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A New Mathematical Model for a Redundancy Allocation Problem with Mixing Components Redundant and Choice of Redundancy Strategies

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Abstract

This paper presents a new mathematical model of a redundancy allocation problem with mixing components redundant in subsystems of a series-parallel system when the redundancy strategy can be chosen for individual subsystems. In practice both active and cold-standby redundancies are used within a particular system design, and the choice of the redundancy strategy becomes an additional decision variable. The proposed model selects the best redundancy strategy, combination of components, and levels of redundancy for each subsystem in order to maximize the system reliability under system-level constraints.

Keywords: Redundancy Allocation Problem; Reliability; Series-Parallel Systems; Redundancy Strategies

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1. Introduction

The primary goal of a reliability design is to improve the system reliability. In the initial design activity, a redundancy allocation is a direct way of enhancing the system reliability. The redundancy allocation problem (RAP) involves in the simultaneous selection of components and a system-level design configuration, which can collectively meet all design constraints in order to optimize some objective functions such as system cost and/or reliability [3]. In this problem, there are several different component types with different levels of cost, reliability, weight, and other characteristics, and the components redundant within the subsystem are the same type. The RAP is an NP-hard problem [2] solved by using many difference optimization approaches for different formulations as summarized in Kuo, *et al.* [10] and Gen, *et al.* [8]. While this problem has been studied in great details, one area, which has not been sufficiently analyzed, is the use of mixing components redundant in subsystems. This kind of the redundancy allocation problem is to select the optimal combination of components and levels of redundancy to collectively meet weight and cost constraints, while maximizing the system reliability [9].

Coit and Smith [4] presented a new formulation and solution method for the redundancy allocation problem with mixing components redundant in subsystems. They used a genetic algorithm (GA) to solve this problem in k-out-of-n:G system. Chen and You [1] also presented the use of an immune algorithm to solve such a problem in a series system. In general, this reliability design problem has been formulated by considering active redundancy. However, the choice of redundancy strategies for each subsystem is much more realistic and it provides a better tool for the designers. This becomes an additional decision variable in a redundancy allocation problem with mixing components redundant in subsystems.

Coit and Liu [5] presented a new formulation and solution method for the RAP when a system design includes multiple subsystems designed with either active or cold-standby redundancy. This solution method assumes that the redundancy strategy (i.e., active or cold-standby) for each subsystem is predetermined. Coit [6] presented a new formulation and solution method to the RAP when there are some subsystems using active redundancy and cold-standby redundancy, or selecting the best redundancy strategy. Furthermore, Tavakkoli-Moghaddam, *et al.* [11] used the genetic algorithm to solve the above-mentioned problem. Unfortunately, the redundancy allocation problem with mixing components in subsystems is not considered when the redundancy strategy can be chosen for individual subsystems. Thus, the problem is to select the best redundancy strategy, combination of components, and redundancy level for each subsystem in order to maximize the system reliability under system-level constraints.

The series-parallel system depicted in Figure 1 is a common system structure that is used in most system designs. Thus in this paper, the series-parallel redundancy allocation problem with mixing components, and the choice of redundancy strategies in subsystems is considered. Finally, a new mathematical model is presented to this problem.

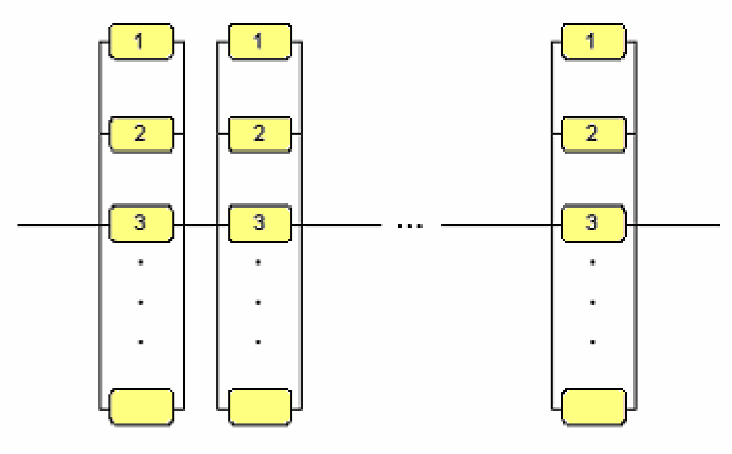

Figure 1. Series-parallel system.

The structure of this paper is organized as follows. Section 2 presents a review on the redundancy strategies. In Section 3, the problem formulation is presented and proposed for a redundancy allocation problem with mixing components in subsystems when either active or cold-standby redundancy can be selected for individual subsystems. Finally, Section 4 presents conclusion.

