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Minimax Estimation of the Scale Parameter of the Selected Gamma Population with Arbitrary Known Shape Parameter

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Abstract

Let X_1, X_2 be independent random variables from gamma populations Π_1, Π_2 with common arbitrary known shape parameter α and unknown scale parameters θ_1, θ_2 respectively. Suppose $X_{(1)}$, $X_{(2)}$ be the order statistics of X_1, X_2 and the population corresponding to largest $X_{(2)}$ observation is selected. In this paper we consider the problem of minimax estimation of the scale parameter θ_M of the selected population under the scale-invariant square-error loss function. We show that the estimator $\frac{X_{(2)}}{\alpha+1}$ which is the analogue of the best scale invariant estimator of θ_2 under the scale-invariant square-error loss, is a minimax estimator of θ_M. Also, the result is extended to a subclass of the scale parameter exponential family and the family of transformed chi- square distributions.

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1 Introduction

Estimating the parameter(s) of the selected population (using a fixed selection rule) is an important estimation problem, having wide application. The man-

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ufacture not only wants to select the most productive machine from available $k(\geq 2)$ machines, but he also wants an estimate of the mean output produced by selected machine. The problem of estimation after selection has received considerable attention by many researches in the past two decades. For a summary of results, as well as a list of references, see Misra et al.(2006 a,b).

In estimation of scale parameter of the selected gamma population, let Π_1, Π_2 be two independent gamma population with associated probability density functions

$$f(x|\theta_i, \alpha) = \frac{1}{\theta_i^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\theta_i}, \quad \alpha > 0, \theta_i > 0, i = 1, 2, \tag{1}$$

respectively where θ_1, θ_2 are unknown scale parameters and α is the common known shape parameter. We observe X_i from $\Pi_i, i = 1, 2$, and let $X_{(1)} \leq X_{(2)}$ denote the order statistics of X_1, X_2. For selecting the population corresponding to the larger (or smaller) θ_i's, we use natural selection rule and select the population corresponding to the $X_{(2)}$ (or $X_{(1)}$). Therefore the scale parameter associated with the larger and smaller selected population are given by

$$\theta_M = \begin{cases} \theta_1 & X_1 \geq X_2 \\ \theta_2 & X_1 < X_2 \end{cases} \tag{2}$$

and

$$\theta_J = \begin{cases} \theta_1 & X_1 < X_2 \\ \theta_2 & X_1 \geq X_2 \end{cases} \tag{3}$$

respectively. Let $\theta = (\theta_1, \theta_2)$ and $h(\theta)$ be a real valued function of $h(\theta)$. For positive integer value shape parameter α, Vellaisamy and Sharma (1988) derived the UMVUE of θ_M and obtained estimators which are admissible (or inadmissible) within a subclass of equivariant estimators under the squared-error loss function

$$L(h(\theta), \delta) = (\delta - h(\theta))^2 \tag{4}$$

and scale-invariant squared-error loss function

$$L(h(\theta), \delta) = (\frac{\delta}{h(\theta)} - 1)^2, \tag{5}$$

where $h(\theta) = \theta_M$. For $k \geq 2$ independent gamma populations and arbitrary shape parameter $\alpha > 0$, Vellaisamy and Sharma (1989) derived the UMVUE of θ_M and showed the inadmissibility of natural estimator, Vellaisamy (1992) obtained estimators which dominates natural estimators under the loss (4) with $h(\theta) = \theta_M$ or θ_J and Vellaisamy (1996) showed that the UMVUE of θ_M (or θ_J)

is inadmissible under the loss (4) and obtained a class of dominating estimators. Misra et al. (2006 a,b) extended the admissibility and inadmissibility results of Vellaisamy and Sharma (1988) to the case of known and arbitrary $\alpha > 0$, but they did not extended the minimaxity results to this case. Also Nematollahi and Motamed-Shariati (2009) derived the Uniformly Minimum Risk Unbiased (UMRU) estimators of θ_M and θ_J for any $k \geq 2$ and $\alpha > 0$, and for $k = 2$ obtained estimators which are admissible (or inadmissible) within a subclass of equivariant estimators under the entropy loss function.

