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Logistic Robust Method to New Generalized Geometric Credit Risk Approach

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Abstract

This paper presents a complementary technique for empirical analysis of financial ratios and bankruptcy risk. Within this new framework, we propose the use of a new measure of risk, the Generalized Risk Box (GRB) measure. This method would be a general methodological guideline associated with financial data, including solving some methodological problems concerning financial ratios such as non-proportionality, non-asymmetry and non-scalability. In this paper, bankruptcy prediction and better accuracy rates obtained with GRB approach in compare to employing common ratios. This paper also suggests a Robust Logit method, which extends the Logit model by taking outlier into account. We employ Logit and Robust Logit Regression to assess our new method and sample forecast performances. Accuracy results show Robust Loigt method is substantially superior to the Logit method in financial studies.

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1 Introduction

Corporate failure prediction has been stimulated both by private and government sectors all over the world (Charitu et al., 2004). Business failure prediction has been one of the major research domains in financial researches to evaluate the financial health of companies (Grice and Dugan, 2001). Moreover, company failure may inflict negative shocks for each of the shareholders, thus the total cost of failure will be large regarding to economic and social costs (Shumway, 2001). Bankruptcy prediction models have been proven necessary to obtain a more accurate statement of firm's financial situation (Keasey and Watson, 1991).

Beaver (1967) predicted corporate failure through the combined use of sophisticated quantitative using selected financial ratios. Altman (1968) extended this narrow interpretation by investigating a set of financial ratios as well as economic ratios as possible determinants of corporate failures using multiple discriminant analysis, named the Z-score model. Since Altman (1968), literature on predicting bankruptcy has witnessed numerous extensions and modifications. However, none of them had a perfect predictor functional form and all procedures utilised use of common ratios without any theoretical basis. Previous researchers all emphasized that financial ratios have significant effect on bankruptcy risk, return, credit risk, commercial risk, market and economic conditions[2].

Trimming the sample ratios, eliminating negative observations, and use of various transformations such as logarithms and square roots to achieve more normal distributions were done by some studies (Canbas et al., 2004). While attempts have been made to solve problems of using accounting-based financial ratios in statistical analysis, none has been entirely successfully developed in quantitative and objective systems for bankruptcy prediction (Andres et al., 2005). However, most of these attempts have utilised use of common ratios, which may exceeded cost of errors in the analysis and problem of mis-specification. In general, no equally convenient or superior alternative transformed ratio has been developed and applied[3] .

Some researchers made correction for univariate non-normality and tried to approximate univariate normality by transforming the variables prior to estimation of their model. Deakin (1976) used logarithmic transformation for the lack of normality for distributions, and then Foster (1986) used square root and lognormal transformation of financial ratios. However, logarithmic and square root transformation may also be arbitrary (So, 1987). The rank transformation used by Kane et al., (1996) reported improvement in fit and less biased results by linear

[2] For more details about financial ratios properties, see Watson (1990) and Tippett (1990).

[3] Some exposition of weaknesses in the use of common ratios such as scaling, proportionality and symmetric effects are provided in Bahiraie et al. (2008) and Azhar and Elliott (2006).

models with transformed data set. Logarithmic and rank transformations and square roots are even more difficult to interpret because they can alter the natural monotonic relationships among data (Canbas et al. 2004). Recently Ooghe, et al., (2005) used Logit transformation to achieve better accuracy.

