A Statistical Test for Time Reversibility of Stationary Finite State Markov Chains

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Abstract

In this paper stationary irreducible aperiodic finite state Markov chains are considered. We investigate time reversibility of these chains and a statistical tool for characterizing their time reversibility is proposed. It is shown that this test has asymptotically the Chi- squared distribution under null hypothesis. Our simulations also confirm the proposed test. Two empirical examples are given, one of them on gasoline price markups, involves observed states, and the other on price level series for different countries.

Mathematics Subject Classification: 60J05, 62M02

Keywords: Markov chain, Time reversible, $\alpha-$mixing, Asymptotic distribution

1 Introduction

A stationary process is said to be time reversible if its finite dimensional distributions are all invariant to the reversal of time indices. Indeed time reversibility implies certain symmetries that are broken in time irreversible processes. If

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we characterize the nature of the observed time reversibility, we will be guided in how to model or use the processes.

Time reversibility is an important concept especially in economics and physics. It has been a topic of great interest ever since the work by Burns and Mitchell (1946). Variety of researches in economics found evidence of time irreversibility in observed time series. For example in the case that inflation is positive, the real prices of goods rise suddenly and fall slowly, then it seems that the fitted model to the prices will be time irreversible.

Ramsey and Rothman (1996) introduced a statistical tool for identifying time irreversible stochastic processes that is named the symmetric-bicovariance function. In the Ramsey and Rothman test, we have to calculate a complicated and asymptotic estimator for variance and the prerequisite of the test is that the data must posses finite sixth moment, but such a condition may be too restrictive for financial data.

Hinich and Rothman (1998) and Robinson (1991) have introduced different tests for time reversibility with special circumstances. Chen, Chou and Kuan (2000) proposed a class of tests for time reversibility that did not have any moment restrictions. Their proposed test is based on characteristic functions, depending on a weighting function. After that Chen and Kuan (2001), Fong (2003) and Noel (2003) are the authors who try to show the time series which they considered are time irreversible.

Cheng (1999) provided a basic theorem which gave a necessary and sufficient condition for time reversibility of stationary linear processes and did not require existence of moments of order higher than two.

Markov chains with some kind of symmetric transition probability matrices, as used by Neftci (1984) and Rothman (1990) are examples of time irreversible processes. Markov chains which are time reversible have different applications with respect to the time irreversible Markov chains and construction of them are in different ways. For example in metropolis algorithm, the Markov chain constructed is reversible and Gibbs samples with a systematic scan, use a Markov chain that is not time reversible. So, identifying the time reversibility of a Markov chain plays an important role.

It is clear that using the test of time reversibility in general is not the best way for Markov chains and it is better to find a test for Markov chains.

McCausland (2007) proposed a decomposition of the matrix of joint probability transition $[P(X_{t-1} = i, X_t = j)]_{i,j}$ of a finite state Markov chain for characterizing the time irreversibility, but testing time reversibility of time series with Markov property has been left.

In this paper a statistical tool for characterizing time reversibility in stationary irreducible Markov chains or in time series with Markov property is introduced. This paper proceeds as follows. In Section 2 we introduce some notations and give some preliminary results. In Section 3 the test statistic

of time reversibility is introduced and its asymptotic distribution is derived. Simulation results are reported in Section 4 and Section 5 presents result from two empirical applications. The first, on gasoline price markups, involves directly observed states and the second is on price level time series in different countries. In Section 6 the power of the proposed test in a special case will be approximated.

2   Preliminary

Let $\{X_t; t = 0, 1, \dots\}$ be an irreducible aperiodic stationary finite state Markov chain with a transition probability matrix P, and the stationary distribution π. Denote by $S = \{1, 2, \dots, m\}$, the state space. Unless otherwise mentioned, to avoid repetition, the terminology of Markov chain will mean irreducible aperiodic discrete time stationary finite state Markov chain throughout the rest of this paper.

Definition 2.1. A discrete time stationary processes $\{X_t; t = 0, 1, \dots\}$ is time reversible if for every positive integer n,

$$(X_0, X_2, \dots, X_n) \stackrel{d}{=} (X_n, X_{n-1}, \dots, X_0).$$

(The notation $\stackrel{d}{=}$ means identical distribution.)

