proportional to

the difference in temperature maps ($(T_i^\nu - T_i^{\nu'}) \propto T_i^\nu$) is in general incorrect because $T_i^{\nu'}$ is not proportional to T_i^ν, the temperature in each pixel being the superposition of many different emissions with different temperatures. Again, we have here the same problem as with the use of templates: the assumption that there is only one temperature along each line of sight, and that the intensity of this emission is describable with a simple average fixed power law multiplying a black body emission with an average temperature. Hansen *et al.* (2006) claim that their method is good because they obtain similar results to Bennett *et al.* (2003b), but this may be due to their similar assumptions.

Foreground contamination residuals are found even for the best available supposed clean CMB maps. Naselsky *et al.* (2006) found some correlation between $\Delta l = 4n$ and $n = 1,\ 2$ spherical harmonic multipole domain, which is caused by a symmetric signal in the Galactic coordinate system. de Oliveira-Costa & Tegmark (2006) find that the alignment of low-l multipoles appears to be rather robust to Galactic cut and different foreground contaminations. It is also relevant that Bernui *et al.* (2007a) found a preferred direction in the fluctuations of the WMAP data while there should be no preferential direction in the sky (isotropy); but the significance of this detection is only at 95% C.L. so it could be just a chance detection, or it could also indicate some kind of contamination. The statistical anisotropy in different circles of the sky is also found by Then (2006). Liu & Zhang (2006) have also studied the cross-correlation between WMAP and Egret γ-ray data and concluded that an unknown source of radiation, most likely of Galactic origin, is implied by their analysis. Verschuur (2007) speculates that this unknown radiation of Galactic origin might take place in the surface of Galactic HI structures moving through interstellar space and/or interacting with one another.

The spatial association found by Verschuur (2007) on scales of 1–2 degrees between interstellar neutral hydrogen, integrated in maps over ranges of 10 km/s, and WMAP-ILC maps, which should be clean of foreground contamination through the ILC methods, is especially significant for the present discussion too. Several extended areas of excess emission at high Galactic latitudes ($b > 30°$) are present in both maps. According to Verschuur (2007), these structures have typical distances from the Sun of the order of 100 pc.

2.3 *Non-gaussianity*

Another aspect to think about is the non-gaussianity of the anisotropies distribution. Standard theories involving inflation generally predict a pattern of gaussian noise; whereas non-standard theories based on symmetry breaking and the generation of defects have more distinctive signatures (Coulson *et al.* 1994). The fact is that a non-gaussian distribution was discovered in the anisotropies against all predictions of current inflationary cosmological models by many different authors with different methods (Ferreira *et al.* 1998; Pando *et al.* 1998; Magueijo 2000a, b; Chiang *et al.* 2003; Coles *et al.* 2004; Eriksen *et al.* 2004a; Park 2004; Vielva *et al.* 2004; Liu & Zhang 2005; Raeth *et al.* 2007; Bernui *et al.* 2007b) and yet the leading cosmologists still claim inflationary cosmology to be valid and to be able to constrain cosmological parameters from the anisotropies. This might mean that the analyses of non-gaussianity are wrong, or that they are not significant enough, or that the inflation model is incorrect, or that some contamination is present in the maps, which are supposed to be clear of any

contamination. Among all these possibilities, maybe more than one applies, and I suspect that one of these is the incorrect subtraction of the Galactic contamination although other factors may also be important. As a matter of fact, Liu & Zhang (2005) show with certain tests that residual foreground contamination in WMAP-cleaned maps may contribute to this non-gaussian features significantly, and Tojeiro *et al.* (2006) have argued that the non-gaussianity is associated with cold spots of unsubtracted foregrounds. The lowest spherical harmonic modes in the map are significantly contaminated with foreground radiation (Chiang *et al.* 2007). However, Galactic contamination may not be the only reason: using the WMAP data and 2MASS galaxy catalog, Cao *et al.* (2006) show that the non-gaussianity of the 2MASS galaxies is imprinted on WMAP maps too. Nonetheless, Rubiño-Martín *et al.* (2006) and McEwen *et al.* (2006) claim that only few regions have such non-gaussian anisotropies due to contamination, e.g., the Corona Borealis supercluster region, and that most regions of the sky are gaussian.