2. Redundancy Strategies

There are two types of redundancy strategies, namely active and standby. If all redundant components operate simultaneously from time zero, even though the system needs only one at any given time, such an arrangement is called active redundancy. There are three variants of the standby redundancy, referred to as cold, warm, and hot. In the cold standby redundancy, the component does not fail before it operates. In the warm standby redundancy, the component is more prone to failure before operation than the cold standby components. In the hot standby redundancy, the failure pattern of component does not depend on whether the component is idle or in operation. The mathematical models for hot standby and active redundancy arrangements are the same. In the standby redundancy arrangement, the redundant components are sequentially used in the system at component failure times. Each redundant component in the standby arrangement can operate only when it is switched on. When the component in operation fails, one of the redundant units is switched on to continue the system operation [7]. In the standby redundancy, there are two scenarios in first detecting failure and then switching to good components. These are classified as Case 1 and Case 2. For the Case 1, the failure detection and switching hardware or software continually monitors the system performance. When it detects a failure, it activates a redundant component. For the Case 2, a failure is only possible when a switch is required. At any time the switch is required, there is a constant probability (ρ_i) that the switching will be successful [6].

This paper considers redundancy strategies consisting of only active (i.e., hot) and cold-standby redundancy. The approach used categorizes all subsystems to four sets according to the following definitions:

N: Set of all subsystems with no redundancy.
A: Set of all subsystems with active redundancy.
S: Set of all subsystems with cold-standby redundancy.
A&S: Set of all subsystems with active or cold-standby redundancy.

3. Problem Formulation

In each subsystem, all assignable component types are sorted in descending order by the component reliability and put in a set of component types. From now, component types are shown by their rank orders in this set. Therefore, components of type 1 are more reliable than components of type 2 and components of type 2 are more reliable than components of type 3, and the like. For increase the total reliability of a system, components of the first type 1 that are more reliable in all component types are used. After all components of type 1 are down and failure, components of type 2 that are more reliable in backup components are used, and so one. Sequential usage of backup components continues until failure of all components.

The new mathematical model of the redundancy allocation problem with mixing components redundant in subsystems for the series-parallel system is presented as the following integer nonlinear programming problem when the redundancy strategy can be chosen for individual subsystems and two separable linear constraints.

3.1. Assumptions

- Two redundancy strategies (i.e., active redundancy, cold standby) are considered.
- The states of components and the system have only two options: good or bad.
- The component attributes (i.e., reliability, cost, and weight) are known and deterministic.
- There is no component repair or preventive maintenance.
- Failures of components are independent events.
- Failed components do not damage the system.

3.2. Indices

i	Index for subsystems $(i = 1,2,...,s)$
j	Index for component type $(j = 1,2,..., m_i)$

k_j Index for number of failure of type j components in each subsystem $(k_j = 1,2,...,n_{ij})$

l Index for number of the allocated component types

z_{il} Index of component choices used for subsystem i, $z_{il} \in \{1,2,...,m_i\}$

z_i Set of z_{il}, $(z_{i1}, z_{i2}, ...)$, for example (1 , 3 , 4)

3.3. Decision Variables

ARS_i Allocated redundancy strategy in subsystem i

$ARS = (ARS_1, ARS_2, ..., ARS_s)$ Vector of allocated redundancy strategy in subsystems

n_{ij} Number of components of type j used in subsystem i,

$n_{ij} \in \{1,2,...,n_{\max,ij}\}$

Type of components

$$
\begin{array}{ccc}
 & \begin{array}{cccccc} 1 & 2 & \cdots & j & \cdots & m \end{array} & \\
n = & \begin{bmatrix}
n_{11} & n_{12} & \cdots & n_{1j} & \cdots & n_{1m} \\
n_{21} & n_{22} & \cdots & n_{2j} & \cdots & n_{2m} \\
. & . & \cdots & . & \cdots & . \\
. & . & \cdots & . & \cdots & . \\
. & . & \cdots & . & \cdots & . \\
n_{i1} & n_{i2} & \cdots & n_{ij} & \cdots & n_{im} \\
. & . & \cdots & . & \cdots & . \\
. & . & \cdots & . & \cdots & . \\
. & . & \cdots & . & \cdots & . \\
n_{s1} & n_{s2} & \cdots & n_{sj} & \cdots & n_{sm}
\end{bmatrix} & \begin{array}{c} 1 \\ 2 \\ . \\ . \\ . \\ i \\ . \\ . \\ . \\ s \end{array}
\end{array}
$$

Subsystem

3.4. Parameters

s Number of subsystems

m_i Number of available component choices for subsystem i, $(i = 1, 2, \ldots, s)$

m Upper bound for m_i, $(m \le m_i \quad \forall i)$

$n_{\max, ij}$ Upper bound for n_{ij}, $(n_{ij} \le n_{\max, ij} \quad \forall ij)$

t Mission time

r_{ij} (*t*) Reliability at time *t* for the *j*th available component for subsystem *i*

c_{ij} , w_{ij} Cost and weight for the *j*th available component for the subsystem *i*

