**Applied Mathematical Sciences, Vol. 3, 2009, no. 52, 2575 - 2594**

# Efficiency Measurement of Multi-Component Decision Making Units Using Data Envelopment Analysis

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**Abstract**

Ordinary DEA models, such as the CCR and BCC models, are not able to measure the efficiency of multi-component decision making units. In management science, there is a need for methods that are capable of carrying out such a measurement, regarding the necessary awareness of the performance of the components of a system. By generalizing and extending multi-component models, this paper provides a model to carry out this task. The model is then used to evaluate 18 Iranian companies producing automobiles and automobile parts. The results are in complete agreement with the existing facts.

**Keywords**: Data envelopment analysis, Decision making unit, Component efficiency

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## 1 Introduction

Since long ago, specifically since near the end of the Second World War, decision making managers have recognized that personal opinions are somehow involved in any decision that has been taken without considering scientific methods, and that such decisions are usually accompanied by serious irreversible mistakes. Therefore, they came to realize the importance of implementing scientific methods in evaluating the efficiency of units. Of utmost importance to managers, in relation to the decision making units (DMUs), is information on the performance of the units under their supervision, in order to direct them. Because of factors such as information complexity, great volume of performance, the effect of extraneous factors, the effect of competing units on performance, limitation of units regarding appropriate decisions (e.g., owing to the units being run by the state), sudden policy changes owing to passive attitude towards problems (such as unemployment, …), managers cannot obtain information on the units under their supervision without scientific methods and tools in order to make appropriate decisions to improve their performance and productivity.

Before 1978, many works of research had been carried out to evaluate the efficiency of decision making units in a system. In such research, by system it is meant the set from which the units under evaluation are chosen. A university, for instance, can be considered as a system in which the departments or colleges are the decision making units. The main part of the research before 1957 led to the development of parametric techniques. Although these techniques were useful in some particular cases, they generally had two major shortcomings, theoretical and practical, as follows:

a) Parametric techniques were utilized for cases of one or more inputs and a single output.
b) Determining the parameters and the parametric function is not generally easy.

To remove these shortcomings, Farrel carried out widespread research in 1957. In fact, he was the first to nonparametrically obtain the estimate of the function under study. Taking into account the five principles, he developed the Production Possibility Set (PPS) and considered a part of its frontier as an estimate of the production function. Any DMU lying on this frontier is considered efficient; otherwise it is inefficient. Later, the CCR model was proposed by Charnes, Cooper, and Rhodes in 1978, which became the basis for a branch of operations research called Data Envelopment Analysis (DEA). The DEA technique is based on mathematical programming and is used in measuring the relative efficiency of homogeneous DMUs. In this technique, the relative efficiency of a DMU is a function of its inputs and outputs, and a DMU is considered technically efficient if it makes best use of its inputs, i.e., it has no waste of inputs, and produces the maximum outputs and has no production shortfall.

By reviewing the present literature, this paper tries to evaluate 18 Iranian companies manufacturing automobiles and automobile parts during 2002-2006. The paper is organized as follows. Section 2 includes a brief introduction on DEA. In Section 3, the DEA model with multi-component units is presented. The above-mentioned Iranian companies are evaluated in Section 4, using the multi-component DEA model. And finally, the conclusion is provided in Section 5.

## 2 Introduction to DEA

In order to evaluate the efficiency of any system in which an activity is performed (productive, economic, educational, …), a criterion called efficiency is utilized. In fact, the efficiency evaluation of any system aims at finding out how well the system is working, whereas in the concept of productivity the focus is more on the quality of work, i.e., how much good work is done. This, however, does not mean that in performance evaluation with efficiency as the criterion, no attention is paid to the quality of work.

Consider a system under evaluation, consisting of $n$ DMUs, $DMU_j \ (j = 1,...,n)$ , in which each $DMU_j$ uses $m$ inputs $x = (x_{1j}, x_{2j},..., x_{mj})$ to produce $s$ outputs $y = (y_{1j}, y_{2j},..., y_{sj})$. The inputs and outputs of every DMU are all nonnegative and every DMU has at least one positive input and one positive output, i.e., $x \geq 0,\ x \neq 0$ and $y \geq 0,\ y \neq 0$. Then, the economic efficiency of $DMU_\rho$ is defined as follows:

$$\text{Efficiency} = \frac{\sum_{r=1}^{s} u_r y_{rp}}{\sum_{i=1}^{m} v_i x_i} = \frac{\text{weighted sum of outputs of } DMU_p}{\text{weighted sum of inputs of } DMU_p} \tag{1}$$

In this case, the DMUs can be easily compared. However, since the input costs and output prices are not always precisely available, DEA models are generally utilized for this purpose.

The CCR model was introduced by Charnes, Cooper and Rhodes in 1978, which became the basis for a branch of operations research called data envelopment analysis (DEA). After the introduction of the CCR model, other models such as the BCC, RAM, SBM, additive, FDH, … models were introduced to enrich DEA. Some of these models are briefly reviewed in this paper.

Considering relation (1), and regarding the indefiniteness of input and output weights, we are trying to find these weights such that the DMU under evaluation has the maximum efficiency possible. So, for any $DMU_p$, the absolute efficiency is denoted by $E_p$ and is defined as follows (for given $u$ and $v$):

$$E_p = \max_{u_r, v_i \geq 0} \frac{\sum_{r=1}^{s} u_r y_{rp}}{\sum_{i=1}^{m} v_i x_{ip}} \tag{2}$$

In order to obtain the efficiency of any DMU as a number between 0 and 1, we introduce the concept of relative efficiency $(RE_p)$ as follows:

$$RE_p = \max_{u_r, v_i \geq 0} \left\{ \left( \sum_{r=1}^{s} u_r y_{rp} \ / \ \sum_{i=1}^{m} v_i x_{ip} \right) \ / \ \max_{1 \leq j \leq n} \left( \sum_{r=1}^{s} u_r y_{rj} \ / \ \sum_{i=1}^{m} v_i x_{ij} \right) \right\} \quad (3)$$

In fact, in model (3) $DMU_p$ is allowed to freely choose the weights $v_i$ and $u_r$ and increase its efficiency as much as possible. This is called freedom of weights in DEA.