Remark 2.1. It is easy to show that a Markov chain is time reversible if and only if for every integers n and m, $(X_n, X_m) \stackrel{d}{=} (X_m, X_n)$ or equivalently the matrix P and the stationary distribution π satisfy the detailed balance equations:

$$\pi(i) P_{ij} = \pi(j) P_{ji} \qquad \forall \, i, j = 1, 2, \dots, m$$

In this paper we try to provide a suitable method for testing H_0 : The Markov chain is time reversible versus H_1 : The Markov chain is time irreversible. To propose a test statistic, first we will investigate properties of the time reversible Markov chains.

For a realization $x_0, x_1, \dots, x_{n-1}, x_n$ of the Markov chain, let n_{ij} be the number of observed direct transitions from i to j and $n_{i.} = \sum_{j=1}^{m} n_{ij}$, from which it follows that $\sum_{i=1}^{m} \sum_{j=1}^{m} n_{ij} = n$. The $m \times m$ matrix $[n_{ij}]_{i,j=1}^{m}$ is called the transition count of the chain.

Bartlett (1950) has shown that the maximum likelihood estimator of P_{ij} in a simple Markov chain is $\hat{P}_{ij} = \frac{n_{ij}}{n_{i.}}$ and $\hat{\pi}(i) = \frac{n_{i.}}{n}$. These estimators are

consistent, thus for consistency of estimation we will have

$$\hat{\pi}(i)\hat{P}_{ij} \xrightarrow{\text{Pr}} \pi(i)\, P_{ij}.$$

($\xrightarrow{\text{Pr}}$ denotes convergence in probability.) Since $\hat{\pi}(i)\hat{P}_{ij} = \frac{n_{ij}}{n}$, it seems reasonable that if the Markov chain is time reversible, $n_{ij} - n_{ji}$ would be small enough. Using this idea we derive a new test statistic of time reversibility.

3   Test Statistic

Let $\{X_t;\ t = 0, 1, \ldots\}$ be a Markov chain on state space S. For $i, j \in S$, define

$$Y_t(i, j) = I_{ij}^t - I_{ji}^t \qquad\qquad t = 1, 2, \ldots \qquad\qquad (3.1)$$

where $I_{ij}^t = I_{\{(i,j)\}}(X_{t-1}, X_t)$, such that $I_{\{(i,j)\}}$ is an indicator function taking the value one at (i, j) and the value zero elsewhere. As $Y_t(i, j)$ is a function of X_t and X_{t-1} and $\{X_t;\ t = 0, 1, \ldots\}$ is a stationary Markov chain, $\{Y_t(i, j)\, ;\, t = 1, 2, \ldots\}$ is a stationary second order Markov chain with state space $\{-1, 0, 1\}$ and $E\left[Y_t(i, j)\right] = \pi(i)\, P_{ij} - \pi(j)\, P_{ji}$.

For a realization of length $n + 1$ from $\{X_t; t = 0, 1, \ldots\}$, $\sum_{t=1}^{n} y_t(i, j) = n_{ij} - n_{ji}$, where $y_1(i, j), y_2(i, j), \ldots, y_n(i, j)$ is the corresponding realization of $\{Y_t(i, j)\,;\ t = 1, 2, \ldots\}$. Hence under time reversibility, we expect a special behavior from $\sum_{t=1}^{n} y_t(i, j)$ and so our main result is finding the asymptotic distribution of the test statistic which will be constructed based on $S_n(i, j)$, where $S_n(i, j) = Y_1(i, j) + \cdots + Y_n(i, j)$.