In this paper we consider minimax estimation of the selected scale parameter of two gamma populations, θ_M given by (2), under the scale-invariant squared-error loss function (5) with $h(\theta) = \theta_M$, where the shape parameter $\alpha > 0$ is not necessary be an integer. To this end, in section 2 we show that $X_{(2)}/(\alpha+1)$, which is the analogue of the Minimum Risk Equivariant (MRE) estimator of θ_2, is minimax for θ_M under the loss (5). In section 3 the result is extended to a subclass of exponential family and also to the family of transformed chi-squer distributions introduced by Rahman and Gupta (1993). Finally a discussion is given in section 4.

2 Minimax Estimation

Let X_1, X_2 be two independent random variables such that $X_i, i = 1, 2$ has a probability density function (p.d.f) as in (1) and $X_{(1)} \leq X_{(2)}$ are the order statistics of X_1, X_2. We want to find minimax estimator of θ_M give by (2) under the loss (5).

Following Sackrowitz and Samuel-Cahn (1987), we first find the minimax estimator in component problem for $\theta_i, i = 1, 2$. So, consider the inverted-gamma prior for $\theta_i, i = 1, 2$, with p.d.f.

$$\pi_i^r(\theta_i) = \frac{\xi^r}{\Gamma(r)\theta_i^{r+1}} e^{-\xi/\theta_i}, \quad \xi > 0,\ r > 0,\ i = 1, 2. \tag{6}$$

It is easy to see that the Bayes estimator of θ_i with respect to prior (6) and under the loss (5) is

$$\delta_{\pi_i^r}(X_i) = \frac{E(\frac{1}{\theta_i}|X)}{E(\frac{1}{\theta_i^2}|X)} = \frac{\xi + X_i}{\alpha + r + 1}, \quad i = 1, 2. \tag{7}$$

Also the posterior risk of $\delta_{\pi_i^r}(X_i)$ is

$$\begin{aligned} r_{\pi_i}(\delta_{\pi_i^r}, x_i) &= E\{(\frac{\delta_{\pi_i^r}(x_i)}{\theta_i} - 1)^2 | x_i\} \\ &= \frac{E^2(\frac{1}{\theta_i}|x_i)}{E(\frac{1}{\theta_i^2}|x_i)} - 2\frac{E^2(\frac{1}{\theta_i}|x_i)}{E(\frac{1}{\theta_i^2}|x_i)} + 1 \end{aligned}$$

$$= \ 1-\frac{E^2(\frac{1}{\theta_i}|x_i)}{E(\frac{1}{\theta_i^2}|x_i)}=\frac{1}{\alpha+r+1}\ . \tag{8}$$

Since the posterior risk does not depend on x_i, therefore the Bayes risk of $\delta_{\pi_i^r}(X_i)$ is

$$r^*(\pi_i^r)=\frac{1}{\alpha+r+1}\ . \tag{9}$$

Now consider the problem of estimation θ_M under the loss (5). Take the i.i.d. priors (6) for θ_1 and θ_2. From (7) and using Lemma 3.1 of Sackrowitz and Samuel-Cahn(1987) , the Bayes estimator of θ_M is

$$\delta^I_{\pi^r}(X_1,X_2)=\frac{X_{(2)}+\xi}{\alpha+r+1}, \tag{10}$$

where $\pi^r=(\pi_1^r,\pi_2^r)$. Since the posterior risk (8) of component problem does not depend on x_i, therefore by Theorem 3.1 of Sackrowitz and Samuel-Cahn (1987), the Bayes risk of $\delta_{\pi^r}\Gamma(X_1,X_2)$ is equal to (9), i.e.,