3. Galactic latitude dependence

A simple way to test whether the Galactic anisotropies are somewhat significant would be to examine their variation with the position in the sky, for example with Galactic latitude dependence (López-Corredoira 1999). Higher Galactic anisotropies are expected towards the plane. Moreover, the lines of sight in the southern Galactic hemisphere should give more Galactic anisotropies than the northern one, because the Sun is approximately 15 pc above the Galactic plane (Hammersley *et al.* 1995; Chyzy *et al.* 2005) and the scale height of the cold dust component is not very high, although unknown in principle. For dust observed in infrared, the scale-height is 40 pc (Unavane *et al.* 1998)[1]. If the cold dust responsible for the microwave fluctuation in the range 60–100 GHz had a scale-height of 40 pc as well, the excess column density in the south over the north $\left(\frac{2 - \exp\left(-15/40 \right)}{\exp\left(-15/40 \right)} - 1 \right)$ would be 91%, for any Galactic latitude. If we assume a cosecant law for Galactic latitude dependence, this absolute value of the asymmetry would be ≈ 3 times higher at $|b| = 20°$ than at $|b| = 90°$. Of course, this is a rough calculation because the scale-height might change in microwaves with respect to the value inferred from the extinction in infrared, there is longitude dependence, flares, the accuracy of the cosecant law for low latitude regions is not good. Cloud sizes and other factors should be taken into account too.

Nonetheless, we are limited by the variations of the anisotropies produced by the own cosmological anisotropies, especially those at large scale, which have a range of possible gradients that could even mimic the Galactic gradients. Multipoles of low-l with $l \geq 2$ are part of the fluctuations, and one might suspect that they are responsible for the gradients and the difference north/south. One way to check whether these variations with Galactic position exist and whether they are within the expected cosmological variation or not is to measure them with real data (WMAP) and compare with realizations.

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[1]Note that, although Unavane *et al.* use near-infrared data, the scale-height of 40 pc does not refer to the old stellar population (the scale-height of the old stellar population is 285 pc; López-Corredoira *et al.* 2002) but to the dust distribution inferred through the extinction maps.

Cross-corr. VxW with mask kp2
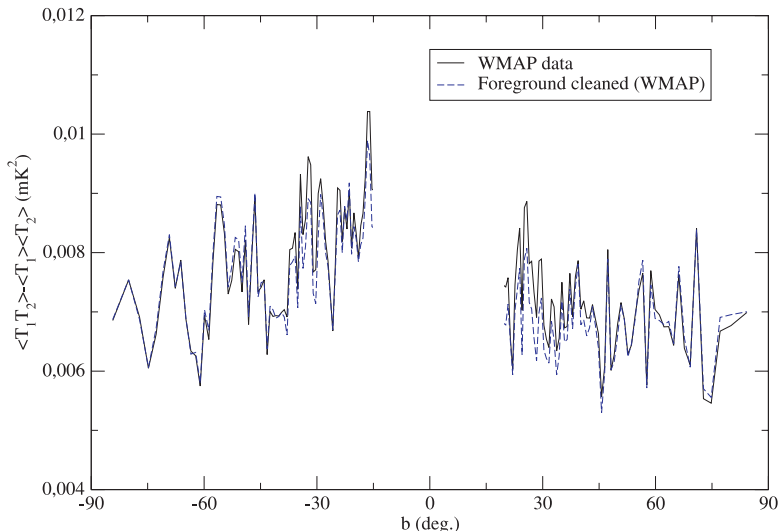
Figure 1. Variance as a function of Galactic latitude in WMAP first-year data (only points with less than 10% of pixels in the mask kp2). Each point in the plot represents the pixels in the ring for all Galactic longitudes and $\Delta(\sin b) = 0.01$.

