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Slip Effects on Peristaltic Transport of Power-Law Fluid through an Inclined Tube

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Abstract

This paper is an analytical study of the flow of a power-law fluid through an inclined tube. The effect of the wall slip condition peristaltic transport are studied. It serves as a model for the study of flow of chyme through small intestines. The long wavelength and low Reynolds number are considered in obtaining solution for the flow. Expressions for the axial velocity and axial pressure gradient are obtained analytically. The pressure rise per wavelength and friction force at the wall are evaluated numerically.

Mathematics Subject Classifications: 76Z05

Keywords: Slip flow; power-law fluid; Inclined Tube; peristaltic transport

1 Introduction

Physiological fluids in animal and human bodies are, in general, pumped by the continuous periodic muscular oscillations of the ducts. These oscillations are presumed to be caused by the progressive transverse contraction waves that propagate along the walls of the ducts. Peristalsis is the mechanism of the fluid transport that occurs generally from a region of lower pressure to higher pressure when a progressive wave of area contraction and expansion travels along the flexible wall of the tube. Peristaltic flow occurs widely in the functioning of the ureter, food mixing and chyme movement in the intestine, movement of eggs in the fallopian tube, the transport of the spermatozoa in the cervical canal, transport of bile in the bile duct, transport of cilia, and circulation of blood in small blood vessels. There are many other important applications of this principle such as the design of roller pumps, which are useful in pumping fluids without contamination due to contact with the pumping machinery.

A number of analytical studies of peristaltic transport obtained by a train of periodic sinusoidal waves in an infinitely long two-dimensional symmetric channel or ax-symmetric tubes containing a Newtonian or non-Newtonian fluid with no-slip wall condition have been investigated in refs. [1-18]. Lew et al [10] suggested chyme as a non-Newtonian material having plastic-like properties.

In several flow problems, the authors assumed adherence, i.e. that the fluid layer next to a rigid surface moves with that surface. Some authors, considered hypotheses involving slippage, i.e. a relative motion of the rigid surface and the fluid next to it. For several fluids including water and mercury, many experiments, some of them beautifully conceived and carefully performed, have indicated that the adherence condition is appropriate even when the fluid does not wet the boundary surface. From time to time, an apparently carefully experiment has seemed to lead to the opposite conclusion but further analysis has revealed theoretical or experimental error.

Curiously enough, there two extremely different type of fluids which appear to slip. One class contains the rarefied gases, the other fluids with much elastic character i.e. fluid with memories that fade very slowly. However, a fluid having as much elastic character, some slippage occur under a large tangential traction. It has been claimed that slippage can occur in non-Newtonian fluids, concentrated polymer solution and, basis of more evidence, molten polymer. Further, in the flow of dilute suspensions of particles a clear layer is sometimes observed next to the wall. Poiseuille, in a work which won a prize in experimental physiology, observed such layer with a microscope in the flow of blood through capillary vessels ref.[19].

Navier [20] proposed boundary conditions that consider the possibility of the fluid slip at a rigid boundary which states the velocity of the fluid at the plate is linearly proportional to the shear stress at the plate. Kwang and Fang [21] studied the peristaltic transport of a Newtonian fluid through a 2D micro channel where the slip effect is present.

With above dissection in mind, we examine the peristaltic flow of a power-law fluid through a deformable inclined tube with the wall slip condition. The considered model of power-law fluid adequately fits the shear stress and shear rate measurements for many non-Newtonian fluids. It serves as a model for the study of flow of chyme through small intestines. The present work extends that Srivastava and Srivastava [18] and Fung, and Yih [7].

2 Formulation and analysis

Consider the flow of an incompressible power-law fluid through an inclined non-uniform tube such that it has a sinusoidal wave traveling down its' wall. The geometry of the wall surfaces is described in Figure 1.