$$r_I^*(\pi^r)=r^*(\pi_i^r)=\frac{1}{\alpha+r+1},\quad i=1,2,$$

and hence

$$\lim_{r\to 0} r_I^*(\pi^r)=\frac{1}{\alpha+1}\ . \tag{11}$$

Now from Theorem 3.2 of Sackrowitz and Samuel-Cahn(1987), the estimator $\delta_M(X_1,X_2)$ is minimax for θ_M if

$$R(\theta_M,\delta_M)<\lim_{r\to 0} r_I^*(\pi^r)=\frac{1}{\alpha+1}\quad for\ \ all\ \ \theta, \tag{12}$$

where R(θ_M,δ_M) is the risk function of δ_M under the loss (5). In the following theorem we find the minimax estimator of θ_M

Theorem 2.1 *let X_1, X_2 be two independent random variables such that $X_i, i=1,2$ has p.d.f. (1). If $X_{(2)}=\max(X_1,X_2)$, then $\delta_M(X_1,X_2)=\frac{X_{(2)}}{\alpha+1}$ is minimax estimator of θ_M under the loss function (5).*

For a proof of Theorem 2.1 we need the following lemma.

Lemma 2.1 *Under the conditions of Theorem 2.1, let $\lambda=\frac{\theta_2}{\theta_1}$, and for* $t>0$

$$G_{m,n}(t)=\frac{1}{B(m,n)}\int_0^t x^{m-1}(1-x)^{n-1}$$

and

$$H_{m,n}(t) = G_{m,n}(t) + G_{m,n}(1-t),$$

then

$$(i) \quad E\left[\left(\frac{X_{(2)}}{\theta_M}\right)^k\right] = \frac{\Gamma(\alpha+k)}{\Gamma(\alpha)} H_{\alpha,\alpha+k}\left(\frac{1}{1+\lambda}\right),$$

$$(ii) \quad H_{\alpha,\alpha+1}(t) = 1 + \frac{1}{\alpha B(\alpha,\alpha+1)} t^{\alpha}(1-t)^{\alpha},$$

$$(iii) \quad H_{\alpha,\alpha+2}(t) = 1 + \frac{1}{\alpha B(\alpha,\alpha+2)} t^{\alpha}(1-t)^{\alpha},$$

where $B(.,.)$ is the Beta function.

Proof (i)

$$\begin{aligned} E\left[\left(\frac{X_{(2)}}{\theta_M}\right)^k\right] &= \int_0^\infty \int_{x_2}^\infty \left(\frac{x_1}{\theta_1}\right)^k \frac{x_1^{\alpha-1} x_2^{\alpha-1}}{\theta_1^\alpha \theta_2^\alpha \Gamma^2(\alpha)} e^{-\frac{x_1}{\theta_1}} e^{-\frac{x_2}{\theta_2}} dx_1 dx_2 \\ &+ \int_0^\infty \int_{x_1}^\infty \left(\frac{x_2}{\theta_2}\right)^k \frac{x_1^{\alpha-1} x_2^{\alpha-1}}{\theta_1^\alpha \theta_2^\alpha \Gamma^2(\alpha)} e^{-\frac{x_1}{\theta_1}} e^{-\frac{x_2}{\theta_2}} dx_2 dx_1 \\ &= I_1 + I_2 \end{aligned}$$

Following Nematollahi and Motamed-Shariati (2009), using the transformation $x_2 = r\theta_2$ in the outer integral and $x_1 = \frac{r\theta_1(1-x)}{x}$ in the inner integral of I_1, it reduces to