C, *W* System-level constraint limits for cost and weight

R(t; n; ARS) System reliability at time *t* for designing matrix *n* and vector *ARS*

ρ_i (*t*) Failure-detection/switching reliability at time *t* (Case 1)

ρ_i Failure-detection/switching success probability (Case 2)

$f_{i,j}^{(k_j)}(t)$ pdf of the k_j th failure times of the type j components for subsystem *i* at time *t*

3.5. Mathematical model

Max *R(t; n; ARS)* (1)

s.t.

$$\sum_{i=1}^{s}\sum_{j=1}^{m_i} c_{ij} n_{ij} \le C \tag{2}$$

$$\sum_{i=1}^{s}\sum_{j=1}^{m_i} w_{ij} n_{ij} \le W \tag{3}$$

With respect to Equation (1), the objective is to determine the redundancy strategy, combination of components, and the quantity of components in each subsystem to achieve the maximum system reliability. Constraints given in Equations (2) and (3) consider the available cost and weight, respectively. To calculate $R(t; n, ARS)$, Equations (4) and (5) are presented for the system reliability in two cases as follows:

Case 1: Continuous detector/switch operation:

$$R(t;n,ARS) = \prod_{i\in N'} r_{ij}(t) \times \prod_{i\in A'}\left(1-\prod_{j=1}^{m_i}\left(1-r_{ij}(t)\right)^{n_{ij}}\right) \times \prod_{i\in S'} r_{iz_{i1}}(t) + \sum_{j\in z_i}\sum_{k_j=1}^{n_{ij}} \int_0^t \rho_i(u)\, f_{ij}^{(k_j)} (u) r_{ij}(t-u)du - \int_0^t \rho_i(u)\, f_{iz_{i1}}^{(1)}(u) r_{iz_{i1}}(t-u)du \Bigg) \tag{4}$$

Case 2: Switch active only in response to a failure:

$$R(t;n,ARS)=\prod_{i\in N'} r_{ij}(t)\times\prod_{i\in A'}\left(1-\prod_{j=1}^{m_i}\left(1-r_{ij}(t)\right)^{n_{ij}}\right)\times\prod_{i\in S'} r_{iz_{i1}}(t)+\sum_{j\in z_i}\sum_{k_j=1}^{n_{ij}}\rho_i^{k_j+\sum_{h=1}^{j}n_{ih}-n_{ij}} \int_0^t f_{ij}^{(k_j)}(u)r_{ij}(t-u)du-\rho_i\int_0^t f_{iz_{i1}}^{(1)}(u)r_{iz_{i1}}(t-u)du \Bigg) \tag{5}$$

After allocating a redundancy strategy to each subsystem and according to the definition of ARS, all subsystems can be classified by the allocated redundancy strategy as follows:

N' : Set of all subsystems with no redundancy.
A' : Set of all subsystems with active redundancy.
S' : Set of all subsystems with cold-standby redundancy.

Therefore, Equation (4) is constructed in three segments. Segments 1, 2, and 3 calculate the reliability of all subsystems with no redundancy, active redundancy, and cold-standby redundancy respectively. In segment 3, the reliability of cold-standby subsystem is the summation of the reliability of components in operation as well as backup components. According to the definition of *j*, at the beginning operation of a cold-standby subsystem, one component of type z_{i1} is operating. When a component in operation fails, one of the backup components is switched on to continue the system operation. Failure in each subsystem is accrued at time *u* with probability of $f_{i,j}^{(k_j)}(u)$ and then the switching will be successful with probability of ($\rho_i(u)$), finally one backup components with reliability of $r_{ij}(t-u)$ is operated. Equation 5 is the same as Equation (4). The only difference is in calculating the reliability of detecting failure and switching system. In this equation, the power of ρ_i is the number of failure in subsystem *i* till time *t*.

4. Conclusion

This paper proposes a new mathematical model for a redundancy allocation problem with mixing components redundant in subsystems for the series-parallel system when either active or cold-standby redundancy is selected for individual subsystems. Most mathematical models of the general, foregoing problems assume that the redundancy strategy for each subsystem is predetermined and known. In general, the active redundancy has received more attention in the past. The choice of redundancy strategies for each subsystem is much more realistic and it provides a better tool for the designers. This problem is formulated as a nonlinear mixed-integer programming model under a number of constraints. This problem is not easy to solve in real cases, especially for large-sized systems. Therefore, the use of heuristic or meta-heuristic methods for solving such a hard problem is proposed for future research.

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