In the other hand, there are many methods to estimate the probability of bankruptcy but none of them have taken the outliers into account when there is a discrete dependent variable. Outliers which can seriously distort the estimated results have been well documented regression model. Some researchers approximate univariate normality by 'trimming' by the method known as 'outlier deletion', which involves segregating outliers with reference to normal distribution (Ezzamel et al., 1990). Although methods and applications that take outliers into account are well known when the dependent variables are continuous (Rousseeuw, 1983; Rousseeuw and Yohai, 1984), few have conducted empirical studies when the dependent variable is binary. Atkinson and Riani (2001), Flores and Garrido (2001) have developed the theoretical foundations as well as the algorithm to obtain consistent estimator in Logit model with outliers, but they do not provide applied studies. If outliers indeed exist when the dependent variable is binary, the conventional Logistic model might be biased. In summary, there is no general guideline concerning the appropriate data representation which is able to solve ratio difficulties. Respectively there is a need of regression method application in order to take outliers into account. Furthermore, none of the previous attempts had perfect prediction in the functional form. While all of procedures utilizing the use of common ratios without considering numerator and denominator of each ratio in specific, which are the most essential factor concerning each ratio value. In view of these shortcomings and the frequently used ratios and statistical techniques in failure prediction modelling based on current knowledge of failing firms, we construct a new type of ratio representation named Generalized Risk Box (GRB). For regression procedure, we employed robust logistic regression in order to take outliers into account in bankruptcy predictions.

Our first objective in this paper is to propose a new approach, which involves data representation, followed by illustrating the use of this methodology for measuring financial risk in ratio analysis and prediction bankruptcies. The second aim of this paper is to predict bankruptcy probability with the consideration of outliers. We apply method of Atkinson and Riani (2001). According to literature, our paper is the first one that using the Robust Logit model for financial data and bankruptcy predictions.

The remainder of this paper proceeds as follows. Section 2 discusses summary of statistical Robust Logit Regression methods of prediction and its general framework. In section 3, we briefly derive our new method, the Generalized Risk Box (GRB). Subsequently, changes in each risk components associated with changes in GRB coordinates will be viewed geometrically. Section 4 illustrates an empirical application of Logit Regression (LR) and Robust Logistic Regression (RLR) as classification methods and we summarise and conclude in Section 5.

2 Robust Logistic regression as Statistical Method of Prediction

Since Altman (1968), MDA is a prevalent technique in bankruptcy prediction in terms of classification or prediction ability among traditional models (Aziz and Dar, 2006). Some studies have found Logit model superior to MDA (Gu, 2002). However, the research by Aziz and Dar, (2006) has shown that the two models are equally efficient. Robust statistics provides an alternative approach to classical statistical methods. Robust methods provide automatic ways of detecting, down weighting (or removing), and flagging outliers, largely removing the need for manual screening. There are various definitions of "a robust statistic". A robust statistic is resistant to errors in the results produced by deviations from assumptions. The median is a robust measure of central tendency, while the mean is not; for instance, the median has a breakdown point of 50%, while the mean has a breakdown point of 0% (Maronna et al., 2006). The median absolute deviation and inter-quartile range are robust measures of statistical dispersion, while the standard deviation and range are not. Trimmed estimators and Winsorised estimators are general methods to make statistics more robust. The basic tools used to describe and measure robustness are, the breakdown point, the influence function and the sensitivity curve. Intuitively, the breakdown point of an estimator is the proportion of incorrect observations an estimator can handle before giving an arbitrarily large result[4].

Historically, several approaches to robust estimation were proposed, including R-estimators and L-estimators. However, M-estimators now appear to dominate the field as a result of their generality, high breakdown point, and their efficiency (Huber, 1981). M-estimators are a generalization of maximum likelihood estimators (MLE). MLE try to maximize $\prod_{i=1}^{n} f(x_i)$ or equivalently minimize $\sum_{i=1}^{n} -\log f(x_i)$. In 1981, Huber proposed to generalize this to the minimization of $\sum_{i=1}^{n} \rho(x_i)$, where ρ is some function. Minimizing $\sum_{i=1}^{n} \rho(x_i)$ can often be done by differentiating ρ and solving $\sum_{i=1}^{n} \psi(x_i) = 0$, where $\psi(x) = \frac{d\rho(x)}{dx}$ if ρ has a derivative[5]. It can be shown that M-estimators are asymptotically normally distributed, so that as long as their standard errors can be computed, an approximate approach to inference is available. It can be shown that the influence function of an M-estimator T is proportional to ψ (Huber,

[4] See Huber, (1981) and Maronna et al. (2006) for more details.