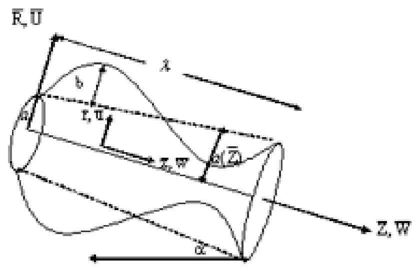

Figure 1: Peristaltic transport in an inclined non-uniform tube.

$$\bar{h}(\bar{Z},\bar{t}) = a\left(\bar{Z}\right) + bSin(\frac{2\pi}{\lambda}(\bar{Z}-c\bar{t})), \tag{1}$$

$$a\left(\bar{Z}\right) = a + k\bar{Z}, \tag{2}$$

where $a\left(\bar{Z}\right)$ is the radius of the tube at any axial distance from inlet, a is the radius of the tube at inlet, k($\ll$1) is a constant whose magnitude depends length of the tube, exit and inlet dimensions, b is the amplitude of the wave, λ is the wavelength, c is the wave speed, α is the inclined angle and $\bar{t}$ is the time .We choose the cylindrical coordinates system $(\bar{R},\bar{Z})$, where $\bar{Z}$- axis lies along the centerline of tube, and $\bar{R}$ is the distance measured radial. In the moving coordinates $(\bar{r},\bar{z})$, which travel in the $\bar{Z}$-direction with the same speed as the wave, the flow in the tube is steady but if we choose the fixed coordinates , the flow in the tube can be treated as unsteady. The coordinate frames are related through:

$$\bar{Z} = \bar{z} + c\bar{t}, \quad \bar{r} = \bar{R}, \tag{3}$$

$$\bar{W} = \bar{w} + c, \ \bar{U} \ = \bar{u}. \tag{4}$$

where $\bar{U},\bar{W}$ and $\bar{u},\bar{w}$ are the velocity components in the radial and axial directions in the fixed and moving coordinates respectively.

Equations of motion and boundary conditions in the fixed coordinates are:

Continuity equation :

$$\frac{1}{\bar{R}}\frac{\partial(\bar{r}\bar{U})}{\partial\bar{R}} + \frac{\partial(\bar{W})}{\partial\bar{Z}} = 0, \tag{5}$$

Navier-Stokes equations :

$$\rho(\frac{\partial \bar{U}}{\partial \bar{t}} + \bar{U}\frac{\partial \bar{U}}{\partial \bar{R}} + \bar{W}\frac{\partial \bar{U}}{\partial \bar{Z}}) = -\frac{\partial \bar{P}}{\partial \bar{R}} - (\frac{1}{\bar{R}}\frac{\partial(\bar{R}\bar{\tau}_{11})}{\partial \bar{R}} + \frac{\partial \bar{\tau}_{31}}{\partial \bar{Z}} - \frac{\bar{\tau}_{22}}{\bar{R}}), \tag{6}$$

$$\rho(\frac{\partial \bar{W}}{\partial \bar{t}} + \bar{U}\frac{\partial \bar{W}}{\partial \bar{R}} + \bar{W}\frac{\partial \bar{W}}{\partial \bar{Z}}) = -\frac{\partial \bar{P}}{\partial \bar{Z}} - (\frac{1}{\bar{R}}\frac{\partial(\bar{R}\bar{\tau}_{13})}{\partial \bar{R}} + \frac{\partial \bar{\tau}_{33}}{\partial \bar{Z}}) + \rho g Sin(\alpha), \tag{7}$$

Constitutive equation:

$$\eta = m\ \bar{\dot{\gamma}}^{n-1} \tag{8}$$

$$\bar{\tau}_{ij} = -\eta \bar{\dot{\gamma}}_{ij}, \tag{9}$$

where ρ is the density, $\bar{\tau}_{ij}$, i, j = 1,2,3 are the components of the extra stress tensor, $\bar{P}$ is pressure, m is the consistency, n is the dimensionless power-law index (When n=1, then $\eta = m$ is the Newtonian viscosity of the fluid) and $\bar{\dot{\gamma}}$ is defined as :

$$\bar{\dot{\gamma}} = \sqrt{\frac{1}{2}\sum_i \sum_j \bar{\dot{\gamma}}_{ij}\bar{\dot{\gamma}}_{ij}} = \sqrt{\frac{1}{2}\Pi_{\bar{\dot{\gamma}}}}, \tag{10}$$