3.1 *WMAP data analysis*

Figure 1 shows the variation with latitude of the variance ($\langle T_1 T_2 \rangle - \langle T_1 \rangle \langle T_2 \rangle$) and with T_1, T_2, the antenna temperature at bands V and W of WMAP (61 and 94 GHz) (Bennett *et al.* 2003a). This variance is free of instrumental noise because it corresponds to the cross-correlation of two independent maps. Each point in the plot represents the cross-variance in the ring for all Galactic longitudes and $\Delta(\sin b) = 0.01$ (which comprises around 15,000 pixels). We used the data with no foreground subtraction with the kp2 mask (Bennett *et al.* 2003a), which removes 15% of the sky, mainly from the Galactic plane, the same mask that was used in the derivation of their WMAP power spectrum. The figure shows two features:

- there is a significant decrease of fluctuations toward the polar caps, especially for the southern cap;
- there is a significant excess in the southern fluctuations ($b < 0$) with respect to the north fluctuations (this fact was already present in COBE-DMR data; López-Corredoira 1999, subsection 3.4).

The least squares linear fits ($y = a + c|b|$) to the ranges $b < -20°$ and $b > 20°$ are respectively:

$$\sigma^2_{\mathrm{cross},b<-20°}(|b|) = (8.99 \pm 0.28) \times 10^{-3} - (27.8 \pm 5.9) \times 10^{-6}|b|(\mathrm{deg})\,\mathrm{mK}^2$$

$$\sigma^2_{\mathrm{cross},b>20°}(|b|) = (7.74 \pm 0.24) \times 10^{-3} - (16.2 \pm 5.1) \times 10^{-6}|b|(\mathrm{deg})\,\mathrm{mK}^2. \quad (1)$$

That is, $c = 0$ is excluded within a $4.7 - \sigma$ level for the south and $3.2 - \sigma$ for the north for the first feature, and $\sigma^2_{\text{cross}}(b = -20°) - \sigma^2_{\text{cross}}(b = 20°) = (1.01 \pm 0.40) \times 10^{-3} \, \text{mK}^2$, i.e., the second feature. Equality between north and south is excluded to within $2.5 - \sigma$.

Bennett *et al.* (2003b, Fig. 10) point out that in the W bands the Galactic contribution constitutes around 10% of the power spectrum. If we examine the same dependence in the same maps once the foregrounds have been cleaned according to their estimations, the results are (see also Fig. 1):

$$\sigma^2_{\text{cross},b<-20°}(|b|) = (8.59 \pm 0.27) \times 10^{-3} - (20.6 \pm 5.6) \times 10^{-6}|b|(\text{deg}) \, \text{mK}^2$$

$$\sigma^2_{\text{cross},b>20°}(|b|) = (6.88 \pm 0.23) \times 10^{-3} + (2.2 \pm 4.7) \times 10^{-6}|b|(\text{deg}) \, \text{mK}^2, \quad (2)$$

and $\sigma^2_{\text{cross}}(b = -20°) - \sigma^2_{\text{cross}}(b = 20°) = (1.25 \pm 0.38) \times 10^{-3} \, \text{mK}^2$. The latitude dependence on the south is less significant ($c = 0$ excluded at $3.7 - \sigma$) and disappears in the north, but the south-north difference is higher than before ($3.3 - \sigma$).

These higher fluctuations in the southern Galactic hemisphere have also been observed by Eriksen *et al.* (2004a), Hansen *et al.* (2004a, b), Chyzy *et al.* (2005) or Bernui *et al.* (2006) with higher asymmetry for lower latitudes (Chyzy *et al.* 2005). The higher asymmetry for lower values of $|b|$ is expected from the Galactic contamination because the dust emission is roughly proportional to $1/\sin(|b|)$.

3.2 *Simulations*

We now do the same measures in 20 random realizations of the expected WMAP sky (Eriksen *et al.* 2004b, 2005) (i.e., 20 measures of the excess south/north, and 40 measures of the slope).