Model (3), which is equivalent to the following model, is called the max-min model.

$$RE_p = \max_{u_r, v_i \geq 0} \min_{1 \leq j \leq n} \left\{ \left( \sum_{r=1}^{s} u_r y_{rp} \ / \ \sum_{i=1}^{m} v_i x_{ip} \right) \ / \ \left( \sum_{r=1}^{s} u_r y_{rj} \ / \ \sum_{i=1}^{m} v_i x_{ij} \right) \right\} \quad (4)$$

Using the change of variable

$$\frac{1}{t} = \max_{1 \leq j \leq n} \left( \sum_{r=1}^{s} u_r y_{rj} \ / \ \sum_{i=1}^{m} v_i x_{ij} \right) \}$$

and then with appropriate change of variable, model (4) can be rewritten (with necessary modifications) as follows:

$$\begin{aligned} E_p &= \max \frac{\sum_{r=1}^{s} u_r y_{rp}}{\sum_{i=1}^{m} v_i c_{ip}} \\ s.t. \quad & \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i c_{ij}} \leq 1 \qquad ; \qquad j = 1,2,\ldots,n \\ & u_r, v_i \geq 0; r = 1,2,\ldots,s; \qquad i = 1,2,\ldots,m \end{aligned} \quad (5)$$

This linear fractional programming model is called the CCR ratio model, which can be linearized using Charnes-Cooper's transformations as follows:

$$\begin{aligned} E_p &= \max \sum_{r=1}^{s} u_r y_{rp} \\ & \sum_{i=1}^{m} v_i x_{ip} = 1 \\ s.t. \quad & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \qquad ; \qquad j = 1,2,\ldots,n \\ & u_r, v_i \geq 0 \qquad ; \qquad r = 1,2,\ldots,s; \qquad i = 1,2,\ldots,m \end{aligned} \quad (6)$$

The model obtained is called the CCR model in multiplier form. The dual of this model, which is called the CCR model in envelopment form, is as follows:

$$
\begin{aligned}
& Min\,\theta \\
s.t.\quad & \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta x_{i\rho} && i = 1,2,\ldots,m \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{r\rho} && r = 1,2,\ldots,s \\
& \lambda_j \ge 0 && j = 1,2,\ldots,n
\end{aligned}
\tag{7}
$$

CCR model in envelopment form

**Theorem 1:** $DMU_o$ is Pareto-Coopmans efficient if and only if in model (7), we have

a) $\theta^* = 1$

b) In any solution of the respective model, $s^{-*} = 0$ and $s^{+*} = 0$ in which $s^{+*}$ and $s^{-*}$ represent the optimal values of the slack variables corresponding to output and input constraints, respectively [8].

## 3 Component efficiency evaluation of DMUs using DEA

In this section, we present a method by which the efficiency of the components of a system with common inputs or outputs can be evaluated. In this method, evaluation is carried out by modifying DEA models, as presented on the next pages.

As stated in Section 1, the DEA technique is based on mathematical programming and is used in measuring the relative efficiency of homogeneous DMUs. It was also mentioned that the relative efficiency of any DMU in this technique is a function of its inputs and outputs, and that a DMU is considered technically efficient if it has made best use of its inputs.

A system consisting of multiple components is considered efficient if all of its components are efficient. In the evaluation of such a system, using ordinary DEA models would not yield the efficiency of system components. It should be noted that in the evaluation of technical efficiency or any type of efficiency including cost-efficiency, revenue efficiency, total efficiency, … performed by ordinary DEA models, component efficiency is not considered.

Cook, for the first time, published a paper on component efficiency in 2001. Also, Beasly carried out some research on the educational and research efficiency of London University, which was published as a paper in 2002. Later, Cook completed his paper in 2003. What follows is an extension and modification to the models presented so far.

### 3-1 Efficiency of two-component DMUs

For the purpose of simplicity, consider a DMU with two components, as represented by $DMU_o$ in the following Figure 1, with one input specific to each component as well as an input commonly used by both components. Moreover, there is one output specific to each component as well as one output produced commonly by both components.

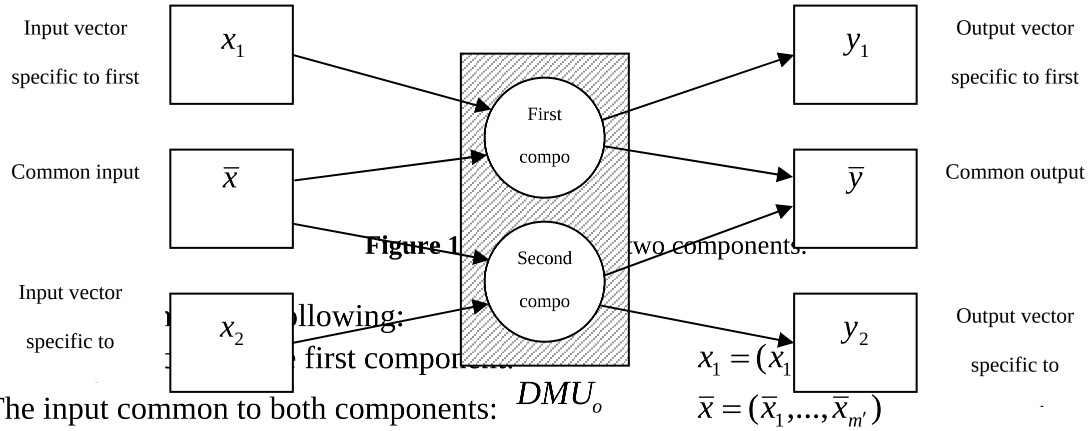


**Figure 1** ... two components.

...llowing:

... first compon... $x_1 = (x_1...$

The input common to both components: $\bar{x} = (\bar{x}_1, \ldots, \bar{x}_{m'})$

The input specific to the second component: $x_2 = (x_{12}, \ldots, x_{m''2})$

The output specific to the first component: $y_1 = (y_{11}, \ldots, x_{s1})$

The output common to both components: $\bar{y} = (\bar{y}_1, \ldots, \bar{y}_{s'})$

The output specific to the second component: $y_2 = (y_{12}, \ldots, x_{s''2})$

Considering what is mentioned above, the input of the first component is

$$\left(x_1, \alpha\bar{x}\right) = (x_{11}, \ldots, x_{m1}\ , \alpha^1 \bar{x}_1, \ldots, \alpha^{m'} \bar{x}_{m'}) \tag{8}$$

in which $0 \le \alpha^i \le 1$ and all α are unknown; and the input of the second component is

$$\left(x_2\ , (1-\alpha)\bar{x}\right) = \left[x_{12}, \ldots, x_{m2}, \left(1-\alpha^1\right)\bar{x}_1, \ldots, \left(1-\alpha^{m'}\right)\bar{x}_{m'}\right] \tag{9}$$

**<u>Important note</u>:** ***$\alpha\bar{x}$ is used as was defined*** in ***relation (8), and does not represent the dot product of the vectors $\alpha$ and $x$.***

Similarly, the output of the second component is

$$(y_1, \beta\ \bar{y}) = (y_{11}, \ldots, y_{s1}, \beta^1 \bar{y}_1, \ldots, \beta^{s'} \bar{y}_{s'}) \tag{10}$$

in which $0 \le \beta^r \le 1$ and all $\beta$ are unknown, and the output vector is

$$(y_2, (1-\beta)\bar{y}) = [y_{12}, \ldots, y_{s2}, (1-\beta^1)\bar{y}_1, \ldots, (1-\beta^{s'})\bar{y}_{s'}] \tag{11}$$

If we consider $e_0^1$ as the efficiency of the first component and $v$ and $\bar{v}$ as the weights of the inputs $x$ and $\bar{x}$, and $u$ and $\bar{u}$ as the weights of the outputs $y$ and $\bar{y}$, respectively, we will have

$$e_0^1 = \frac{\sum_{r=1}^{s} u_r y_{r1} + \sum_{r=1}^{s'} \bar{u}_r \beta^r \bar{y}_r}{\sum_{i=1}^{m} v_i x_{i1} + \sum_{i=1}^{m'} \bar{v}_i \alpha^i \bar{x}_i}$$

or

$$e_0^1 = \frac{u^t y_1 + \bar{u}^t \beta \bar{y}}{v^t x_1 + \bar{v}^t \alpha \bar{x}} \tag{12}$$

In a similar manner, if we consider $e_0^2$ as the efficiency of the second component and $v'$ and $u'$ as the weights of the input $x_2$ and the output $y_2$, respectively, we will have

$$e_0^2 = \frac{\sum_{r=1}^{s} u'_r y_{r2} + \sum_{r=1}^{s'} \bar{u}'_r (1-\beta^r) \bar{y}_r}{\sum_{i=1}^{m''} v'_i x_{i1} + \sum_{i=1}^{m'} \bar{v}'_i (1-\alpha^i) \bar{x}_i}$$

or

$$e_0^2 = \frac{u''^t y_2 + \bar{u}''^t (1-\beta) \bar{y}}{v''^t x_2 + \bar{v}'(1-\alpha) \bar{x}} \tag{13}$$

(Once again, it is necessary to emphasize the concept of $(1-\alpha)\bar{x}, (1-\beta)\bar{y}_r$ )

If $e$ denotes the efficiency of $DMU_o$, aggregate efficiency, $e_0^\alpha$, is defined as follows:

$$e_0^\alpha = \frac{\sum_{r=1}^{s} u_r y_{r1} + \sum_{r=1}^{s'} \bar{u}_r \beta^r \bar{y}_r + \sum_{r=1}^{s} u'_r y_{r2} + \sum_{r=1}^{s} \bar{u}'_r (1-\beta^r) \bar{y}_r}{\sum_{i=1}^{m} v_i x_{i1} + \sum_{i=1}^{m'} \bar{v}_i \alpha^i \bar{x}_i + \sum_{i=1}^{m''} v'_i x_{i2} + \sum_{i=1}^{m'} \bar{v}'_i (1-\alpha^i) \bar{x}_i}$$

and

$$e_0^\alpha = \frac{u^t y_1 + \bar{u}^t \beta \bar{y} + u''^t y_2 + \bar{u}'(1-\beta)\bar{y}}{v^t x_1 + \bar{v}\alpha \bar{x} + v''^t x_2 + \bar{v}'(1-\alpha)\bar{x}} \tag{14}$$

Thus, efficiency evaluation of $DMU_o$ is carried out using the following model, called the MCDEA model:

$$\begin{aligned} & \max e_0^a \\ s.t.\quad & e_0^1 \le 1 \\ & e_0^2 \le 1 \\ & \alpha^i \ge 0 \qquad i = 1,\dots,m' \\ & \beta^r \ge 0 \qquad r = 1,\dots,s' \end{aligned} \tag{15}$$

The above model is nonlinear, which is linearized by the following transformations:

$$\bar{u}^t \beta = u^1 \qquad \bar{u}'(1-\beta) = u^2$$

$$\bar{v}\alpha = v^1 \qquad \bar{v}'(1-\alpha) = v^2$$

Problem (15) is, then, converted to the following:

$$\begin{aligned}
\max \quad & u^t y_1 + u^1 \bar{y} + u'^t y_2 + u^2 \bar{y} \\
s.t. \quad & v^t x_1 + v^1 \bar{x} + v' x_2 + v^2 \bar{x} = 1 \\
& u^t y_1 + u^{1t} y - v^t x_1 - v^{1t} x \le 0 \\
& u'^t y_2 + u^{2^t} \bar{y} - v'^t x_2 - v^2 \bar{x} \le 0
\end{aligned} \tag{16}$$

in which all variables are non-negative.
(In transforming the above model, Charnes-Cooper's transformation has also been used)
Note that each DMU in the above model has only two components.