where $\Pi_{\bar{\dot{\gamma}}}$ is second invariant of strain-rate tensor $\bar{\dot{\gamma}}_{ij}$.
let the velocity vector be$\vec{V}\left(\bar{U}, 0, \bar{W}\right)$,where $\bar{U}$and $\bar{W}$ are functions of $(\bar{R}, \bar{Z}, \bar{t})$, then the rate of strain tensor, has the components:

$$\bar{\dot{\gamma}}_{11} = 2\frac{\partial \bar{U}}{\partial \bar{R}},\ \bar{\dot{\gamma}}_{22} = 2\frac{\bar{U}}{\bar{R}},\ \bar{\dot{\gamma}}_{33} = 2\frac{\partial \bar{W}}{\partial \bar{Z}}\ and\ \bar{\dot{\gamma}}_{13} = \bar{\dot{\gamma}}_{31} = \frac{\partial \bar{U}}{\partial \bar{Z}} + \frac{\partial \bar{W}}{\partial \bar{R}} \tag{11}$$

With the boundary conditions:

$$\begin{aligned} &\bar{U} = 0, \tfrac{\partial \bar{W}}{\partial \bar{R}} = 0, && \bar{R} = 0, \\ &\bar{W} = -\Lambda\tfrac{\partial \bar{W}}{\partial \bar{R}}, \bar{U} = -c\tfrac{d\bar{h}}{d\bar{Z}}, && \bar{R} = \bar{h} = a\left(\bar{Z}\right) + bSin(\tfrac{2\pi}{\lambda}(\bar{Z} - c\bar{t})), \end{aligned} \tag{12}$$

where Λ is the slip parameter.

We will non-dimensionalize the variables appearing in equations (2.1-2.12) introducing Reynolds number (Re), wave number (δ), Knudsen number (kn) and Froud number (Fr) as follows:

$r=\frac{\bar{r}}{a},\quad R=\frac{\bar{R}}{a},\quad z=\frac{\bar{z}}{\lambda},\quad Z=\frac{\bar{Z}}{\lambda},\quad u=\frac{\lambda\bar{u}}{a\ c},$

$U=\frac{\lambda\bar{U}}{a\ c},\quad w=\frac{\bar{w}}{c},\quad W=\frac{\bar{W}}{c},\quad t=\frac{c\bar{t}}{\lambda},\quad P=\frac{a^{n+1}\bar{P}}{m\ c^{n}\ \lambda},$

$\tau_{ij}=\frac{a^{n}\bar{\tau}_{ij}}{m\ c^{n}},\quad \dot{\gamma}_{ij}=\frac{a\bar{\dot{\gamma}}_{ij}}{c},\quad \dot{\gamma}=\frac{a\bar{\dot{\gamma}}}{c},\quad Re=\frac{\rho\ a^{n}}{m\ c^{n-2}},\quad \delta=\frac{a}{\lambda},$

$kn=\frac{\Lambda}{a}\quad Fr=\frac{c^{2}}{ag},\quad f=\frac{Fr}{Re}\quad and\ \ h=\frac{\bar{h}}{a}=a(Z)+\phi\ Sin(2\pi(Z)-t),$

$a(Z)=1+\frac{\lambda k}{a}Z,$

where $\phi=\frac{b}{a}<1$ is the amplitude ratio.
Equations of motion, boundary conditions, rate of strain tensor components and extra stress tensor components in the dimensionless form take the following form: Continuity equation becomes :

$$\frac{1}{R}\frac{\partial(R\ U)}{\partial R}+\frac{\partial W}{\partial Z}=0, \tag{13}$$