The average slope c (of a linear law $\sigma^2(|b|) = a - c|b|$) is $+11 \times 10^{-6} \, \text{mK}^2/\text{deg}$ and the r.m.s. is $21 \times 10^{-6} \, \text{mK}^2/\text{deg}$, so our measured values of $+27.8 \times 10^{-6} \, \text{mK}^2/\text{deg}$ and $+16.2 \times 10^{-6} \, \text{mK}^2/\text{deg}$ for the south and the north respectively are indeed within the expected values only for cosmological fluctuations. The fact that we have a non-null value for the average c ($\langle c \rangle = +11.3 \pm 3.3 \times 10^{-6}$) is due to the way the fluctuations are measured in different rings with different latitudes; all rings have the same number of points but different shapes; for instance, toward the polar caps the areas are different from the rings near the plane.

The difference south/north in the same realizations is on average $0.33 \pm 0.28 \times 10^{-3} \, \text{mK}^2$ (compatible with zero, which is the exact expected value, since there are no reasons for an asymmetry north/south) and the r.m.s. is $1.3 \times 10^{-3} \, \text{mK}^2$. Again, our measured value in WMAP of $1.25 \times 10^{-3} \, \text{mK}^2$ is within the expected statistical values.

3.3 *Latitude dependence on small angular scales*

We might think that the variations among zones could be reduced if we used only a high-l (small angular scale) range of the power spectrum with the same shape for all selected regions instead of circular rings of variable thickness and height parallel to the Galactic equator. If we do the analysis of $[l(l + 1)C_l/(2\pi)]$ for $316 \leq l \leq 354$ with 28 points distributed with different Galactic longitudes and latitudes $\pm36°$, $\pm54°$, $\pm72°$, $\pm90°$

in circles of radius of $9°$ which are not overlapping to each other, we get

$$[l(l+1)C_l/(2\pi)](|b| = 90°) = 3.0(\pm 0.6) \times 10^{-3} \, mK^2/deg \qquad (3)$$

$$\frac{d[(l(l+1)C_l/(2\pi)]}{d|b|} = +3(\pm 10) \times 10^{-6} \, mK^2/deg, \qquad (4)$$

that is, the error bar is larger than the measured slope.

3.4 *Constraints on the maximum Galactic contamination*

All angles: We are therefore very limited in ascertaining the Galactic contamination through this kind of analysis. The values of the present simulations allow us to put a higher limit but these numbers permit a very wide range of Galactic contamination amplitudes. On average, for both hemispheres, it should be less than $+43 \times 10^{-6} \, mK^2/deg(1\sigma)[+64 \times 10^{-6} \, mK^2/deg(2\sigma)]$. Since Bennett *et al.*'s (2003b) foreground corrections reduce the value of the slope c by an average of $13 \times 10^{-6} \, mK^2/deg$ and they claim that this is representative of 10% of contamination by the galaxy, an estimate of the maximum limit of contamination by the galaxy is less than 33% (1σ) [49% (2σ)].

Another way to analyse it would be to suppose that the Galactic contamination follows a cosecant law in the dependence on Galactic latitude. Then $c_{gal}/a_{gal} = 9.5 \times 10^{-3}$ (derived from a linear fit in the same conditions as for the analysis of the WMAP data); so the excess over the average in the simulation ($\langle c \rangle = +11.3 \times 10^{-6}$) in the south of $c_{gal} = \Delta c = 27.8 \times 10^{-6} - 11.3 \times 10^{-6} \pm 21 \times 10^{-6} \, mK^2/deg = 16.5 \pm 21 \times 10^{-6} \, mK^2/deg$ implies that $a_{gal,\,south} = 1.7 \pm 2.2 \times 10^{-3}$, i.e., $19 \pm 25\%$ of the total emission ($a = 8.9 \times 10^{-6}$). This means that the Galactic emission in the south is less than 44% (1σ) [69% (2σ)]. The same analysis for the northern hemisphere gives: less than 35% (1σ) [64% (2σ)]. Given that, as said above with regard to the asymmetry south/north, the contamination in the south should be 1.94 times the contamination in the north, we can further reduce the constraint in the north: less than 26% (1σ) [41% (2σ)]. Therefore, on average in the north and the south, the contamination is less than 35% (1σ) [55% (2σ)].