It is easy to show that

$$\begin{aligned}
Var\left(S_n(i, j)\right) \;=\; & n\left[\pi(i)\, P_{ij} + \pi(j)\, P_{ji} - \left(\pi(i)\, P_{ij} - \pi(j)\, P_{ji}\right)^2\right] \qquad (3.2)\\
& - 2\sum_{k=1}^{n-1}(n - k)\left[\pi(i)\, P_{ij} P_{ji}^{(k-1)} P_{ij} + \pi(j)\, P_{ji} P_{ij}^{(k-1)} P_{ji}\right.\\
& - \pi(i)\, P_{ij} P_{jj}^{(k-1)} P_{ji} - \pi(j)\, P_{ji} P_{ii}^{(k-1)} P_{ij} - \left.\left(\pi(i)\, P_{ij} - \pi(j)\, P_{ji}\right)^2\right].
\end{aligned}$$

Where $P_{ij}^{(k)}$ denotes the probability that the chain goes from state i to state j in k transitions.

Definition 3.1 Let $\{X_t;\ t = 0, 1, \ldots\}$ be a stochastic process and α_n be a number such that

$$|P(A \cap B) - P(A)\, P(B)| \leq \alpha_n,$$

for $A \in \sigma(X_0, \ldots, X_k)$, the σ-field generated by $X_0, \ldots, X_k$, and $B \in \sigma(X_{n+k}, \ldots)$ and $k \geq 0$, $n \geq 1$. Suppose that $\alpha_n \to 0$ as $n \to \infty$. In this case $\{X_t; \ t = 0, 1, \ldots\}$ is said to be $\alpha-$mixing. The idea being that X_k and X_{n+k} are approximately independent for large n.

In the next proposition by $\alpha-$mixing property of $\{Y_t(i,j); t = 0, 1, \ldots\}$ we will find asymptotic distribution of $S_n(i,j)$.

Proposition 3.1. For the stationary second order Markov chain $\{Y_t(i,j)\}_{t \geq 1}$, defined by (3.1), let $S_n(i,j) = Y_1(i,j) + \cdots + Y_n(i,j)$, then

$$\frac{1}{n} Var(S_n(i,j)) \to \sigma^2(i,j) = Var(Y_1(i,j)) + 2\sum_{k=1}^{\infty} Cov(Y_1(i,j), Y_{k+1}(i,j)),$$

$$(3.3)$$

where the series converges absolutely. Moreover,

    i) If $\sigma(i,j) > 0$, then $[S_n(i,j) - E(S_n(i,j))]/\sqrt{n} \xrightarrow{L} N(0, \sigma^2(i,j))$.

    ii) If $\sigma(i,j) = 0$, then $[S_n(i,j) - E(S_n(i,j))]/\sqrt{n} \xrightarrow{Pr} 0$.

($\xrightarrow{L}$ denotes convergence in distribution.)

Proof. The Markov chain $\{X_t; t = 0, 1, \ldots\}$ is $\alpha-$mixing with $\alpha_n = m\rho^n$ such that ρ is a constant and $0 < \rho < 1$, [Billingsley, 1998, page 364]. By this fact we can show that for fix $i, j \in S$, the stationary second order Markov chain $\{Y_t(i,j); t = 0, 1, \ldots\}$ defined by (3.1) is $\alpha-$mixing with $\alpha_n = m\rho^{n-1}$. Indeed if $A \in \sigma(Y_1, \ldots, Y_k)$ and $B \in \sigma(Y_{n+k}, Y_{n+k+1}, \ldots)$, there exists $A^- \in \sigma(X_0, \ldots, X_k)$ and $B^- \in \sigma(X_{n+k-1}, X_{n+k}, \ldots)$ such that

$$|P((Y_1, \ldots, Y_k) \in A, (Y_{n+k}, Y_{n+k+1}, \ldots) \in B) - P((Y_1, \ldots, Y_k) \in A)$$
$$\times \quad P((Y_{n+k}, Y_{n+k+1}, \ldots) \in B)|$$
$$= \quad |P((X_0, \ldots, X_k) \in A^-, (X_{n+k-1}, X_{n+k}, \ldots) \in B) - P((X_0, \ldots, X_k) \in A^-)$$
$$\times \quad P((X_{n+k-1}, X_{n+k}, \ldots) \in B^-)|$$
$$\leq \quad m\rho^{n-1}.$$

The fact that $\alpha_n = O(n^{-5})$ and using Theorem (27.4) of Billingsley (1995), imply the equation (3.3) and part (i), and Chebychev's inequality proves part (ii). ∎