### 3-2 Efficiency of multi-component DMUs

Having elaborated on two-component DMUs, now we consider DMUs with $k \ge 3$ components. Suppose $x^i = (x_1^i, ..., x_m^i)$ $\quad i = 1, ..., k$ is the input specific to the $i$th DMU. $\bar{x} = (\bar{x}_1, ..., \bar{x}_{m'})$ is the input common to all components of the DMU.
Assume that

$\alpha^j \bar{x}_j = (\alpha_1^j \bar{x}_1, \alpha_2^j \bar{x}_2, ..., \alpha_{m'}^j \bar{x}_{m'}) \qquad j = 1, ..., k$

$\equiv \alpha_r^j \bar{x}_j$ is the $r$th component's share of the $j$th input of the common input vector.
Note that

$$\sum_{r=1}^{k} \alpha_r^j = 1 \qquad \alpha_r^j \ge 0 \qquad j = 1, ..., m'$$
$$r = 1, ..., k$$

For the purpose of illustration, consider the following matrix $\alpha$ :

$$\alpha = \begin{bmatrix} \alpha_1^1 & \alpha_2^1 & \dots & \alpha_{m'}^1 \\ \alpha_1^2 & \alpha_2^2 & \dots & \alpha_{m'}^2 \\ \vdots & & & \vdots \\ \alpha_1^k & \alpha_2^k & \dots & \alpha_{m'}^k \end{bmatrix} = [\alpha_1, ..., \alpha_{m'}] \ge 0$$

Note: The set of the column elements of this matrix is equal to 1, that is:

$\underline{1}\alpha_e = 1 \qquad e = 1, ..., m'$

Similarly, if we consider

$\bar{y} = (\bar{y}_1, ..., \bar{y}_{s'})$

and

$\beta^j \bar{y}_j = (\beta_1^j \bar{y}_j, ..., \beta_k^j y_j) \qquad j = 1, ..., s'$

$\equiv \beta_r^j \bar{y}_j$ is the $r$th component's share of the $j$th output of the common output vector.

$$\beta = \begin{bmatrix} \beta_1^1 & \beta_2^1 & \cdots & \beta_{s'}^1 \\ \beta_1^2 & \beta_2^2 & \cdots & \beta_{s'}^2 \\ \vdots & & & \vdots \\ \beta_1^k & \beta_2^k & \cdots & \beta_{s'}^k \end{bmatrix} = [\beta_1, \ldots, \beta_{s'}] \geq 0$$

Here again $\underline{1}\beta_\ell = 1 \qquad \ell = 1, \ldots, s'$.

With regard to what has been mentioned so far, the input of the $\ell$ th component is

$(x^\ell, \alpha^\ell \bar{x}) = (x_1^\ell, \ldots, x_m^\ell, \alpha_1^\ell \bar{x}_1, \ldots, \alpha_{m'}^\ell \bar{x}_{m'}) \qquad (\ell = 1, \ldots, k)$

By the same token, the output of the $\ell$ th component is

$(y^\ell, \beta^\ell \bar{y}) = (y_1^\ell, \ldots, y_s^\ell, \beta_1^\ell \bar{y}_1, \ldots, \beta_{s'}^\ell \bar{y}_{s'}) \qquad (\ell = 1, \ldots, k)$

If $e_0^\ell$ denotes the efficiency of the $\ell$ th component of $DMU_o$, we have:

$$e_0^\ell = \frac{\sum_{r=1}^{s} u_r^\ell y_r^\ell + \sum_{r=1}^{s'} u'^\ell \beta_r \bar{y}_r}{\sum_{i=1}^{m} v_i^\ell x_i^\ell + \sum_{i=1}^{m'} \alpha_i^\ell v_i' \bar{x}_i} \qquad \ell = 1, \ldots, k$$

$$e_0^\ell = \frac{u^{\ell t} y^\ell + u'^{t\ell} \beta_r \bar{y}}{v^{\ell t} x^\ell + v'^t \alpha^\ell \bar{x}} \qquad \ell = 1, \ldots, \ell$$

and the aggregate efficiency of $DMU_o$ is obtained from the following relation:

$$e_0^a = \frac{\sum_{\ell=1}^{k} u^{\ell t} y^\ell + \sum_{\ell=1}^{k} u'^{\ell t} \beta^\ell \bar{y}}{\sum_{\ell=1}^{k} v^{\ell t} x^\ell + \sum_{\ell=1}^{k} v'^{\ell t} \alpha^\ell \bar{x}} \tag{16}$$

The model under consideration will thus be as follows:

$$\begin{aligned} & \max e_0^a \\ s.t. \quad & e_0^\ell \leq 1 \qquad \ell = 1, \ldots, k \\ & e_j^a \leq 1 \qquad j = 1, \ldots, n \\ & \underline{1}\alpha_\ell = 1 \qquad \ell = 1, \ldots, m' \\ & \underline{1}\beta_\ell = 1 \qquad \ell = 1, \ldots, s' \end{aligned} \tag{17}$$

in which all variables are non-negative.

Using the following transformations, as well as Charnes-Cooper's transformation, Model (17) can be converted to a linear model, as follows.

