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Spatial Damping of Linear Compressional Magnetoacoustic Waves in Quiescent Prominences

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Abstract. We study the spatial damping of magnetoacoustic waves in an unbounded quiescent prominence invoking the technique of MHD seismology. We consider Newtonian radiation in the energy equation and derive a fourth order general dispersion relation in terms of wavenumber k. Numerical solution of dispersion relation suggests that slow mode is more affected by radiation. The high frequency waves have been found to be highly damped. The uncertainty in the radiative relaxation time, however, does not allow us to conclude if the radiation is a dominant damping mechanism in quiescent prominence.

Key words. Sun: prominences—oscillations—magnetohydrodynamics radiation: thermal.

1. Introduction

Prominences are masses of relatively cool ($T \sim 10^4$ K) and dense material suspended in the corona ($T \sim 10^6$ K). The spectra of prominences hold the key to understanding the physical conditions, e.g., temperatures, densities, pressures, etc. The internal structure and physical properties of prominences, however, can be studied through a new tool of prominence seismology. Magnetohydrodynamic (MHD) waves and oscillations of the solar prominence have been carried out both from the ground and from space (Patsourakos & Vial 2002). Small amplitude waves (or oscillations) with velocity amplitudes from 0.1 km s^{-1} to 2–3 km s^{-1} have been observed (e.g., Bashkirtsev & Mashnich 1984; Molowny-Horas *et al.* 1999). The velocity field oscillations are classified into three main categories: short-period oscillations ($8 \leq t \leq 20$ min), intermediate-period class ($10 \leq t \leq 40$ min), and long period oscillations ($40 \leq t \leq 90$ min). A very short period oscillation of 30 s has been observed simultaneously from two telescopes (Balthasar *et al.* 1993).

Using the VTT telescope at Sac Peak, Molowny-Horas *et al.* (1999) found velocity perturbations with periods between 28 and 95 min at different locations in a prominence and observed that the amplitude of the oscillations decreases in time with damping times between 101 and 377 min. Terradas *et al.* (2002) investigated the temporal and spatial variations of oscillations and reported the strong damping of oscillations with damping times between two and three times the wave period. Terradas *et al.* (2001) considered the Newtonian radiation in the energy models and found only slow mode waves being affected by damping leaving fast mode waves almost undamped. Carbonell *et al.* (2004) studied the damping of MHD waves in an unbounded medium by considering the effect of prominence-corona transition region (PCTR) and neglecting the adiabaticity. Terradas *et al.* (2005) investigated the damping of MHD waves in an inhomogeneous and bounded medium by neglecting the adiabaticity. They found out that only slow mode waves are affected leaving fast mode waves almost undamped. This is in agreement with Terradas *et al.* (2001). Recently, Ballai (2003) has reviewed some of the possible mechanisms that can work in the prominence to explain the spatial damping of linear compressional waves.

In this paper, we consider the Newtonian radiation to discuss the spatial damping of MHD waves. MHD equations and dispersion relation are presented in section 2; results and discussions are given in the last section.

2. MHD equations and dispersion relation

Considering a homogeneous equilibrium configuration, which is unbounded in all directions with magnetic field in the x-direction and neglecting the effect of gravity, we have

$$p_0 = \text{cons}\,t, \quad \rho_0 = \text{cons}\,t, \quad T_0 = \text{cons}\,t, \quad \boldsymbol{B}_0 = B_0\hat{\boldsymbol{x}}, \quad \boldsymbol{v}_0 = 0, \tag{1}$$

where p_0, ρ_0, T_0 and B_0 are equilibrium values of pressure, density, temperature and magnetic field.

The relevant MHD equations are given by:

$$\frac{D\rho}{Dt} + \rho\nabla\cdot\boldsymbol{v} = 0, \tag{2}$$

$$\rho\frac{Dv}{Dt} = -\nabla p + \frac{1}{\mu}(\nabla\times\boldsymbol{B})\times\boldsymbol{B} + \rho\boldsymbol{g}, \tag{3}$$

$$\frac{\rho^{\hat{\gamma}}}{\hat{\gamma}-1}\frac{D}{Dt}\left(\frac{P}{\rho^{\hat{\gamma}}}\right) = 0, \tag{4}$$

$$\frac{\partial\boldsymbol{B}}{\partial t} = \nabla\times(\boldsymbol{v}\times\boldsymbol{B}), \tag{5}$$

$$\nabla\cdot\boldsymbol{B} = 0, \tag{6}$$

$$p = \rho RT, \tag{7}$$

where $\hat{\gamma}$ is the complex ratio of specific heats.