[5] Notice that M-estimators do not necessarily relate to a probability density function. As such, off-the-shelf approaches to inference that arise from likelihood theory can not, in general, be used.

1981). Which means we can derive the properties of such an estimator (such as its rejection point, gross-error sensitivity or local-shift sensitivity) when we know its ψ function. The classical MLE for generalized linear models can be highly influenced by outliers. In all of the above models the explanatory vectors x_i can be highly influential outliers.

The Robust Library in S-Plus software enables us to robustly fit Generalized Linear Models (GLIM) for response observations y_i, $i = 1, 2, ...n$, which may follow one of the Poisson or Binomial distributions. The Binomial Distribution is $P(y_i = j) = \binom{n_i}{j} \mu_i^j (1-\mu_i^j)^{(n_i - j)}$ for $j = 0, 1, ...n_i$ where $0 \le \mu_i \le 1$ and n_i is the number of binomial trials for observation y_i. When $n_i = 1$, the observations are called y_i Bernoulli trials. The expected value of y_i for the Binomial distribution is related to μ_i by $E\left(\frac{y_i}{n_i}\right) = \mu_i$. Then we have a vector $x_i^T = (x_{i1,} x_{i2}, ..., x_{ip})$ of P independent explanatory variables, and corresponding vector $\beta^T = (\beta_{1,} \beta_2, ..., \beta_p)$ of unknown regression coefficients, from which software form the linear predictor $\eta = x_i^T \beta$. The linear predictor η and the expected value μ_i are related through the link function g which maps μ_i to $\eta = g(\mu_i)$. The inverse link transformation g^{-1} maps η to $\mu_i = g^{-1}(\eta)$. Following Binomial Model (Logit), we have $\eta = g(\mu_i) = \log\left(\frac{\mu_i}{1-\mu_i}\right)$ which $0 < \mu_i < 1$ with inverse transformation $\mu_i = g^{-1}(\eta) = \log\left(\frac{\exp(\eta)}{1+\exp(\eta)}\right)$ which $-\infty < \eta < +\infty$. For the Binomial model there is conditional expectation $E_\beta(y_i \,|\, x_i\rangle = n_i \times \mu_i = n_i\left(\frac{\exp(x_i^T\beta)}{1+\exp(x_i^T\beta)}\right)$. In the Bernoulli distributions, the response y_i is either 0 or 1, and so can not be an outlier. In the general Binomial model when n_i is large, the y_i can also be outliers in cases where the expected values of $\frac{y_i}{n}$ are small. Thus, in the general Binomial cases, influential y_i outliers need for a robust alternative to the MLE. Regarding misclassification results which are important in our research we used misclassification model approach to estimate β_i instead of Cubif or Mallows approaches, as a solution of the estimating equation $\sum_{i=1}^{n} w_i^{mc}.x_i.(y_i - F(x_i^T\beta, \gamma)) = 0$. F is given by the mis-classification model

$P(y_i = 1, x_i) = g^{-1}(x_i^T \beta) + \gamma \times \left[1 - 2g^{-1}(x_i^T \beta)\right] = F(x_i^T \beta, \gamma)$ with g^{-1}. This estimator, introduced by Copas (1988), has properties similar to those of the Mallows-type unbiased bounded influence estimates.

