Applied Mathematical Sciences, Vol. 4, 2010, no. 3, 119 - 131

An Efficient Algorithm for Solving System of Nonlinear Equations

M. M. Hosseini and B. Kafash

Department of Mathematics, Yazd University
P. O. Box 89195-741, Yazd, Iran.
hosse_m@yazduni.ac.ir

Abstract

In this paper, the accounting of the answers of a system of nonlinear equations is considered and a new algorithm based on the method of Adomian decomposition convergence basis for solving functional equations is presented.
This algorithm is performed for some different examples and obtained results show the usage of the recommended method in this paper for solving these equations. In addition to the methods against Adomian which yield one real answer form convergent system in this algorithm can account for all the real answers of a system if a suitable and primary approximation is chosen.

Mathematics Subject Classification: 65R20

Keywords: System of Non-Linear Equations; Convergence Conditions of Adomian Decomposition Method

1. Introduction

Adomian decomposition method(ADM) was presented by Adomian in 1981. This method and rectification of it, have a good usage in solving the differential, differential- algebraic and integral equations. In addition “Abbaoui” and “Cherruault” [1] have used ADM for solving equation of type $f(x)=0$ where $f(x)$ is a nonlinear function. “Kaya”[7] and” Babolian “ [2,3] have applied this method for solving system of nonlinear algebraic equations and “Jafari”[6] has improved these methods.
In this paper, a new and useful algorithm based on ADM convergence conditions for solving functional equations [5] is presented. By this algorithm we are able to solve a system of nonlinear equations in a suitable method. In addition in this method we can account for all the answers of a system by using a suitable and

primary approximation, while the existing ADMs just yield an answer of these systems. In this paper, some numerical examples have fulfilled with maple software and accounting these examples and obtained results show the usage of the suggested method in this paper in comparison with the existing ADMs.

2. An efficient algorithm for solving a System of Non-Linear Equation

Here, the review of the standard ADM is presented to solve a system of nonlinear equation. For this reason, consider the system of nonlinear equation,

$$f(X) = 0, \tag{1}$$

where $f(X) = (f_1(X), f_2(X), \ldots, f_m(X))^T = 0$ and $X = (x_1, x_2, \ldots, x_m)^T$.

We can rewrite the equations in the form

$$x_i = F_i(x_1, x_2, \ldots, x_m) + c_i\ , \quad i = 1, 2, \ldots, m, \tag{2}$$

where F is a nonlinear function and c is a real constant. The ADM decomposes the solution x_i by an infinite series of components,

$$x_i = \sum_{j=0}^{+\infty} x_{i,j}\ , \qquad i = 1, \ldots, m \tag{3}$$

and the nonlinear term $F_i(x_1, x_2, \ldots, x_m)$ by an infinite series,

$$F_i(x_1, x_2, \ldots, x_m) = \sum_{j=0}^{\infty} A_{i,j}\ , \qquad i = 1, 2, \ldots, m \tag{4}$$

where the components of $A_{i,j}$ are the so-called Adomian polynomials, and for each i, $A_{i,j}$ depends on $x_{1,0}, \ldots, x_{1,j};\ x_{2,0}, \ldots, x_{2,j}; x_{m,0}, \ldots, x_{m,j}$ only. Substituting (3) and (4) into (2) yields,

$$\sum_{j=0}^{\infty} x_{i,j} = c_i + \sum_{j=0}^{\infty} A_{i,j}\ , \qquad i = 1, 2, \ldots, m. \tag{5}$$

Now, we define

$$\begin{aligned} x_{i,0} &= c_i, \\ x_{i,k+1} &= A_{i,k}\ , \qquad k \ge 0 \end{aligned} \tag{6}$$

If the series converges in a suitable way, then it can be seen that

$$x_i = \lim_{M \to +\infty} \Psi_{i,M}(x), \tag{7}$$

where

$$\Psi_{i,k} = \sum_{j=0}^{k-1} x_{i,j} \tag{8}$$

Now, we require an expression for the $A_{i,j}$. Specific algorithms are given in [9,10] to formulate Adomian polynomials. The following formula:

$$A_{i,n}(x_{1,0}, \ldots, x_{1,n}; x_{2,0}, \ldots, x_{2,n}; \ldots; x_{m,0}, \ldots, x_{m,n}) = \frac{1}{n!} \frac{d^n}{d\lambda^n} F_i(x_1, \ldots, x_m) \bigg|_{\lambda = 0} \tag{9}$$

can be used to construct Adomian polynomials of a system with nonlinear term $F_i(X)$.The theoretical treatment of the convergence of Adomian decomposition method has been considered in [1, 4, 5, 8]. As stated in [5], $\sum_{i=0}^{\infty} x_i$, which is obtained by (6), converges to the exact solution x, when,

$$\exists\ 0 \le \alpha < 1, \quad |x_{k+1}| \le \alpha |x_k|, \quad \forall k \in N \cup \{0\}. \tag{10}$$

Now, by considering (10), we construct an efficient algorithm to solve the system of nonlinear equations (1). For this reason, we need an initial approximation. This initial approximation must be guessed. But it can be computed if the ith equation of system (1) has a real constant term as below,

$$F_i(x_1, x_2, \dots, x_m) + c_i = 0, \qquad i = 1, 2, \dots, m. \tag{11}$$

where F_i's are nonlinear functions and c_i's are real constants. In this case, we add $\beta_i x_i$ on both sides of (11). So, we have,

$$\beta_i x_i = F_i(x_1, x_2, \dots, x_m) + \beta_i x_i + c_i, \qquad i = 1, 2, \dots, m. \tag{12}$$

Here, β_i's are unknown real constants and these will be determined such that the convergence conditions (10) will be held.