We take perturbation in the MHD equations as $\exp i(\omega t + \boldsymbol{k}\cdot\boldsymbol{r})$ and ω is the frequency of oscillations. The effect of Newtonian radiation can be incorporated by keeping the form of the energy equation adiabatic with complex $\hat{\gamma}$ (Bunte and Bogdan 1994), which is given by

$$\hat{\gamma} = \frac{1 + i\omega\tau_R\gamma}{1 + i\omega\tau_R}. \tag{8}$$

We can generalize the ideal MHD case by replacing sound speed C_s by complex $\hat{C}_s$, which is given by

$$\hat{C}_s^2 = \frac{\hat{\gamma} p_0}{\rho_0}, \tag{9}$$

where $\hat{\gamma} = \gamma$ in the adiabatic limit and $\hat{\gamma} = 1$ in the isothermal limit.

Linearising the MHD equations (2–7), we obtain a fourth order dispersion relation as

$$a_4 k^4 + a_3 k^3 + a_2 k^2 + a_1 k + a_0 = 0, \tag{10}$$

where the coefficients in the dimensionless form are given by

$$\begin{aligned}
a_0 &= \omega^5 - \frac{i\omega^4}{\tau_R}, \\
a_1 &= 0, \\
a_2 &= -\omega^3(1 + V_A^2) + \frac{i\omega^2}{\tau_R}\left(\frac{1}{\gamma} + V_A^2\right), \\
a_3 &= 0, \\
a_4 &= \cos^2\theta V_A^2 \omega - \frac{i\cos^2\theta V_A^2}{\gamma\tau_R}.
\end{aligned}$$

For spatial damping, we take ω to be real and k to be complex as $k_R + ik_I$. The dispersion relation has four roots, which reveal the nature of magnetoacoustic modes due to damping. The roots of the dispersion relation have been obtained numerically by making use of the physical parameters in prominences (Terradas *et al.* 2001), namely $B_{0x} = 5G$, $\rho_0 = 1.2 \times 10^{-10}\,\mathrm{kg\,m^{-3}}$, $T = 7 \times 10^3\,\mathrm{K}$. We have calculated damping per wavelength, $D_L (= k_I/k_R)$ for different cases.

3. Results and discussion

The real and imaginary parts of complex wavenumber give damping per wavelength, D_L for slow and fast modes. Slow mode is not present when there is no horizontal wavenumber. The dispersion relation gives higher damping per wavelength for slow mode wave compared to fast mode. For a given value of frequency ω, the damping per wavelength first increases as a function of τ_R and attains some maximum value then decreases as we vary τ_R from 10^{-5} to 10^{+5} (in dimensionless units) (Fig. 1). As the radiative relaxation time increases, the damping per wavelength increases which shows greater wave damping at low values of radiative time and then attains some maximum value and then decreases.

The slow mode wave has higher values of damping per wavelength, showing higher levels of damping due to radiation. For $\tau_R \to \infty$, the wave takes infinite time to damp and therefore travels very long distances.

The value of radiative relaxation time in prominences is not known. According to Terradas *et al.* (2001), the radiative relaxation time could be between 10^{-4} and 10^4.

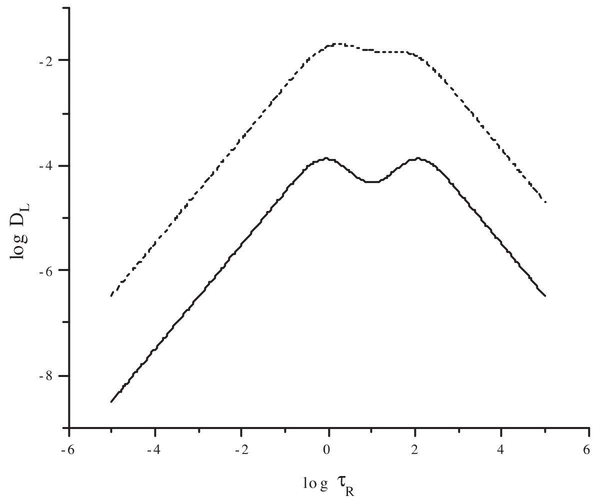

Figure 1. Damping per wavelength, $D_L (= \mathrm{Im}(k)/\mathrm{Re}(k))$ for the magnetoacoustic modes with $\omega = 10^{-1}$ (dotted line: slow mode; solid line: fast mode).