3 Methodology: Generalized Risk Box Method (GRB)

Following the static framework proposed by Bahiraie et al. (2008) which is a two-dimensional box, we introduce a new generalized geometric device named the Generalized Risk Box (GRB). This new approach allows visualization of evolution of transformation that is associated with ratio values in which pair values of each risk ratios (X_i, Y_i) are represented as Cartesian coordinates. For expositional purposes suppose our proxy for risk chosen is employed by X_i as numerator and Y_i as denominator values of $\frac{X_i}{Y_i}$ ratio. For any number of firms, $\forall i = 1, 2, 3, ..., n$, proposed Generalized Risk Box GRB_i is defined as a function of X_i and Y_i. Consider a square two-dimensional space that captures all changes in numerator X_i and denominator Y_i, for any firm *i* and any period *t* where *X* and *Y* can be positive, negative or zero[6]. Let the risk flows for any hypothetical firm *i* consist of the set of all X and Y for *n* years $\forall t = 1, 2, 3, ..., n$. Ratio values are usually available at uniform discrete time intervals, annually, quarterly. The dimensions of the GRB are central with respect to $\max(X)$ and $\max(Y)$. The essential ingredient is that the length of any side is set at two times the maximum of largest absolute changes value of whichever is bigger from the numerator or denominator values recorded during the considerable period *t*. Correspondingly, the total area of GRB for $i \in t$ is $2 \times \max(\max|X_i|) = 2L$ if the largest absolute value is from X_i or $2 \times \max(\max|Y_i|) = 2L$ if the largest value is from Y_i values where *L* is the length of one side of a GRB. Y_i values are depicted on the vertical axis $(\pm Y)$ and X_i values on the horizontal axis $(\pm X)$ as labeled in Figure 1 as $(\pm X_{\max})$ and $(\pm Y_{\max})$. When comes down to it, the actual values of $\pm X_{\max}$ and $\pm Y_{\max}$ depends on which of the two is largest, and this value will then be applied to both axes to ensure a perfect square. One of the primary innovations of the GRB index is the scaling and ranking ability factor that stems directly from the GRB construction and is two times the absolute maximum of the largest change for the period of study which is 2(*L*). Please note that X or Y value in the denominator and numerator will only be equal when either X or Y is also the largest value during the period of study.

[6] It is applicable to any level of aggregation such as cross-country studies, cross sector, and ratios.

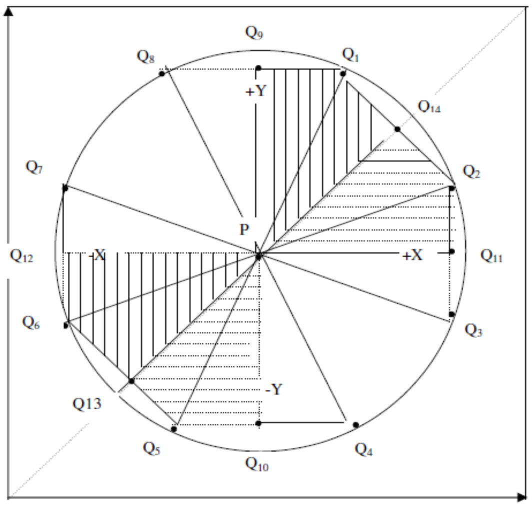

Figure 1: Generalized Risk Box

Assume that

(i) Changes are a monotonically increasing function

(ii) Risk values requirements for both X_i values and Y_i values are equal.

Measure of Generalized Risk Box that satisfies for n firms $\forall i = 1,2,3,...,n$ is given by:

$$GRB_i = \frac{1}{2L}(X_i - Y_i) = \frac{X_i - Y_i}{2(\max\{\max|X|_n, \max|Y|_n\})}$$

Thus the GRB index has a range $(-1 < GRB_i < 1)$.

In order to show the Geometrical relation between (X_i, Y_i) and GRB consider the proposed measure of Generalized Risk Box: $GRB_i = \dfrac{X_i - Y_i}{2(\max\{\max|X_n|, \max|Y_n|\})}$

which can be rewritten as:

$$GRB_i = \frac{X_i}{2(\max\{\max|X_n|, \max|Y_n|\})} - \frac{Y_i}{2(\max\{\max|X_n|, \max|Y_n|\})}$$

Hence, we will have:

$$X_i = Y_i + 2(\max\{\max|X_n|, \max|Y_n|\}) \times (GRB_i) = Y_i + 2L \times (GRB_i).$$

In order to have scaled measure, scaling will be done by the largest value for a given data set that allows us to observe the progress of risk changes. We have: $GRB_i = \frac{X_i - Y_i}{2(\max\{\max|X_n|, \max|Y_n|\})} = \frac{X_i - Y_i}{2L}$ which is a scaled measure.