Setting

$u'^\ell \beta^\ell = \bar{u}^\ell \qquad \ell = 1, \ldots, k$

$\alpha^\ell v'^\ell = \bar{v}^\ell \qquad \ell = 1, \ldots, k$

Problem (17) is converted to the following LP

$$
\begin{aligned}
\max \quad & \sum_{\ell=1}^{k} u^{\ell t} y^{\ell} + \sum_{\ell=1}^{k} \bar{u}^{\ell} \bar{y} \\
s.t. \quad & \sum_{\ell=1}^{k} v^{\ell t} x^{\ell} + \sum_{\ell=1}^{k} \bar{v}^{\ell} \bar{x} = 1 \\
s.t. \quad & u^{\ell t} y^{\ell} + \bar{u}^{\ell t} \bar{y} - v^{\ell t} x^{\ell} - \bar{v}^{\ell t} \bar{x} \le 0 \qquad \ell = 1,\dots,k \\
& \bar{u}^{\ell} \ge \underline{1}\varepsilon \qquad \ell = 1,\dots,k \\
& u^{\ell} \ge \underline{1}\varepsilon \qquad \ell = 1,\dots,k \\
& \bar{v}^{\ell} \ge \underline{1}\varepsilon \qquad \ell = 1,\dots,k \\
& v^{\ell} \ge \underline{1}\varepsilon \qquad \ell = 1,\dots,k
\end{aligned}
\tag{18}
$$

in which $\varepsilon$ is a non-Archimedean. Model (18) is a linear model which yields component efficiency, as well as the elements of matrices $\alpha$ and $\beta$.

## 4 Applied example

As was stated in Section 1, since DEA-based models have a mathematical basis, they require accurate information. In order to make sure the information is accurate, we had to undergo some limitations in the selection of the companies under consideration (DMUs), as well as the input and output information. Thus, out of the companies manufacturing automobiles and automobile parts, we selected those accepted in Tehran Stock Exchange. This was because the information provided by such companies in terms of finance, production, and commerce is clearer, more accurate, and more reliable than those not accepted in Tehran Stock Exchange. Moreover, regarding the standards required by Tehran Stock Exchange for accepting these companies and since the information on these companies is publicly available, such information is acceptably homogeneous.

The total companies manufacturing automobiles and automobile parts that have been accepted in Tehran Stock Exchange are 19 companies, of which one company, Fanar Sazi-e Zar, manufacturing automobile leaf springs, had not provided the Stock Exchange with any information on its activity during 2005 (one of the years under consideration in this study), due to the problems it had undergone during the year. The company was, thus, removed from the list of companies under evaluation, and consequently 18 companies were considered for the purpose of evaluation. The following table contains the names of the companies, as well as the numbers assigned to them regardless of any priority, which will replace their names throughout the evaluation process.

| No. | Company name |
| --- | --- |
| 1 | Saipa Azin |
| 2 | Ahangari-e Tractor |
| 3 | Iran Khodro Diesel |
| 4 | Bahman Group |
| 5 | Pars Khodro |
| 6 | Rikhtegari-e Tractor |
| 7 | Ghata'at-e Automobile |
| 8 | Charkheshgar |
| 9 | Sanaye-e Rikhtegari |
| 10 | Zamiad |
| 11 | Saipa |
| 12 | Fanar Sazi-e Khavar |
| 13 | Carburetor-e Iran |
| 14 | Saipa Diesel |
| 15 | Komak Fanar-e Indamin |
| 16 | Niroo Moharrekeh |
| 17 | Iran Khodro |
| 18 | Mehvar-Sazan |

From among the existing indices, six were chosen to be used in the computations for efficiency evaluation: stocks, total assets, sales, registered capital, capital, and net profit.

In the next stage, we had to separate the input and output components from among the indices and information obtained. Thus, capital, total assets, and stocks were determined as the inputs and sales, net profit, and total equity were considered as outputs.

Since MCDEA was chosen for the analysis, which is a component efficiency evaluation technique, we divided the activities of each DMU into two components: production and administration. Figure 2 demonstrates this division into two components, as well as the inputs and outputs of each component.

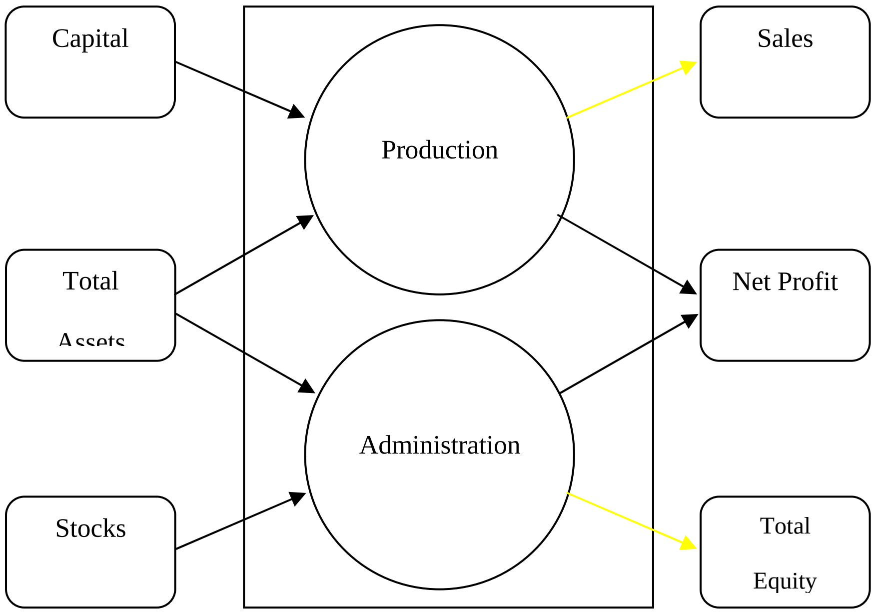


**Figure 2.** Representation of a DMU with two components: production and administration

1- ***First component: Production:*** Regarding the fact that, like other manufacturing companies, the main aim of automobile and automobile parts manufacturing companies is to optimally manufacture their products, the major part of the expected results in realizing the strategies of such companies can be defined in terms of production-oriented inputs and outputs. Therefore, the production component is considered as one of the components in the evaluation of each DMU. Also, the indices capital and total assets are considered as the inputs, and the indices sales and, net profit are considered as the outputs of this component.

1-1- First input: Capital: This input demonstrates the ability of the company in possessing the required financial resources for production.