If $\hat{\sigma}^2(i,j)$ is a consistent estimator of $\sigma^2(i,j)$, Proposition 3.1 and Slutsky Theorem imply

$$\frac{S_n(i,j) - E(S_n(i,j))}{\sqrt{n}\hat{\sigma}(i,j)} \xrightarrow{L} N(0,1). \tag{3.4}$$

In the case that the chain is time reversible, we have

$$\sigma^2 (i,j) = 2\pi (i) P_{ij}[1 + P_{ij} \sum_{n=0}^{\infty}(P_{ji}^{(n)} - P_{ii}^{(n)}) + P_{ji} \sum_{n=0}^{\infty}(P_{ij}^{(n)} - P_{jj}^{(n)})]. \quad (3.5)$$

Where two series converge absolutely. In fact there is a constant $a > 0$ such that

$$\left| P_{ji}^{(n)} - P_{ii}^{(n)} \right| \leq \left| P_{ji}^{(n)} - \pi(i) \right| + \left| P_{ii}^{(n)} - \pi(i) \right| \leq a\rho^n.$$

Also $E(S_n(i,j)) = 0$, therefore

$$\frac{S_n(i,j)}{\sqrt{n}\hat{\sigma}(i,j)} \xrightarrow{L} N(0,1).$$

This fact and the next proposition guide us to the proposed test statistic.

Proposition 3.2. Suppose that the Markov chain $\{X_t; \ t = 0, 1, \dots\}$ is time reversible and $S_n(.,.)$ is defined as Proposition 3.1. If for $i < j$ and $k < l$ in the state space of S, $\{i,j\} \neq \{l,k\}$ and $\sigma(i,j)\sigma(k,l) > 0$, then

$$\lim_{n\to\infty} \frac{Cov(S_n(i,j), S_n(k,l))}{n} = 2\pi(k)P_{kl}[P_{ij} \sum_{n=0}^{\infty}(P_{li}^{(n)} - P_{ki}^{(n)}) - P_{ji} \sum_{n=0}^{\infty}(P_{lj}^{(n)} - P_{kj}^{(n)})].$$
$$(3.6)$$

Proof. The proof is similar argument as led to (3.3) in Proposition 3.1. In this case if we set $X_t := Y_t(i,j) - Y_t(k,l)$ in Theorem (27.4) of Billingsley (1995), the limiting $Var(S_n(i,j) + S_n(k,l))/n$ is achieved. Make using the limit of $Var(S_n(.,.))/n$ in Proposition 3.1 and time reversibility of the chain, the equality is proved.∎

Theorem 3.1. Suppose that the Markov chain $\{X_t; t = 0, 1, \dots\}$ is time reversible and $S_n(.,.)$ is defined as Proposition 3.1. Also suppose that $T = \{(i,j) \in S; \ i < j, \ \sigma(i,j) = 0\}$. Then the random vector $S'_n = (S_n(i,j)/\sqrt{n};$ $i = 1, 2, \cdots, m{-}1, j = i{+}1, \cdots, m, (i,j) \notin T)$ is asymptotically d-dimensional normal, $N_d(0, \Delta_d)$, for which $d = \frac{m(m-1)}{2} - z$, where z is the number of members of T, and Δ_d is asymptotic covariance matrix of S_n which can be achieved by Proposition 3.1 and 3.2. (The prime means transpose.)

Proof. It is enough to show that any linear combinations of the components of S'_n, $(b, S'_n) = \sum_{i,j} b_{i,j} S_n(i,j)/\sqrt{n}$ is asymptotically normal. The proof

is similar argument as led to part (i) in Proposition 3.1. In this case, in Theorem (27.4) of Billingsley (1995), if we take $X_t := \sum_{i,j} b_{i,j} Y_t(i,j)$, the theorem is proved.■

Now suppose that $\hat{\Delta}_d$ is a consistent estimator for Δ_d. Under conditions of Theorem 3.1,

$$X^2 = S'_n \hat{\Delta}_d^{-1} S_n \tag{3.7}$$

has asymptotically the Chi-squared distribution with d degree of freedom. X^2 is our proposed test statistic.