Small angular scales: With the numbers at small angular scales given in subsection 3.3, we can set the constraint: $\delta[(l(l+1)C_l/(2\pi)](|b| = 20°) - [l(l+1)C_l/(2\pi)](|b| = 90°)] < 0.49 \times 10^{-3} \, mK^2/deg(1\sigma)$ [$< 1.2 \times 10^{-3} \, mK^2/deg(2\sigma)$], which implies, assuming a dust emission proportional to $1/\sin(|b|)$, that $[l(l+1)C_l/(2\pi)](|b| = 90°) < 2.5 \times 10^{-4}(1\sigma)[< 6.2 \times 10^{-4}(2\sigma)]$. Comparing with equation (3), we get the constraint that the Galactic emission must be lower than 8% (1σ) [20% (2σ)]. We see, then, that the restriction of values of l is more efficient for constraining the Galactic contamination.

4. The frequency dependence

In my opinion, the most important proof presented to claim that most of the off-plane microwave anisotropies have a cosmological origin is the frequency dependence. The near independence in the range of 50–90 GHz is not conclusive because this could

be produced by the galaxy too, mainly as a combination of thermal dust and rotational dust emission (López-Corredoira 1999). However, the surveys with a larger range of frequencies, such as that at 170 GHz by Ganga *et al.* (1993), BOOMERANG (Netterfield *et al.* 2002) or ARCHEOPS (Tristam *et al.* 2005), have shown that the power spectrum is practically the same in amplitude and shape with respect to WMAP measures and is constant over the frequency range 50–250 GHz. The spectral analysis of FIRAS + WMAP by Fixsen (2003) also shows that the anisotropies are Planckian, although Fixsen (2003) assumes that the frequency and spatial dependences of the galaxy are separable and modelled by templates, which is not a trustworthy assumption. Since Galactic dust thermal emission of 7–20 K temperature grains should have some frequency dependence, we must conclude that they are not predominant. We have, within the error ranges of the calibration and within the error bars of the power spectrum (around a 10% in total), a constant amplitude over a factor 5 in frequency.

The amplitude of the anisotropies depends on two factors (López-Corredoira 1999): the variation of the mean flux with frequency, and the ratio of diffuse and cloud emission as a function of frequency; but in any case, the variation of $(\Delta T)^2$ should be at least twice as high at 250 GHz than at 50 GHz. This minimum factor roughly equals to two stems from the multiplication of the first two factors specified in López-Corredoira (1999, equation 2):

$$\frac{(\Delta T)^2(\nu = 250\,\mathrm{GHz})}{(\Delta T)^2(\nu = 50\,\mathrm{GHz})} = \left\langle \frac{T(\nu = 250\,\mathrm{GHz})}{T(\nu = 50\,\mathrm{GHz})} \right\rangle^2 \left\langle \frac{\frac{\delta T}{T}(\nu = 250\,\mathrm{GHz})}{\frac{\delta T}{T}(\nu = 50\,\mathrm{GHz})} \right\rangle^2 . \quad (5)$$

$\left\langle T(\nu = 250\,\mathrm{GHz})/T(\nu = 50\,\mathrm{GHz}) \right\rangle \approx 5$ (López-Corredoira 1999, Fig. 5), and $\left\langle \frac{\delta T}{T}(\nu = 50\,\mathrm{GHz})/\frac{\delta T}{T}(\nu = 250\,\mathrm{GHz}) \right\rangle < \approx 3.3$ (López-Corredoira 1999, equation 12) so $(\Delta T)^2(\nu = 250\,\mathrm{GHz})/(\Delta T)^2(\nu = 50\,\mathrm{GHz}) > \approx 2.3$.

Assuming this roughly minimum factor of two in the increase in Galactic anisotropy at 250 GHz with respect to 50 GHz, if we observe a maximum of 10% excess in the total contribution, it means that the Galactic emission should be lower than a 10% of the total contribution of the anisotropies in average in the range of frequencies.