The radiative relaxation time in PCTR varies from 10–10^3 s (Terradas *et al.* 2001). Taking the radiative term as

$$L_r = \rho^2 \chi^* T^\alpha,$$

the radiative relaxation time τ_R is given by

$$\tau_R = \frac{\gamma p}{(\gamma - 1)\chi^* \rho^2 T^\alpha},$$

which for different prominence regimes mentioned in Carbonell *et al.* (2004) is between 10^2 and 10^4 s both for prominence and PCTR. The theoretical value of radiative relaxation time in the corona typically is 6×10^4 seconds (Priest *et al.* 1991). The value of radiative relaxation time calculated using opacity values of ($4 \times 10^3\,\mathrm{cm}^2\,\mathrm{g}^{-1}$). Weigert & Wendker (1996) are 1.1×10^{-4} s (Ballai 2003). For a given value of radiative relaxation time, the damping per wavelength increases almost linearly with frequency (Fig. 2). This shows that damping of magnetoacoustic waves could be directly proportional to frequency and higher frequency waves could be damped more in quiescent prominences.

The magnetoacoustic waves can be spatially damped by a few more dissipative mechanisms such as classical viscosity/resistivity, magnetic diffusivity, and thermal conduction and electrical conduction. The kinematic viscosity/resistivity gives very high damping lengths, which shows that the effect of classical viscosity in the damping of magnetoacoustic waves is very small. Using ion collisional rate and Larmor frequency, Ballai (2003) concluded that the nature of viscosity could be isotropic and that of thermal conduction could be anisotropic. Thermal conduction can affect the slow mode waves provided the wavelengths are short.

Singh (2006), studied the spatial damping of the magnetoacoustic waves using the energy losses through Newtonian cooling and MHD turbulence. It was found that the slow mode wave has shorter damping length compared to the fast mode wave. From

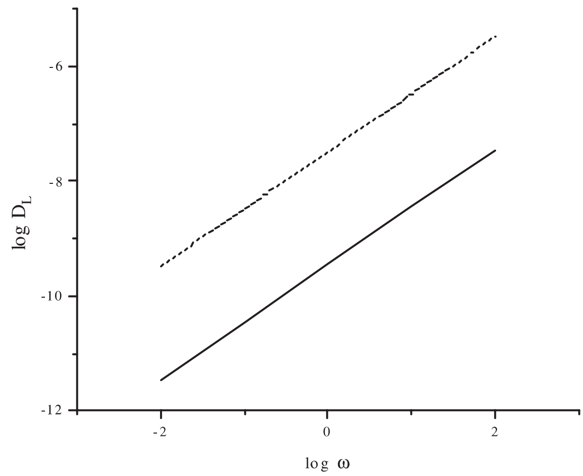

Figure 2. Damping per wavelength, $D_L (= \text{Im}(k)/\text{Re}(k))$ for the magnetoacoustic modes with $\tau_R = 10^{-7}$ (dotted line: slow mode; solid line: fast mode).

prominence seismology, the opacity and turbulent kinematic viscosity is inferred. It is found that the turbulent viscosity is orders of magnitude higher than the Spitzer's one. Also, it has been found that the calculated turbulent viscosity can reproduce the observed damping time and damping length in prominences. The convective disturbances may travel up in the solar atmosphere through the magnetic field lines that are anchored in the photosphere and cause the MHD turbulence in prominences.

In conclusion, I find that the Newtonian radiation alone is inadequate to explain the spatial damping of both slow and fast mode waves and gives acceptable damping lengths for certain values of radiative relaxation time. The presence of density inhomogeneity may lead to phase mixing and resonant absorption of MHD waves. The uncertainty in the radiative relaxation time in prominences does not allow us to reach a definite conclusion as to whether the oscillations are damped by Newtonian radiation alone. In order to explain the damping of both slow and fast modes, some additional mechanisms such as turbulent viscosity, ion-neutral damping, wave leakage, phase-mixing, resonant absorption, etc. are worth considering.

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