Note that determining d and invertibility of $\hat{\Delta}_d$ must be considered. S_n must be restricted to those indices i and j for which $n_{ij} \neq 0$ or $n_{ji} \neq 0$ and the consistent estimator $\hat{\Delta}_d$ must be invertible. This test statistic under null hypothesis have the Chi-squared distribution with d degrees of freedom. So for testing the time reversibility hypothesis of Markov chains, we reject H_0 if $X^2 > \chi^2_{1-\alpha}(d)$.

4 Simulation

In this section, we investigate the finite sample performance of the proposed test statistic X^2 by simulation. Δ_d has been estimated by replacing P_{ij} and $\pi(i)$ with $\hat{P}_{ij}$ and $\hat{\pi}(i)$, respectively in (3.6) and (3.7) and $\hat{P}_{ij}^{(k)}$ will be the (i,j)th element of the matrix $\hat{P}^k$, the kth power of the matrix $\hat{P}$. So the elements of $\hat{\Delta}_d$ are as follows,

$$\hat{\Delta}_d((i,j),(i,j)) = 2\hat{\pi}(i)\hat{P}_{ij}[1 + \hat{P}_{ij}\sum_{n=0}^{\infty}(\hat{P}_{ji}^{(n)} - \hat{P}_{ii}^{(n)}) + \hat{P}_{ji}\sum_{n=1}^{\infty}(\hat{P}_{ij}^{(n)} - \hat{P}_{jj}^{(n)})],$$

$$\hat{\Delta}_d((i,j),(k,l)) = 2\pi(k)\hat{P}_{kl}[\hat{P}_{ij}\sum_{n=0}^{\infty}(\hat{P}_{li}^{(n)} - \hat{P}_{ki}^{(n)}) - \hat{P}_{ji}\sum_{n=0}^{\infty}(\hat{P}_{lj}^{(n)} - \hat{P}_{kj}^{(n)})].$$

First, we generate a stationary finite state Markov chain of lenght $n+1$ with the known transition probability matrix $P = [P_{ij}]$ and the stationary distribution $\pi = [\pi]_{i,j}$. Second, we compute the transition count $n = [n_{ij}]$ and calculate our proposed test statistic X^2 and its corresponding p-value.

Example 4.1. Consider an Ehrenfest model of diffusion with $P_{i,i+1} = 1 - \frac{i}{m}$, $P_{i,i-1} = \frac{i}{m}$ for $0 \leq i \leq 5$ which is is a known reversible Markov chain.

We are interested in testing the time reversibility of this chain by using our statistic. We generate the chain of lenght $n+1$, and compute its corresponding X^2 and compare it with $\chi^2(5)$. The results are given in Table 1. As expected, the time reversibility isn't rejected with p-value more than 0.995.

Table 1: The results of the Ehrenfest
model of Example 1.

n+1	X^2	p-value
50	0.0615	> 0.995
100	0.1991	> 0.995
200	3.89×10^{-2}	> 0.995
500	9.97×10^{-3}	> 0.995
1000	1.8×10^{-2}	> 0.995

Example 4.2. Our simulation has been based on $n+1$ iterations of the irreversible Markov chain with following transition probability matrix :

$$P = \begin{bmatrix} 0 & 0.6 & 0.4 \\ 0.1 & 0.8 & 0.1 \\ 0.5 & 0 & 0.5 \end{bmatrix}$$

The results of simulations are collected in Table 2.

Table 2: The results in the irreversible
Markov chain of Example 2.

n+1	X^2	p-value
50	19.7911	< 0.005
100	25.208	< 0.005
200	32.821	< 0.005
500	443.37	< 0.005
1000	175.51	< 0.005

As we see, the null hypothesis of time reversibility of this irreversible Markov chain is rejected at less than the 0.005 level.

Example 4.3. Consider the random walk on an undirected weighted graph with nodes labelled $E = \{1, 2, \dots, 10\}$ and weights $w(i,j) = i+j$ for $1 \le i, j \le 10$. We simulate $n+1$ times and we observe the results in Table 3.