5. Other contaminants

Although it is not the matter for this paper, it is important to point out that, apart from the cosmological and the Galactic signal, there may be other contaminants: either from closer sources (in the solar system or the solar neighbourhood), or extragalactic sources much closer than $z = 1000 - 1500$ (as is supposed for the cosmological origin).

Copi *et al.* (2006, 2007) show that the two lowest cosmologically interesting multipoles, $l = 2$ and 3, are not statistically isotropic. The planes of the quadrupole and the octopole are unexpectedly aligned (de Oliveira-Costa 2004; Copi *et al.* 2006, 2007; Land & Magueijo 2007; Rakić & Schwarz 2007). Indeed, the combined quadrupole plus octopole is surprisingly aligned with the geometry and direction of motion of the solar system: the plane they define is perpendicular to the ecliptic plane and to the plane defined by the dipole direction, and the ecliptic plane carefully separates stronger from weaker extrema, running within a couple of degrees of the null-contour between a maximum and a minimum over more than $120°$ of the sky. The axis of maximum

asymmetry of the WMAP data tends to lie close to the ecliptic axis (Eriksen *et al.* 2004a). There is alignment of the quadrupole and octopole with each other and they are correlated with each other: 99.6% C.L. (Copi *et al.* 2007). There are also statistically significant correlations with local geometry, especially that of the solar system (> 99.9% C.L.; Copi *et al.* 2006). Moreover, the angular two-point correlation function at scales > 60 degrees in the regions outside the Galactic cut is approximately zero in all wavebands and is discrepant with the best fit ΛCDM inflationary model: 99.97% C.L. for the discrepancy (Copi *et al.* 2007; Rakić & Schwarz 2007). Eriksen *et al.* (2004a) also found that the ratio of the large-scale fluctuation amplitudes in the southern ecliptic hemisphere is high at the level 98–99% , with an absence of large-scale power in the vicinity of the north ecliptic pole. This asymmetry is stable with respect to frequency and sky coverage. This seems to point to the presence of some contamination in relation with the solar system or neighbourhood, although part of the effect may be due to non-uniform observational time sky coverage (Chyzy *et al.* 2005). Abramo *et al.* (2006) claim that if a hypothetic foreground produced by a cold spot in the Local Supercluster is subtracted from the CMB data, the amplitude of the quadrupole is substantially increased, and the statistically improbable alignment of the quadrupole with the octopole is substantially weakened, but this does not explain the coincidence of the alignment with the ecliptic.

The extragalactic contamination is still not very clear. In this respect, the discovery that the hard X-ray background measured by the HEAO-1 satellite and the number counts of radio galaxies in the NVSS survey are correlated with the WMAP microwave fluxes (Boughn & Crittenden 2004) should be pointed out. One could raise the objection that microwave anisotropies are produced over a very wide range of angles, which include structures of several degrees and largest structures while galaxies and clusters are generally smaller. However, if one considers the very large structures of filaments, walls, hyperclusters, etc., one can find in the sky structures of even several tens of degrees; that is, excess of densities of galaxies over regions with several tens of degrees. As a matter of fact, Narlikar *et al.* (2003) explain the peak at $l = 200$ and other peaks too in the power spectrum in terms of rich clusters of galaxies. The galaxy counts in López-Corredoira & Betancort-Rijo (2004, Fig. 1) also suggest structures of this kind and size. I did the cross-correlation of these near-infrared galaxy counts with COBE-DMR-90 GHz and found nothing significant (cross-corr. for $|b| > 20°$: $5.5(\pm 11.2) \times 10^{-7}$ K). However, as said in subsection 2.3, using the WMAP data and 2MASS galaxy catalog, Cao *et al.* (2006) show that the non-gaussianity of the 2MASS galaxies is imprinted on WMAP maps, and Cabre *et al.* (2006) cross-correlate the third-year-WMAP data with optical galaxy samples extracted from the SDSS-DR4 giving a positive signal-to-noise of about 4.7.