Navier-Stokes equations become:

$$Re\ \delta^{3}\left(\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial R}+W\frac{\partial U}{\partial Z}\right)=-\frac{\partial P}{\partial R}-\delta\ \frac{1}{R}\frac{\partial(R\tau_{11})}{\partial R}-\delta^{2}\frac{\partial\tau_{31}}{\partial Z}+\delta\frac{\tau_{22}}{R}, \tag{14}$$

$$Re\ \delta\left(\frac{\partial W}{\partial t}+U\frac{\partial W}{\partial R}+W\frac{\partial W}{\partial Z}\right)=-\frac{\partial P}{\partial Z}-\frac{1}{R}\frac{\partial(R\tau_{13})}{\partial R}-\delta\frac{\partial\tau_{33}}{\partial Z}+\frac{Sin(\alpha)}{f}, \tag{15}$$

$$\tau_{ij}=-\dot{\gamma}^{n-1}\dot{\gamma}_{ij}, \tag{16}$$

with the dimensionless boundary conditions:

$$\begin{aligned} &U=0,\frac{\partial W}{\partial R}=0, && \bar{R}=0,\\ &W=-kn\frac{\partial W}{\partial R},U=-\frac{dh}{dZ}, && R=h=1+\frac{\lambda k}{a}Z+\phi Sin(2\pi(Z-t)), \end{aligned} \tag{17}$$

the components of rate of strain tensor in the dimensionless form become:

$$\dot{\gamma}_{11}=2\delta\frac{\partial U}{\partial R},\ \dot{\gamma}_{22}=2\delta\frac{U}{R},\ \dot{\gamma}_{33}=2\delta\frac{\partial W}{\partial Z}\ and\ \dot{\gamma}_{13}=\dot{\gamma}_{31}=\frac{\partial W}{\partial R}+\delta^{2}\frac{\partial U}{\partial Z}. \tag{18}$$

Also,$\dot{\gamma}$ is defined in the dimensionless form as following:

$$\dot{\gamma}=\delta\sqrt{\frac{1}{2}\sum_{i}\sum_{j}\dot{\gamma}_{ij}\dot{\gamma}_{ij}}\ \ for\ i=j, \tag{19}$$

$$\dot{\gamma} = \sqrt{\frac{1}{2}\sum_i \sum_j \dot{\gamma}_{ij}\dot{\gamma}_{ij}} \;\; for\ i \neq j, \tag{20}$$

Using the long wavelength approximation and neglecting the wave number then from equations (16), (18) and (20) shearing extra stress tensor becomes.

$$\tau_{13} = \tau_{31} = (-\frac{\partial W}{\partial R})^n, \tag{21}$$

Using the long wavelength approximation and neglecting the wave number then from equations (16), (18) and (20) shearing extra stress tensor becomes.

$$\tau_{13} = \tau_{31} = (-\frac{\partial W}{\partial R})^n, \tag{22}$$

Navier-Stokes equations reduce to:

$$\frac{\partial P}{\partial Z} = -\frac{1}{R}\frac{\partial (R\tau_{13})}{\partial R} + \frac{Sin(\alpha)}{f}, \tag{23}$$

$$\frac{\partial P}{\partial R} = 0. \tag{24}$$

The instantaneous volume flow rate in the fixed coordinate system is given by

$$\bar{Q} = 2\pi \int_0^{\bar{h}} \bar{W}\bar{R}\ d\bar{R}, \tag{25}$$

where $\bar{h}$ is a function of $\bar{Z}$ and $\bar{t}$. On substituting equations (3) and (4) into equation (25), and then integrating, one obtains:

$$\bar{Q} = \bar{q} + \pi c\bar{h}^2, \tag{26}$$

where:

$$\bar{q} = 2\pi \int_0^{\bar{h}} \bar{w}\bar{r}\ d\bar{r}, \tag{27}$$

is the volume flow rate in the moving coordinate system and is independent of time. Here, $\bar{h}$ is a function of $\bar{z}$ alone. Using the dimensionless variables, we find:

$$F = \frac{\bar{Q}}{2\pi a^2 c} = \int_0^h RW\ dR\ . \tag{28}$$

The time-mean flow over a period $T = \frac{\lambda}{c}$ at a fixed $\bar{Z}$-position is defined as:

$$Q = \frac{1}{T}\int_0^T \bar{Q}\ d\bar{t}\ . \tag{29}$$

Using equations (26) in equation (29) and integrating, we get:

$$\frac{\bar{Q}}{2\pi ca^2} = \frac{Q}{2\pi ca^2} - \frac{\phi^2}{4} + \phi Sin(2\pi(Z-t)) + \frac{\lambda k}{a} Z\phi Sin(2\pi(Z-t)) + \frac{\phi^2}{2} Sin^2(2\pi(Z-t)). \tag{30}$$