$$\begin{aligned} I_1 &= \frac{\Gamma(2\alpha+k)}{\Gamma^2(\alpha)} \int_0^{\frac{\theta_1}{\theta_1+\theta_2}} x^{\alpha-1}(1-x)^{\alpha+k-1} \left[\int_0^\infty \frac{r^{2\alpha+k-1} e^{-\frac{r}{x}}}{x^{2\alpha+k}\Gamma(2\alpha+k)} dr\right] dx \\ &= \frac{\Gamma(\alpha)\Gamma(\alpha+k)}{\Gamma^2(\alpha)} . \frac{\Gamma(2\alpha+k)}{\Gamma(\alpha)\Gamma(\alpha+k)} \left[\int_0^{\frac{1}{1+\lambda}} x^{\alpha-1}(1-x)^{\alpha+k-1} dx\right] \\ &= \frac{\Gamma(\alpha+k)}{\Gamma(\alpha)} G_{\alpha,\alpha+k}\left(\frac{1}{1+\lambda}\right). \end{aligned}$$

Similarly

$$I_2 = \frac{\Gamma(\alpha+k)}{\Gamma(\alpha)} G_{\alpha,\alpha+k}\left(\frac{\lambda}{1+\lambda}\right),$$

which completes the proof of part (i).

(ii),(iii) See the proof of Lemma 3.1.(iii) of Nematollahi and Motamed-Shariati (2009).

Proof of Theorem 2.1 To construct the minimaxity of δ_M, we must show that $R(\theta_M, \delta_M)$ satisfies (12). But from Lemma 2.1 we have

$$\begin{aligned}
R(\theta_M, \delta_M) &= E\left[\left(\frac{X_{(2)}}{(\alpha+1)\theta_M} - 1\right)^2\right] \\
&= \frac{1}{(\alpha+1)^2} E\left[\left(\frac{X_{(2)}}{\theta_M}\right)^2\right] - \frac{2}{\alpha+1} E\left(\frac{X_{(2)}}{\theta_M}\right) + 1 \\
&= \frac{1}{(\alpha+1)^2} \frac{\Gamma(\alpha+2)}{\Gamma(\alpha)} H_{\alpha,\alpha+2}\left(\frac{1}{1+\lambda}\right) \\
&\quad - \frac{2}{\alpha+1} \frac{\Gamma(\alpha+1)}{\Gamma(\alpha)} H_{\alpha,\alpha+1}\left(\frac{1}{1+\lambda}\right) + 1 \\
&= \frac{\alpha}{\alpha+1} H_{\alpha,\alpha+2}\left(\frac{1}{1+\lambda}\right) - \frac{2\alpha}{\alpha+1} H_{\alpha,\alpha+1}\left(\frac{1}{1+\lambda}\right) + 1 \\
&= \frac{\alpha}{\alpha+1}\left\{1 + \frac{1}{\alpha B(\alpha,\alpha+2)}\ \frac{\lambda^\alpha}{(1+\lambda)^{2\alpha}}\right\} \\
&\quad - \frac{2\alpha}{\alpha+1}\left\{1 + \frac{1}{\alpha B(\alpha,\alpha+1)}\ \frac{\lambda^\alpha}{(1+\lambda)^{2\alpha}}\right\} + 1 \\
&= \frac{1}{\alpha+1}\left\{1 + \frac{\lambda^\alpha}{\Gamma(\alpha)(1+\lambda)^{2\alpha}}\left[\frac{\Gamma(2\alpha+2)}{\Gamma(\alpha+2)} - \frac{2\Gamma(2\alpha+1)}{\Gamma(\alpha+1)}\right]\right\} \\
&= \frac{1}{\alpha+1}\left\{1 - \frac{2}{(\alpha+1)B(\alpha,\alpha)}\ \frac{\lambda^\alpha}{(1+\lambda)^{2\alpha}}\right\}.
\end{aligned}$$

So,

$$R(\theta_M, \delta_M) < \frac{1}{\alpha+1},$$

which completes the proof.

Remark 2.1 *Vellaisamy and Sharma (1988) showed that* $\delta_M(X_1, X_2) = \frac{X_{(2)}}{\alpha+1}$ *is minimax for* θ_M *for integer value* $\alpha > 0$*. Their proof is only valid for integer value* α*. But the above proof is based on arbitrary* $\alpha > 0$.