For partial presentation we will have $\left(\frac{\partial GRB_i}{\partial X_i}\right) = \frac{1}{2(\max\{\max|X_n|, \max|Y_n|\})}$ and $\left(\frac{\partial GRB_i}{\partial Y_i}\right) = \frac{-1}{2\max(\max\{|X_n|, |Y_n|\})}$, which can verify the rate of changes of GRB in $\left(\frac{\partial GRB_i}{\partial Y_i}\right)$ is similar but opposite to that in the lower sector $\left(\frac{\partial GRB_i}{\partial X_i}\right)$. Hence, the GRB index exhibits proportional scaled.

4 Illustrative Empirical Application

4.1 Data collection and mean comparison

The database used in our illustrative empirical study consists of 200 Iranian companies from Tehran Stock Exchange (TSE). 50 companies went bankrupt under bankruptcy law of Iranian companies' act 1970, which a firm is bankrupt when its total value of retained earning is equal or greater than 50% of its listed capital. 150 companies are "matched" companies from the same period of listing 1998-2005. Bankrupt companies are indicated as 1 and non-failed companies as 0. Thus, a firm will have a higher failure probability and will be classified into failing group if its score is higher than cut-off point in each approach.

In this study base on the financial ratios successfully identified by past studies and availability, 40 indices been built by using balance-sheet data. Ratios and significances on mean differences for each group is tested and presented in Table 1. These indices reflect different aspects of firm structure and performance such as liquidity, turnover, operating structure and efficiency, capitalization and finally profitability.

Table 1: Variables employed and comparison of means in two groups

#	Definition of variables	Original Ratios			Transformed Ratios		
		Means of non-bankrupt companies	Means of bankrupt companies	TEGM (Sig level)	Means of non-bankrupt companies	Means of bankrupt companies	TEGM (Sig level)
1	EAIT/TA	0.21985	0.05165	0.000	-0.39008	-0.47417	0.025
2	TD/SE	2.32591	2.99969	0.051	0.17897	0.33310	0.043
8	R/S	0.53916	0.01808	0.000	-0.29721	-0.49609	0.023
4	TD/TA	0.64600	0.78450	0.011	-0.17700	-0.10775	0.000
5	CL/SE	2.07355	2.60760	0.874	0.13713	0.28837	0.211
6	CL/TD	0.87258	0.83419	0.234	-0.06371	-0.08290	0.323
7	OA/TA	0.54037	0.62549	0.201	-0.22981	-0.18725	0.083
8	R/S	0.64792	0.40207	0.445	-0.28176	-0.31233	0.527
9	R/Inv	64191.96287	60.03362	0.000	-0.00444	-0.12682	0.000
10	SE/TD	0.81727	0.33380	0.000	-0.17897	-0.33310	0.025
11	E/TA	0.37868	0.24421	0.041	-0.31066	-0.37789	0.000
12	CA/CL	1.37059	1.13940	0.567	0.07046	0.03709	0.000
13	QA/CL	0.88108	0.49283	0.002	-0.14017	-0.25456	0.311
14	QA//CA	0.59121	0.44456	0.001	-0.20439	-0.27772	0.000
15	NFA/TA	0.22169	0.22309	0.976	-0.38916	-0.38846	0.005
16	WC/TA	0.11022	0.06320	0.696	-0.44489	-0.46840	0.313
17	CL/TA	0.56389	0.65641	0.000	-0.21806	-0.17179	0.000
18	POC/SE	0.53201	0.57998	0.199	-0.23447	-0.10467	0.008
19	RE/TA	0.06492	-0.02391	0.000	-0.46754	-0.51196	0.078
20	EAIT/SE	0.53080	0.17283	0.410	-0.24864	-0.46834	0.000
21	EAIT/S	0.27192	-0.04296	0.000	-0.36405	-0.50608	0.000
22	EBIT/TA	0.17862	0.00639	0.000	-0.41069	-0.49680	0.000
23	D/EAIT	2.02476	0.92434	0.311	-0.11523	0.24383	0.072
24	OI/S	0.28441	-0.01012	0.000	-0.35780	-0.49572	0.874
25	MVE/TA	0.04992	0.05746	0.008	-0.47504	-0.47127	0.006
26	EBIT/IE	4496.20577	-43.01149	0.000	0.59907	0.55253	0.213
27	OI/TA	0.19620	0.02240	0.000	-0.40190	-0.48880	0.107
28	Ca/S	0.18568	0.05238	0.000	-0.43579	-0.47381	0.000
29	GP/S	0.35047	0.09577	0.000	-0.32476	-0.45211	0.214
30	S/SE	3.01240	3.06662	0.072	0.20837	0.29016	0.844
31	S/NFA	10.53526	5.98830	0.893	0.33491	0.31069	0.034
32	S/CA	1.37378	1.07683	0.006	0.06508	0.00171	0.000
33	S/WC	14.68814	5.10868	0.213	0.40842	0.44656	0.008
34	S/TA	0.88013	0.75620	0.107	-0.08629	-0.12527	0.002
35	S/Ca	37.35053	121.39542	0.005	0.43579	0.47381	0.000
36	IE/GP	-0.32201	-1.87164	0.087	-0.57508	-0.60523	0.405
37	Ca/CL	0.17422	0.05219	0.002	-0.41614	-0.47391	0.292
38	Ca/TA	0.08993	0.03416	0.009	-0.45503	-0.48292	0.023
39	S/GP	4.81397	24.35715	0.000	0.32476	0.45211	0.125
40	BVD/MVE	81.75837	73.27468	0.032	0.46128	0.46254	0.043