1-2- Second input (common input): Total assets: This input, which is common to the area of administration, as well, indicates the ability of the company from the production point of view, to secure the required resources (land, buildings, movable and immovable property, machinery, patents, …) for production.

1-3- First output: Sales: In the area of production, this input is a measure indicating the success of the company in providing the planned share of market needs and customer demands, as well as obtaining revenues necessary for keeping up the production process.

1-4- Second output (common output component): Net profit: This index somehow demonstrates the financial productivity of the company, and in fact it is the most important part of the motivation of the stockholders, and the most fundamental index of decision making for implementation and continuation of investment.

***2- Second component: Administration:*** Administration and the administrative section of a system is the basic and key part in productivity and gaining acceptable and even outstanding results from the resources of the organization. The effect of this section on the performance of the organization is such that it can change a company with magnificent performance in the area of production into a loss-making company. On the other hand, it can set a company with limited financial abilities, resources, and facilities at maximum productivity through administrative methods such as expanding participation, providing financial resources, training and optimal use of human force, and establishing advanced systems in different areas. Therefore, we considered administration, or the administrative section, as the second component for the DMUs under evaluation. Also, the indices stocks and total assets were considered as inputs and the indices total equity and net profit as outputs.

2-1- First input: Stocks: According to the rules of the stock market, the number of stocks of a company indicates the capital of the company with regard to the base price of each share at 1000 rials. The major reasons for choosing this index as one of the inputs of the administration component are: maintaining the value of stocks, the fact that an increase in the capital leads to an increase in the number of stocks, and the number of stocks being inclusive of the total assets utilized.

2-2- Second input (common component): Total assets: This is one of the key indices in determining the value of a company, which also affects the ability of the company in starting new plans and projects from the viewpoint of the investors and employers and indicates the capability of the company to enter challenging ventures. This index includes the total fixed assets, current assets, investments, stock goods, orders and down payments, long-term investments, and other assets. In the area of administration, the index shows the extent of the managers' capability in and attention to maximal use of resources and facilities, increasing the risking power, creating a balance between receipts and payments, and timely decision in investments.

2-3- First output: Total equity: This component indicates the assets belonging to the stockholders of the company, which in some way demonstrates the productivity created by the management for the stockholders, using the capital, facilities, and resources of the company.

2-4 Second output (common component): Net profit: The most valuable result of the activity of an economic institution (productive/ service providing) is the amount of net profit. Both in the production and administration, this profit indicates the desirable or undesirable performance of the company in appropriate and timely utilization and management of the resources, as well as effectively directing the company's business processes. The amount of profit allocable/ payable to stockholders is a number which, in the outermost layers of the company and for all persons and the market, is considered as a sign of success or failure of the companies, and based on which the company is judged.

The above-mentioned data was used in the model based on the explanations above, for the years 2005 and 2006, to obtain the component efficiency and aggregate efficiency of each DMU. The data and the results of computations are presented in the following tables.

| DMUs / Company name | Total Share | Total Assets | Sales | Capital | Total Equity | Net Profit |
|---|---|---|---|---|---|---|
| Saipa Azin | 100,000,000 | 617,522 | 657,630 | 100,000 | 159,169 | 61,299 |
| Ahangary Tractor | 40,000,000 | 290,498 | 263,218 | 40,000 | 109,962 | 47,539 |
| Iran Khodro Diesel | 972,000,000 | 12,380,003 | 10,027,979 | 972,000 | 1,314,892 | 1,101,491 |
| Bahman Group | 3,200,000,000 | 8,621,396 | 2,737,907 | 3,200,000 | 5,152,599 | 1,768,521 |
| Pars khodro | 1,200,000,000 | 4,989,371 | 3,477,336 | 1,200,000 | 1,296,560 | 319,702 |
| Rikhtegary Tractor | 142,000,000 | 479,611 | 454,293 | 142,000 | 173,923 | 49,152 |
| Ghatate Automobile Iran | 120,000,000 | 355,318 | 147,233 | 120,000 | 177,743 | 169,943 |
| Charkheshgar | 72,000,000 | 902,996 | 687,261 | 72,000 | 137,872 | 69,265 |
| Sanaye Rikhtegary Iran | 10,000,000 | 206,248 | 156,447 | 10,000 | 28,386 | 20,739 |
| Zamyad | 300,000,000 | 4,209,861 | 4,164,860 | 300,000 | 730,422 | 657,018 |
| Saipa | 5,250,000,000 | 23,688,837 | 24,066,626 | 5,250,000 | 11,747,508 | 6,814,114 |
| Fanarsazy-e-Khavar | 60,000,000 | 245,749 | 130,743 | 60,000 | 76,987 | 16,630 |
| Iran Carbrator | 34,000,000 | 164,631 | 178,340 | 34,000 | 41,188 | 22,587 |
| Saipa Diesel | 600,000,000 | 13,223,609 | 7,457,854 | 600,000 | 1,293,222 | 572,180 |
| Komakfanar-e-Indamin | 13,000,000 | 144,673 | 174,628 | 13,000 | 16,361 | 5,075 |
| Niroo Mohareke | 170,100,000 | 1,446,430 | 1,590,012 | 170,100 | 395,420 | 110,143 |
| Iran Khodro | 4,500,000,000 | 56,595,173 | 45,039,116 | 4,500,000 | 9,267,337 | 4,164,647 |
| Mehvar Sazan | 150,000,000 | 846,744 | 1,629,222 | 150,000 | 194,875 | 87,375 |