Remark 2.2 *If a random sample* $X_{ij}, j = 1, \cdots, n$ *derived from population* $\Pi_i, i = 1, 2$ *and* Π_i *has associated p.d.f. of the from (1) and* $\boldsymbol{X}_i = (X_{i1}, \cdots, X_{in}), i = 1, 2,$ *then* $T_i = T_i(\boldsymbol{X}_i) = \sum_{j=1}^{n} X_{ij}, i = 1, 2$ *is complete sufficient statistics for* θ_i *and* T_i *has a gamma distribution with parameters* $(n\alpha, \theta_i)$*. So, the result of Theorem 2.1 holds for this case with replacing* α *by* $n\alpha$ *and* X_i *by* $T_i, i = 1, 2$.

3 Extension to Some Subclass of Exponential Family

Following Nematollahi and Motamed-Shariati (2009), let $X_{i1}, X_{i2}, \cdots, X_{in}$ be a random sample of size n from the i-th population $\Pi_i, i = 1, 2$, with the joint scale probability density function

$$f(\mathbf{x}_i; \tau_i) = \frac{1}{\tau_i^n} f\left(\frac{\mathbf{x}_i}{\tau_i}\right), \qquad i = 1, 2,$$

where $\mathbf{x}_i = (x_{i1}, \cdots, x_{in})$. In some cases the above model reduces to

$$f(\mathbf{x}_i; \theta_i) = c(\mathbf{x}_i, n)\theta_i^{-\nu} e^{-\frac{T_i(\mathbf{X}_i)}{\theta_i}}, \qquad i = 1, 2, \tag{13}$$

where $c(\mathbf{x}_i, n)$ is a function of $\mathbf{x}_i$ and n, $\theta_i = \tau_i^r$ for some $r > 0$, ν is a function of n and $T_i(\mathbf{X}_i)$ is a complete sufficient statistic for θ_i with $Gamma(\nu, \theta_i)$-distribution. Examples of distributions of the from (13) are

1. $Exponential(\beta_i)$ with $\theta_i = \beta_i, \nu = n, T_i(\mathbf{X}_i) = \sum_{j=1}^{n} X_{ij}, c(\mathbf{x}_i, n) = 1$,

2. $Gamma(\alpha, \beta_i)$ with known α and $\theta_i = \beta_i, \nu = n\alpha$, $T_i(\mathbf{X}_i) = \sum_{j=1}^{n} X_{ij}$,

   $c(\mathbf{x}_i, n) = \dfrac{\left(\prod_{i=1}^{n} x_{ij}^{\alpha-1}\right)}{[\Gamma(\alpha)]^n}$,

3. $Inverse\ Gaussian(\infty, \lambda_i)$ with $\theta_i = \dfrac{1}{\lambda_i}$, $\nu = \dfrac{n}{2}$, $T_i(\mathbf{X}_i) = \dfrac{1}{2}\sum_{j=1}^{n} \dfrac{1}{X_{ij}}$,

   $c(\mathbf{x}_i, n) = \left(\prod_{i=1}^{n} 2x_{ij}^3\right)^{-\frac{1}{2}}$,

4. $Normal(0, \sigma_i^2)$ with $\theta_i = \sigma_i^2$, $\nu = \dfrac{n}{2}$, $T_i(\mathbf{X}_i) = \dfrac{1}{2}\sum_{j=1}^{n} X_{ij}^2$, $c(\mathbf{x}_i, n) = (2\pi)^{-\frac{n}{2}}$,

5. $Weibull(\eta_i, \beta)$ with known β and $\theta_i = \eta_i^\beta$, $\nu = n$, $T_i(\mathbf{X}_i) = \sum_{j=1}^{n} X_{ij}^\beta$,

   $c(\mathbf{x}_i, n) = \beta^n \prod_{j=1}^{n} x_{ij}^{\beta-1}$,

6. $Rayleigh(\beta_i)$ with $\theta_i = \beta_i^2$, $\nu = n$, $T_i(\mathbf{X}_i) = \dfrac{1}{2}\sum_{j=1}^{n} X_{ij}^2, c(\mathbf{x}_i, n) = \prod_{j=1}^{n} x_{ij}$.