Table 3: The results of the random
walk Markov chain of Example 3.

n+1	X^2	p-value
200	42.65	> 0.995
500	43.3	> 0.995
1000	50.69	> 0.995
5000	44.8	> 0.995
10000	44.53	> 0.995

The random walk on an undirected weighted graph is time reversible (Cover and Thomas, 1991, page 68). As expected, the proposed test statistic can not reject the null hypothesis of reversibility at level of more than 0.995 that is a strong evidence of time reversibility in this Markov chain.

5 The Real Data

The first real data investigates the time irreversibility of gasoline price mark-ups. The data are the same as those used in McCausland (2007) that collected by the government of Ontario. They are 270 weekly observations of retail price r_t and wholesale price w_t for gasoline from November 27, 1989 to September 25, 1994. r_t is an average for a sample of gasoline stations in Windsor, Ontario and Canada. w_t is the price charged for large scale purchases of unbranded gasoline at the terminal in Toronto and Ontario.

McCausland (2007) divided the mark-up r_t/w_t into six bins and modelled the evolution of the mark-up bin s_t as a stationary regular 6-state Markov chain. We use these computations and calculate this following transition count:

$$n = \begin{bmatrix} 5 & 3 & 1 & 0 & 0 & 0 \\ 4 & 23 & 1 & 13 & 2 & 1 \\ 0 & 15 & 43 & 19 & 1 & 1 \\ 0 & 1 & 32 & 68 & 9 & 1 \\ 0 & 1 & 2 & 11 & 4 & 1 \\ 0 & 0 & 0 & 1 & 3 & 3 \end{bmatrix}$$

According to the proposed statistic, $X^2 = 98.65$ and by comparison it with $\chi^2(0.95, 12) = 21.0$, the null hypothesis is rejected. Also p-value is less than 0.005 that it is a strong evidence of rejection the reversibility hypothesis.

The second real data investigates the time reversibility of Backus and Kehoe international data set. We examined price level series for different countries. These four countries were: Australia, Canada, the United kingdom and

the United States. Data sources and data for each country can be found in the appendix of Backus and Kehoe (1992). First we compute raw first differences and selected the part of series that can model them as a stationary regular Markov chain , then divide them into some bins according to the range of raw first difference and model the evaluation of the price bin as a stationary regular finite state Markov chain. For each series, we calculate the proposed test statistic and its corresponding p-value. The results are given in Table 4.

Table 4: The result of test statistic for the price level time series.

Country	Year	Number of state	X^2	df	P-value
Australia	1861-1972	12	48.88	66	0.96
Canada	1870-1973	7	19.37	17	0.25
The United Kingdom	1870-1980	12	33.94	23	0.07
The United States	1869-1974	7	11.53	15	0.71

As we see in Table 4, the price levels in the related period which are stationary, are time reversible at significant level 0.05.

6   Power of the test in special cases

In this section we approximate the power of the proposed test in simulated data. As the hypotheses of the test is complex and the distribution of the test statistic under irreversibility is unknown, computing the power is not possible. So the power of test has been approximated in simple cases. We have performed 100 replicates for each data set with n size. For each one we have used the testing procedure and then have counted the number of rejecting.

In Example 4.1 the reversibility of an Ehrenfest Markov chain has been investigated. In this part we simulate a realization with size n from a Markov chain with the following transition probability matrix:

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{1}{5} & 0 & \frac{3}{5} & \frac{1}{5} & 0 & 0 \\ 0 & \frac{2}{5} & 0 & \frac{3}{5} & 0 & 0 \\ 0 & 0 & \frac{3}{5} & 0 & \frac{2}{5} & 0 \\ 0 & 0 & 0 & \frac{4}{5} & 0 & \frac{1}{5} \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

which is irreversible but close to the reversible Ehrenfest Markov chain. In every replication, we produce n realization of Markov chain and calculate the proposed test statistic and compare the corresponding p-value to 0.05. This

steps has been replicated 100 times. In every step, the time reversibility hypothesis r times has been rejected and with this way, we estimate the power of the test by $\frac{r}{100}$. The observations in Table 5 shows that the proposed test statistic is a powerful test.

Table 5: The result of the power of rejection in
the changed Ehrenfest model.

n	500	1000	2000
$\frac{r}{100}$	0.74	0.92	1

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