**Table 1.** Data of the year 2005

| DMUs / Company name | Total Share | Total Assets | Sales | Capital | Total Equity | Net Profit |
|---|---|---|---|---|---|---|
| Saipa Azin | 100,000,000 | 492,726 | 844,851 | 100,000 | 167,683 | 43,000 |
| Ahangary Tractor | 40,000,000 | 321,486 | 298,007 | 40,000 | 115,972 | 48,949 |
| Iran Khodro Diesel | 972,000,000 | 10,660,537 | 11,462,010 | 972,000 | 1,909,307 | 840,107 |
| Bahman Group | 3,200,000,000 | 10,186,199 | 2,414,483 | 3,200,000 | 5,636,671 | 1,259,910 |
| Pars khodro | 1,200,000,000 | 6,043,419 | 4,796,789 | 1,200,000 | 2,244,373 | 912,000 |
| Rikhtegary Tractor | 142,000,000 | 611,790 | 579,128 | 142,000 | 293,385 | 65,024 |
| Ghatate Automobile Iran | 120,000,000 | 359,500 | 2,147,552 | 120,000 | 293,076 | 162,858 |
| Charkheshgar | 72,000,000 | 829,380 | 687,994 | 72,000 | 129,203 | 5,040 |
| Sanaye Rikhtegary Iran | 10,000,000 | 193,570 | 135,061 | 10,000 | 36,841 | 1,300 |
| Zamyad | 600,000,000 | 5,369,319 | 5,167,457 | 600,000 | 1,568,362 | 720,000 |
| Saipa | 7,000,000,000 | 29,649,891 | 32,886,940 | 7,000,000 | 15,588,292 | 12,263,233 |
| Fanarsazy-e-Khavar | 60,000,000 | 204,703 | 129,664 | 60,000 | 66,156 | 2,288 |
| Iran Carbrator | 34,000,000 | 167,676 | 172,681 | 34,000 | 55,498 | 35,747 |
| Saipa Diesel | 600,000,000 | 12,355,705 | 6,034,301 | 600,000 | 879,072 | 684,072 |
| Komakfanar-e-Indamin | 13,000,000 | 148,175 | 218,298 | 13,000 | 21,059 | 5,200 |
| Niroo Mohareke | 170,100,000 | 1,257,490 | 1,679,620 | 170,100 | 414,424 | 93,555 |
| Iran Khodro | 4,500,000,000 | 108,277,619 | 73,914,600 | 4,500,000 | 8,755,929 | 4,279,392 |
| Mehvar Sazan | 150,000,000 | 820,490 | 1,600,077 | 150,000 | 163,426 | -664 |

**Table 2.** Data of the year 2006

Regarding the above data, the following results are obtained.

| Row | Company Name/DMUs | Aggregated Efficiency | First Component Efficiency | Second Component Efficiency |
|---|---|---|---|---|
| 1 | Saipa Azin | 0.78893 | 0.63352 | 1.00000 |
| 2 | Ahangary Tractor | 1.00000 | 1.00000 | 1.00000 |
| 3 | Iran Khodro Diesel | 0.75674 | 1.00000 | 0.12691 |
| 4 | Bahman Group | 1.00000 | 1.00000 | 1.00000 |
| 5 | Pars khodro | 0.58272 | 1.00000 | 0.43971 |
| 6 | Rikhtegary Tractor | 0.80575 | 1.00000 | 0.70269 |
| 7 | Ghatate Automobile Iran | 1.00000 | 1.00000 | 1.00000 |
| 8 | Charkheshgar | 0.77174 | 0.86214 | 0.68244 |
| 9 | Sanaye Rikhtegary Iran | 1.00000 | 1.00000 | 1.00000 |
| 10 | Zamyad | 1.00000 | 1.00000 | 1.00000 |
| 11 | Saipa | 1.00000 | 1.00000 | 1.00000 |
| 12 | Fanarsazy-e-Khavar | 0.61271 | 0.00016 | 0.90710 |
| 13 | Iran Carbrator | 0.71811 | 1.00000 | 0.30963 |
| 14 | Saipa Diesel | 0.79299 | 0.79381 | 0.55773 |
| 15 | Komakfanar-e-Indamin | 1.00000 | 1.00000 | 1.00000 |
| 16 | Niroo Mohareke | 0.99771 | 0.99545 | 1.00000 |
| 17 | Iran Khodro | 0.79785 | 0.59458 | 1.00000 |
| 18 | Mehvar Sazan | 1.00000 | 1.00000 | 1.00000 |

Component efficiency and aggregate efficiency in the year 2005

| Row | Company Name/DMUs | Aggregated Efficiency | First Component Efficiency | Second Component Efficiency |
|---|---|---|---|---|
| 1 | Saipa Azin | 0.64057 | 0.00064 | 1.00000 |
| 2 | Ahangary Tractor | 1.00000 | 1.00000 | 1.00000 |
| 3 | Iran Khodro Diesel | 0.73525 | 1.00000 | 0.70912 |
| 4 | Bahman Group | 0.70461 | 0.01402 | 0.79234 |
| 5 | Pars khodro | 0.70998 | 0.01082 | 0.85351 |
| 6 | Rikhtegary Tractor | 0.80626 | 0.00122 | 0.95073 |
| 7 | Ghatate Automobile Iran | 1.00000 | 1.00000 | 1.00000 |
| 8 | Charkheshgar | 0.59923 | 1.00000 | 0.37410 |
| 9 | Sanaye Rikhtegary Iran | 1.00000 | 1.00000 | 1.00000 |
| 10 | Zamyad | 0.93965 | 1.00000 | 0.90030 |
| 11 | Saipa | 1.00000 | 1.00000 | 1.00000 |
| 12 | Fanarsazy-e-Khavar | 0.44447 | 0.00025 | 0.50770 |
| 13 | Iran Carbrator | 0.67377 | 1.00000 | 0.63922 |
| 14 | Saipa Diesel | 0.75887 | 1.00000 | 0.71340 |
| 15 | Komakfanar-e-Indamin | 0.93825 | 1.00000 | 0.00029 |
| 16 | Niroo Mohareke | 0.86405 | 0.17009 | 1.00000 |
| 17 | Iran Khodro | 0.56933 | 1.00000 | 0.50103 |
| 18 | Mehvar Sazan | 0.59576 | 1.00000 | 0.00026 |

Component efficiency and aggregate efficiency in the year 2006

## 5 Data analysis

The results indicate that during the years of this study, some of the units have been inefficient owing to inefficiency in one component or both. Meanwhile, in 2005, eight companies, 2,7,9,11,4,10,15, and 18, were efficient, out of which only four companies, 2,7,9, and 11, remained efficient in 2006. Thus, by comparing the performance of the companies in 2005 and 2006 we conclude that companies 4,10,15,18 have had regression in 2006. Moreover, during the two years of the study, none of the companies have been in a better condition in 2006 than in 2005. In other words, none of the companies have had any progress in 2006.