On defining the dimensionless time-mean flow as :

$$\Theta = \frac{Q}{2\pi ca^2}, \tag{31}$$

we rewrite (30)as:

$$F = \Theta - \frac{\phi^2}{4} + \phi Sin(2\pi(Z-t)) + \frac{\lambda k}{a} Z\phi Sin(2\pi(Z-t)) + \frac{\phi^2}{2} Sin^2(2\pi(Z-t)) \tag{32}$$

solving equation (23) by using equation (24) and boundary condition (17), we obtain

$$W = \frac{n}{n+1}\left[\frac{1}{2}\left(-\frac{dP}{dZ} + \frac{Sin(\alpha)}{f}\right)\right]^{\frac{1}{n}}\left[h^{\frac{n+1}{n}} - R^{\frac{n+1}{n}} + \frac{n+1}{n} kn\ h^{\frac{1}{n}}\right]. \tag{33}$$

Substituting from equation (33) in equation (28) and solving the result in $\frac{dP}{dZ}$ we get:

$$\frac{dP}{dZ} = -2\left(\frac{3n+1}{n}\right)^n \frac{(2F)^n}{h^{3n+1}}\left[1 + \frac{3n+1}{n\ h} kn\right]. \tag{34}$$

By putting α=0 and kn=0 we obtain the results obtained by Sirvastava and Sirvastava [17]. The pressure rise $\Delta P_\lambda(t)$ and the friction force $F_\lambda(t)$ (at the wall) in the tube of length λ, in their non-dimensional forms, are given by:

$$\Delta P_\lambda(t) = \int_0^1 \frac{dP}{dZ}\, dZ, \tag{35}$$

$$F_\lambda(t) = \int_0^1 h^2(-\frac{dP}{dZ})\, dZ. \tag{36}$$

3 Numerical results and discussion

To discuss the results obtained above quantitatively, we shall assume the form of the flow rate F, in period (Z-t) as in Srivastava and Srivastava [17]:
$F^n = \Theta^n - \frac{\phi^2}{4} + \phi Sin(2\pi(Z-t)) + \frac{\lambda k}{a} Z\phi Sin(2\pi(Z-t)) + \frac{\phi^2}{2} Sin^2(2\pi(Z-t))$
This form has been assumed in view of the fact that the constant values of Θ gives always ΔP_λ negative, and hence there will be no pumping action. This theoretical analysis of peristaltic flow in an axisymmetric non-uniform tube is applicable the transport of chyme through the small intestine. The value

of various parameter for the transport of chyme in male small intestine as reported in Srivastava and Srivastava [17]:

$$a = 1.25cm, c = 2cm\ /min,\ \lambda = 8.01cm\ and\ k = 3a/\lambda.$$

It may be noted that the theory of long wavelength of the present investigation remain applicable here as the radius of the small intestine at inlet a=1.25 cm, is small compared with the wavelength λ=8.01 cm. It has also, observed by Lew et al. [10] that Reynolds number in the small intestine was very small. The average pressure rise $\Delta\bar{P}_\lambda$ is evaluated by averaging $\Delta P_\lambda(t)$ over one period of the wave and so the friction force F_λ. As the integral in equations (35) and (36) are not integrable in the closed form, they are evaluated numerically using a digital computer.

Figure 2 represent the variation of dimensionless pressure rise with time for fixed α, ϕ, kn, f and different values of power law index and flow rate. We can see that the pressure rise is increases when power-law index (n) increases. Further, the pressure rise for Newtonian fluid (n=1), is greater than it for non-Newtonian fluid (n=2/3, n=12). It can also be seen that the effect of increasing the flow rate is to reduce the pressure rise.