Let $T_i = T_i(\mathbf{X}_i), i = 1, 2$, and $T_{(1)} \leq T_{(2)}$ denote the order statistics of T_1, T_2. To select the population with the larger (or smaller) θ_i's, we naturally select the population corresponding to the $T_{(2)}$ (or $T_{(1)}$). Therefore the parameter associated with the larger or smaller selected population are given by

$$\theta_M = \begin{cases} \theta_1 & T_1 \geq T_2 \\ \theta_2 & T_1 < T_2 \end{cases} \tag{14}$$

and

$$\theta_J = \begin{cases} \theta_1 & T_1 < T_2 \\ \theta_2 & T_1 \geq T_2 \end{cases} \tag{15}$$

respectively. Since $T_i, i = 1, 2$ has a $Gamma(\nu, \theta_i)$ distribution, therefore we can use Remark 2.2 and extend the result of section 2 to the subclass of exponential family (13) by replacing α by ν and X_i by T_i. Hence a minimax estimator of θ_M is give by $\dfrac{T_{(2)}}{\nu+1}$. For example in $weibull(\eta_i, \beta)$ distribution, a minimax estimator of

$$\eta_M^\beta = \begin{cases} \eta_1^\beta & \sum_{j=1}^n X_{1j}^\beta \geq \sum_{j=1}^n X_{2j}^\beta \\ \eta_2^\beta & \sum_{j=1}^n X_{1j}^\beta < \sum_{j=1}^n X_{2j}^\beta \end{cases}$$

is $\dfrac{\max\left(\sum_{j=1}^n X_{1j}^\beta, \sum_{j=1}^n X_{2j}^\beta\right)}{n+1}$.

The result of section 2 can be extended to some other families of distributions which do not necessarily belong to a scale family, such as Pareto or Beta distributions. A family of distributions that includes these distributions as special cases, is the family of transformed chi-square distributions which is originally introduced by Rahman and Gupta (1993). They considered the one parameter exponential family

$$f(\mathbf{x}_i, \eta_i) = e^{a_i(\mathbf{X}_i)b(\eta_i)+c(\eta_i)+h(\mathbf{X}_i)}, \qquad i = 1, 2 \tag{16}$$

and showed that $-2a_i(\mathbf{X}_i)b(\eta_i)$ has a $Gamma(\frac{m}{2}, 2)$- distribution if and only if

$$\frac{2c'(\eta_i)b(\eta_i)}{b'(\eta_i)} = m. \tag{17}$$

When m is an integer, $-2a_i(\mathbf{X}_i)b(\eta_i)$ follows a chi-square distribution with m degrees of freedom. They called the one parameter exponential family (16) which satisfies (17), the family of transformed chi-square distributions. For example Beta, Pareto, Exponential, Lognormal and some other distributions belong to this family of distributions (see Table 1 of Rahman and Gupta, 1993).

Now it is easy to show that if condition (17) holds then the one parameter exponential family (16) is in the form of the scale parameter exponential family (13) with $\nu = \frac{m}{2}$, $T_i(\mathbf{X}_i) = a_i(\mathbf{X}_i)$ and $\theta_i = -\frac{1}{b(\eta_i)}$. Hence with these substitutions, we can extend the result of section 2 to the family of transformed chi-square distributions.

4 Discussion

In previous sections we find a minimax estimator of the scale parameter θ_M of the selected gamma population under the scale-invariant squared-error loss function (5) when the common shape parameter α is arbitrary and known.

Another parameter that is interested in selection problems, is θ_J which is given in (3) and (15). Unfortunately, we can not derive a minimax estimator for this parameter under the scale-invariant squared-error loss function (5).

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