The formula $k = \frac{\theta_{t+1}}{\theta_t}$, in which $k$ indicates the progress or regression of a unit in a period of time compared to the previous period, is applicable for both aggregate efficiency and component efficiency. If $k > 1$, the unit is said to have progress, and if $k < 1$, then the unit has regression. Table 3 demonstrates the progress and regression status, in 2005 and 2006, of the eight units that were efficient in 2005, based on the aggregate efficiency and component efficiency of both their components. As can be observed from the table, ignoring two units Zamiad and Komak Fanar-e Indamin with 7% regression, two units Bahman Group and Mehvar-Sazan with 30% and 41% regression, respectively, in 2006 have been in a very unfavorable state. In order to closely investigate the reasons for their regression, we turn to the columns indicating the comparison of their respective component efficiency in 2005 and 2006.

| Row | DMU/Cmpany Name | Condition in 2005 | Condition in 2006 | $k_A = \frac{\theta_{t+1}}{\theta_t}$ | $k_{e1}$ | $k_{e2}$ |
|---|---|---|---|---|---|---|
| 2 | Ahangary Tractor | Efficient | Efficient | 1 | 1 | 1 |
| 7 | Ghatate Automobile Iran | Efficient | Efficient | 1 | 1 | 1 |
| 9 | Sanaye Rikhtegary Iran | Efficient | Efficient | 1 | 1 | 1 |
| 11 | Saipa | Efficient | Efficient | 1 | 1 | 1 |
| 4 | Bahman Group | Efficient | Inefficient | 0.7 | 0.014 | 0.79 |
| 10 | Zamyad | Efficient | Inefficient | 0.93 | 1 | 0.9 |
| 15 | Komakfanar-e-Indamin | Efficient | Inefficient | 0.93 | 1 | 0.00029 |
| 18 | Mehvar Sazan | Efficient | Inefficient | 0.59 | 1 | 0.00026 |

**Table 3**

In Table 3 above, $k_{e1}$, $k_{e2}$, and $k_A = \frac{\theta_{t+1}}{\theta_t}$ indicate the progress or regression coefficient of the first component, the progress or regression coefficient of the second component, and the aggregate progress or regression coefficient, respectively. As can be observed from columns $k_{e1}$ and $k_{e2}$ above, there are totally different reasons for the regression of these two units. The performance regression or efficiency decrease of Mehvar-Sazan has been in the second component, and it has been efficient in the first component during both years, while the regression of Bahman Group has been due to regression in both components. The two units should, therefore, be investigated from two different viewpoints. Finally, the effect of the inputs and outputs specific to each

component and the common inputs and outputs should be taken into account, so that suggestions can be made regarding the necessary changes in the two units.
We categorize the final results of these investigations, which are in complete agreement with the existing facts, as follows.

a) Except for the eight units presented in the table, the rest of the units have been inefficient in both components, and thus totally inefficient. These units must fundamentally revise their planning.
b) The regression in 2006 of four out of the eight efficient units in 2005 indicates that the general economic conditions of the country in 2006 have not been in support of production.
c) The considerable regression in the second component of Mehvar-Sazan, taking the inputs and outputs of this component into account, has been greatly affected by the decrease in net profit (and even loss making in 2006) of the company and the decrease in total equity. The reasons for these decreases must be investigated more closely. Such a condition holds for some inefficient units, as well.
d) The regression in Bahman Group is greater in the first of its two components, which, taking the inputs and outputs of this component into account, is mainly due to the company's decrease in sales and net profit. Regarding the fact that there has been no price cut in the company's products in 2006, the decrease in sales of the company is due to its uncompetitiveness against imported automobiles (which greatly increased during 2006). Further investigation is warranted regarding some manufacturing units taking advantage of customs tariffs. Uncompetitiveness of domestic products versus their foreign counterparts in terms of manufacturing technology and finished price is another issue that requires further consideration.

## 6 Discussion and conclusion

In this paper, the efficiency of 18 Iranian automobile and automobile parts manufacturing companies accepted in Tehran Stock Exchange was evaluated using DEA models designed based on component structure. To do so, the efficiencies of the companies in the production and administration components were computed separately, based on which the aggregate efficiency was calculated for each company.

Since MCDEA evaluates the efficiency of a DMU in different components, in addition to its high accuracy, it also indicates the strengths and weaknesses with greater discrimination power and provides the possibility for presenting solutions to the problem in each component. In other words, useful information can be obtained from this index since it can be divided into its constituents such as administration efficiency, production efficiency, etc., which facilitates policy making for the management.

In this paper the MCDEA model was employed to analyze and evaluate the performance of 18 Iranian automobile and automobile parts manufacturing companies accepted in Tehran Stock Exchange. Based on the results obtained from the analysis, each company can assess its own performance in the production and administration components compared to those in the previous year, and plan to increase or decrease the inputs and outputs of each component directly or indirectly, based on the effect they have on the results obtained.

It can be generally stated that implementing the MCDEA model and the multi-component evaluation of the decision making units makes it possible to analyze them more accurately in order to evaluate their efficiency. This accuracy, in turn, allows for presenting specific suggestions regarding changes in system inputs and outputs. That is to say, we can say that the evaluation carried out in this model has not been a mere comparison, but it has also been accompanied by presenting a solutions and patterns.

Concerning the analytical investigation of the results and the way solutions and appropriate patterns are provided, an independent study is under way using the Malmquist Productivity Index, which will be presented in future.

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Received: May, 2009