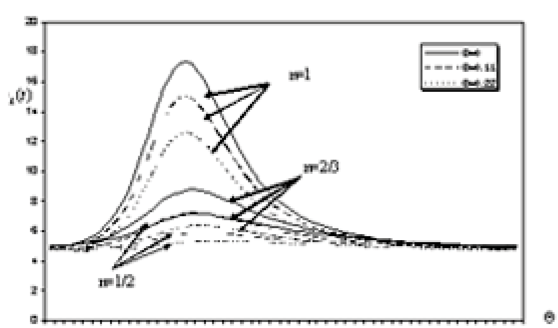

Figure 2: The pressure rise versus time at ϕ= 0.6, $\alpha = \frac{\pi}{6}$, kn = 0.01 and f=0.1.

The effects of inclined angle α and the slip boundary conditions on the pressure rise appears through figure 3. we can notice that the slip boundary conditions is greatly affected on the pressure rise in the case of Newtonian fluid and non-Newtonian. We observe that the pressure rise decreases with increasing knudsen number. Also, the effect of increasing inclined angle α is to increase the pressure rise. Further, for the fixed at Θ, n, α and f the pressure rise becomes smaller as ϕ decreases from 0.6 and it becomes constant with time when the there is no peristalsis (ϕ=0) (see figure 4).

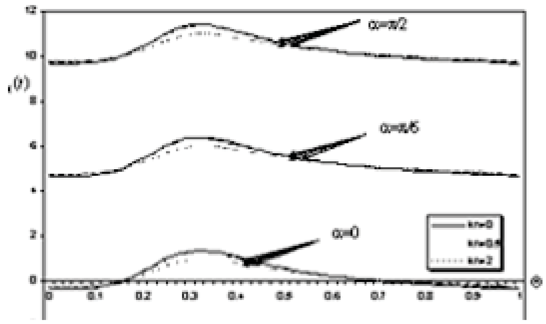

Figure 3: The pressure rise versus time at ϕ= 0.6, n=$\frac{2}{3}$, Θ= 0.22 and f=0.1.

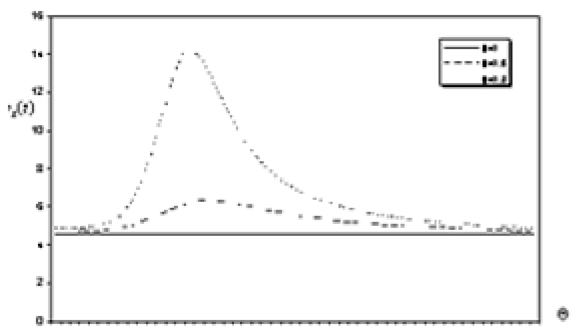

Figure 4: The pressure rise versus flow rate at Θ= 0.22 , n=2/3, $\alpha = \pi/6$, kn =0.01 and f=0.1.

The average pressure rise versus flow rate is plotted for different values of n, kn and α in figures (5-7). As expected the fugues show a linear relation between pressure rise and flow rate for Newtonian fluid (n=1) and non-linear for non-Newtonian (n=2/3, 1/2). Further, it is clear that an increase in flow rat and knudsen number decreases the pressure rise and thus, maximal pressure rise is achieved at zero flow rate. Also, an increase the inclined angle α increases the pressure rise. Furthermore the pressure rise is independent of n and kn at certain values of flow rate but it is depend on inclined angle for different values of flow rate. Moreover, In the case of vertical tube (α=π/2) with high occlusion (ϕ=0.6), the peristaltic pumping, where $\Delta\bar{P}_\lambda > 0$ and $\Theta > 0$, occur for different values of flow rate and there is no augmented pumping region, but the augmented pumping, where $\Theta > 0$ and $\Delta\bar{P}_\lambda < 0$, occurs in the case of horizontal tube (α=0) at $0.3 < \Theta \leq 1$ (figure.7).

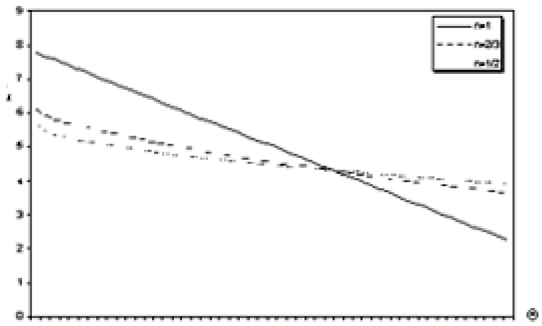

Figure 5: The average pressure rise versus flow rate at ϕ= 0.6, α= π /6, kn = 0.01and f=0.1.