BVD: Book Value of Dept.; CA: Current assets; EAIT: Earning after income and taxes; GP: Gross profit; Inv: Inventory; MVE: Marked value of equity; NI: Net income; OI: Operational income; QA: Quick assets; RE: Retained earnings; SC: Stock capital; TA: Total assets; Ca: Cash flow; CL: Current liabilities; EBIT: Earnings before interest and taxes; IE: Interest expenses; LA: Liquid assets; NFA: Net Fixed assets; OA: Operating asset; POC: Paid on capital; R: Receivables; S: Sales; SE: Shareholders' equity; TEGM: Test of equity of group mean.

Following recent research by Bahiraie et al. (2009), for primary variable selection and testing each variable's effectiveness on discriminating power, CartProEx V.6.0 software with Mahalanobis D^2 measure was used. Table 2 reports selected variables that produced greatest effectiveness on separation for each groups to have more stable and well-balanced model.

Table 2: Significant variables in each sample

Original Ratios	GRB Method
CR/TA	EBIT/S
QA/CA	QA/CA
OI/TA	TD/TA
CF/GP	MVE/TA
SE/TA	

4.2 Regression results

Subsequently, selected variables coefficients are regressed using Logistic and Robust Logistic Regression to illustrate that this new transformation will produce more accurate prediction equation and can be used as an alternative for common ratios. Results show that robust logit model outperforms logit model in both data sets. Table 3 report the estimated results using the Logit and the Robust Logit models, respectively. When the Logit model is used, less coefficients show are significant compare to Robust Logit model. Alongside, the psudo-R^2 is higher for the Robust Logit models in both approaches, suggesting that in-sample fitting is much better in the Robust Logit model than in the Logit model.