Another consideration that could point to the importance of the non-cosmological extragalactic contamination over photons of cosmological origin is that the simulations of gravitational lensing of the microwave background by galaxy clusters at $z < 1$ under any plausible Big Bang model variation produces far more dispersion in the angular size of the primary acoustic peaks than WMAP observations allow (Lieu & Mittaz 2005). When all the effects are taken together, it is difficult to understand how WMAP could reveal no evidence whatsoever of lensing by groups and clusters. Cool spots in the microwave background are too uniform in size to have travelled from $z = 1000$–1500 to us. There should be a spread of sizes around the average, with some of these cool spots noticeably larger and others noticeably smaller. But this dispersion of sizes

is not seen in the data. Too many cool spots have the same size. Moreover, the observed WMAP Sunyaev–Zel'dovich effect caused by the clusters only accounts for about 1/4 of the expected decrement (Lieu *et al.* 2006); although the level of Sunyaev–Zel'dovich effect is observed as in the predictions in radio (Bonamente *et al.* 2006). Effects like a central cooling flow in clusters, the abundance of hot cluster gas, large scale radial decline in the temperature, uncertainties in the β-model and the role played by cluster radio sources are too weak to change the estimation (Lieu *et al.* 2006). Under Big Bang premises, this implies that the cosmological parameters (including the Hubble constant, the amount of dark matter, etc.) used to predict the original, pre-lensed sizes of the cool and hot spots in the microwave background might be wrong or some of these cool spot structures are caused by nearby physical processes and are not really remnants of the creation of the Universe; or there is some other, unknown factor damping the effects of dispersion and focusing. It was speculated that the large-scale curvature of space may not entirely be an initial value problem related to inflation. The absence of gravitational lensing of the CMB points to the possibility that even effects on light caused by wrinkles in the space of the late (nearby) Universe have been compensated for, beyond some distance scale, by a mechanism that maintains a flat geometry over such scales. Or high energy electrons may synchrotron radiate in the intracluster magnetic field of strength B $<\sim 1\mu$G to produce cluster microwave emissions in the WMAP passbands that account for the missing Sunyaev–Zel'dovich effect flux (Lieu & Quenby 2006). However, one is also tempted to interpret this in terms of the importance of extragalactic non-cosmological contribution in the microwave anisotropies.

6. Summary and conclusions

Summing up, CMBR anisotropies are not totally free from solar system, Galactic or extragalactic contamination, and an accurate way of correcting all these contributions is still to be devised since we do not have accurate information on the microwave emission of any of these contributions.

Considering only the Galactic dust component, the main topic of this paper, it was shown that the methods used to remove it (templates, cross-correlations, assumption of a Galactic power spectrum, MEM, ILC, etc.) are all inaccurate and one should not expect to produce maps clean of Galactic dust contamination by applying them. The analysis of Galactic latitude dependence of these anisotropies and the fact that the power spectrum is nearly independent of the frequency over a range 50–250 GHz can be considered at least as a proof that the Galactic dust emission is lower than a 10% on the $\sim$ 1 degree scale [or double of this value considered to 2σ], possibly higher for lower l multipoles. This uncertainty in the Galactic contamination may produce important systematic errors in some cosmological parameters. The determination of the error bars in the cosmological parameters taking into account the possible foreground contamination values is beyond the scope of this paper. Further research is needed in this respect. In any case, one thing is clear: the present error bars calculated for the cosmological parameters (e.g., Spergel *et al.* 2007) are very significantly underestimated and the range of possible values is not as small as indicated by the "precision cosmology" claim.

Therefore, I conclude that the most pessimistic possibilities among those claimed in López-Corredoira (1999) (that Galactic contamination could be dominant) is discarded, but I keep my position that we are far from achieving a "precision cosmology"

with CMBR data. My suggestion with regard to the problem would be that rather than performing an incorrect subtraction of the foreground and claiming that a clean map is obtained that is purely cosmological, we might use better the map without corrections or only with a first-order approximation subtraction, and calculate the cosmological parameters with the appropriate error bars taking into account the uncertainties in the foregrounds (Galactic or any other), to be constrained only by the frequency dependence.

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