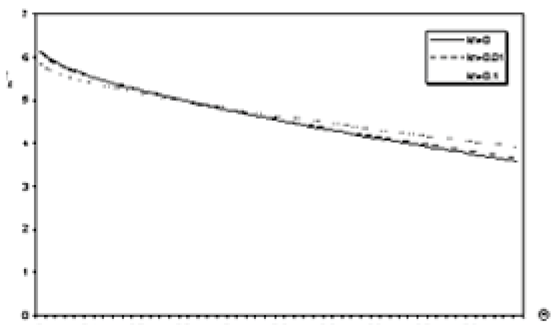

Figure 6: The average pressure rise versus flow rate at ϕ= 0.6, α = π /6, n=2/3 and f=0.1.

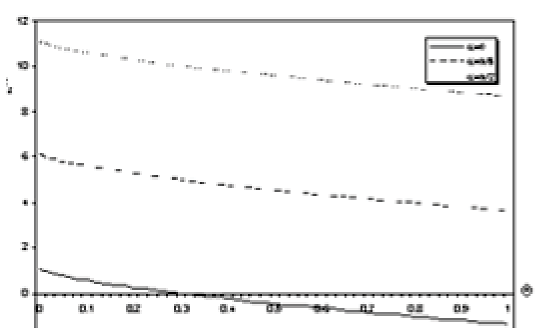

Figure 7: The average pressure rise versus flow rate at ϕ= 0.6, kn = 0.01, n=2/3 and f=0.1.

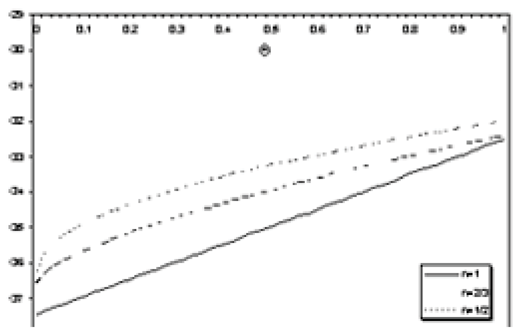

Figure 8: The friction force versus flow rate at ϕ= 0.6, α = $\pi/6$, kn = 0.01 and f =0.1.

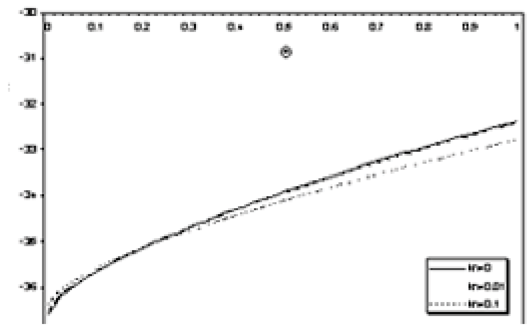

Figure 9: The friction force versus flow rate at ϕ= 0.6, $\alpha = \pi/6$, n=2/3 and f=0.1.

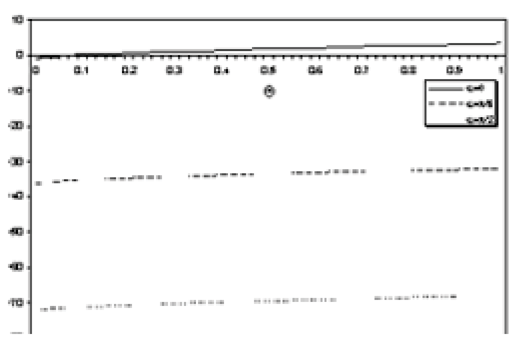

Figure 10: The friction force versus flow rate at at ϕ= 0.6, kn = 0.01, n=2/3 and f=0.1.

Finally the friction force have been plotted in Figures (8-10), which show an opposite character in comparison to pressure rise.

Our results obtained in this study, with no slip condition (kn=0) and horizontal tube (α=0), agree with those obtained by Srivastava and Srivastava [17].

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