Table 3: Estimated Results for Logit and Robust Logit models

		Logit		Robust Logit	
	Models	Coefficient	t-Value	Coefficient	t-Value
Original Ratios	Constant	-0.3600	-0.7506	17.0487**	2.1627
	CR/TA	1.6195	1.1766	10.2357*	1.7913
	QA/CA	-13.1535***	-4.1651	-34.2707**	-2.2311
	OI/TA	-0.5519**	-2.0683	-2.1146**	-2.3319
	CF/GP	-0.4227	-0.5858	-12.3312**	-2.0225
	SE/TA	0.6539**	2.0013	2.344***	4.6586
	psudo-R^2	0.5941		0.7539	
GRB Method	Constant	0.2134	0.0342	1.4303 *	1.9953
	EBIT/S	1.6349 **	2.1142	8.3259 **	2.9488
	QA/CA	5.7633 *	1.5935	6.5205 **	2.3285
	TD/TA	-2.5894	-0.4968	-1.8580 ***	-5.7351
	MVE/TA	7.5318	0.1936	5.7025	0.2132
	psudo-R^2	0.6816		0.8936	

(*, **, *** denote significant at 10%, 5% and 1% level, respectively)

Specific company is classified as distressed if the calculated probability from models is more than 0.5, otherwise it would be non-distressed.

4.3 K-fold cross-validation

In order to observe the effects of biasness, we conduct the K-fold cross validation procedure. Each one of the subsets is then in turn as testing set after all other sets combined have been training set on which a tree has been built. This cross validation procedure allows mean error rates to be calculated which gives a useful insight into classifiers decision. This technique is simply k-fold cross validation whereby *k* is number of data instances. This has advantage of allowing the largest amount of training data to be used in each run and conversely means that the testing procedure is deterministic. With large data sets, this is computationally infeasible however and in certain situations, the deterministic nature of testing results in weir errors. Further, k-fold crosses validation primary method for estimating turning parameters, dividing the data into *k* equal parts. For each $k = 1, 2, ..., k$ fit the model with parameters to the other *k-1* parts and the *k*th part as testing sample. In our experiment, we set our sample to 5-fold accuracy results. Table 4 represents the comparison of 5-fold accuracy results.

Table 4: The transformed ratios still outperform original ratios

	Original Ratios		GRB Approach	
Items	Logit	Robust Logit	Logit	Robust Logit
1	57.23 %	69.30 %	66.15 %	82.27 %
2	56.17 %	69.73 %	65.48 %	82.13 %
3	57.94 %	68.61 %	63.74 %	82.51 %
4	57.29 %	70.52 %	66.03 %	81.94 %
5	57.71 %	69.89 %	66.34 %	82.72 %
Average	57.26 %	69.61 %	65.54 %	82.31 %

Results highlight the following evidences that under transformation process better classification accuracy results achieved while Robust Logit model outperforms Logit model.

5 Conclusion

The properties of GRB methodology may be a general guideline for ratios analysis, financial analysis and bankruptcy prediction in which there is no arbitrary conditioning, because the numbers of transformations are equal the number of observations. Furthermore, the natural distribution of GRB transformation ensures data are not skewed and should be more robust to the assumptions of Gaussian statistical methods. In addition, GRB can be applied equally to variety of distributional forms, thus making the technique particularly useful in ratio analysis where a diverse set of distributional functions have been identified. Because new transformations GRB is naturally bounded and unaffected by distance between observations, outlier effect if present will be reduced. Similarly, distance data containing white noise and the sensitivity and power of statistical test are improved. Negative values will be transformed to specific variation, thus removing the necessity of deletion of negative data used in previous studies. Besides, proportionality is a theoretical assumption that may not in fact hold and the degree of departure varies across industries and size classes. Thus if the relationship between elements of a ratio is constant over time, size and industry, then the proportionality effect will be satisfied for ratios by using GRB method. Finally, with the use of pooled data across time, the GRB method will reduce effects of history and maturation across population. In summary, as we can observe from the prediction results in next section, we suggest the use of this new methodology for ratio analysis, which can provide a conceptual and complimentary methodological solution to many problems associated with the use of ratios.

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