Equation (12) implies that,

$$x_i = \frac{F_i(x_1, x_2, \dots, x_m) + \beta_i x_i}{\beta_i} + \frac{c_i}{\beta_i}, \qquad i = 1, 2, \dots, m. \tag{13}$$

Here, we put :

$$x_{i,0} = \frac{c_i}{\beta_i}, \tag{14.a}$$

And

$$x_{i,1} = \frac{F_i(x_{1,0}, x_{2,0}, \dots, x_{m,0})}{\beta_i} + x_{i,0}, \tag{14.b}$$

where $i = 1, 2, \dots, m$. Now, choose an arbitrary number α, $0 < \alpha < 1$, and by attention to (10), we consider m equations $x_{i,1} = \alpha\, x_{i,0}$, $i = 1, 2, \dots, m$, and with (14), we have:

$$F_i(x_{1,0}, x_{2,0}, \dots, x_{m,0}) = (\alpha - 1) c_i, \qquad i = 1, 2, \dots, m, \tag{15}$$

In addition, Taylor expansion of F_i's, at $X = (\overbrace{0, 0, \dots, 0}^{m})$ is:

$$F_i(x_{1,0}, x_{2,0}, \dots, x_{m,0}) \approx F_i(0,0,\dots,0) + x_{1,0} \frac{\partial}{\partial x_1} F_i(0,0,\dots,0) + \dots + x_{m,0} \frac{\partial}{\partial x_m} F_i(0,0,\dots,0), \quad i = 1, 2, \dots, m. \tag{16}$$

Substituting (16) into (15) yields,

$$\begin{cases} x_{1,0}\dfrac{\partial}{\partial x_1}F_1(0,0,\ldots,0)+\ldots+x_{m,0}\dfrac{\partial}{\partial x_m}F_1(0,0,\ldots,0)=(\alpha-1)\,c_1-F_1(0,0,\ldots,0) \\ \vdots \\ x_{1,0}\dfrac{\partial}{\partial x_1}F_m(0,0,\ldots,0)+\ldots+x_{m,0}\dfrac{\partial}{\partial x_m}F_m(0,0,\ldots,0)=(\alpha-1)\,c_m-F_m(0,0,\ldots,0) \end{cases} \tag{17}$$

Then $x_{1,0}, x_{2,0}, \ldots, x_{m,0}$ are obtained with solving the above linear system of equations, and substituting these values into (14.a) yields β_i's, that speed convergence of ADM. Now, we are able to compute the initial approximation for the system of nonlinear equations (11) by using part (I) of algorithm (2.1).

To compute an initial approximation:

Algorithm (2.1): Part (I)

Input: System of nonlinear equations $0 = f_i(X) = F_i(X) + c_i$, $i = 1,2,\ldots,m$, and $\alpha (0 < \alpha < 1)$.

Output : Initial approximation, $\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_m$.

***Step 1*:**

Solve the system of linear equations (17), and substitute $x_{1,0}, x_{2,0}, \ldots, x_{m,0}$ into

$$\beta_i = \frac{c_i}{x_{i,0}}, \qquad i = 1,2,\ldots,m$$

to produce β_i's

Step 2:

Consider Adomian's polynomials $A_{i,j}$,

$$\frac{F_i(x_1, x_2, \ldots, x_m) + \beta_i x_i}{\beta_i} = \sum_{j=0}^{+\infty} A_{i,j}, \qquad i = 1,2,\ldots,m$$

and set:

$$\begin{cases} x_{1,0} = \dfrac{c_1}{\beta_1} \\ x_{1,1} = A_{1,0}, \\ \vdots \\ x_{1,5} = A_{1,4} \end{cases} \qquad \begin{cases} x_{2,0} = \dfrac{c_2}{\beta_2} \\ x_{2,1} = A_{2,0}, \\ \vdots \\ x_{2,5} = A_{2,4} \end{cases} \qquad \ldots, \qquad \begin{cases} x_{m,0} = \dfrac{c_m}{\beta_m} \\ x_{m,1} = A_{m,0}. \\ \vdots \\ x_{m,5} = A_{m,4} \end{cases}$$

For $1 \le i \le m$, choose the largest p_i that $1 \le p_i \le 5$ and the conditions

$\left|x_{i,1}\right| < \left|x_{i,2}\right| < \cdots < \left|x_{i,p_i}\right|$ are held.

Now, set

$\overline{x}_1 = x_{1,0} + x_{1,1} + \cdots + x_{1,p_1}$,

$\overline{x}_2 = x_{2,0} + x_{2,1} + \cdots + x_{2,p_2}$,

$\vdots$

$\overline{x}_m = x_{m,0} + x_{m,1} + \cdots + x_{m,p_m}$ □

Now, we have the system of nonlinear equations (1) and its initial approximation, $\overline{\mathrm{x}}$, which is obtained by guessing or by using algorithm (2.1), Part (I). To continue, we want to improve the initial approximation, $\overline{X}$. For this reason, we rewrite (1) as below,

$$x_i = \frac{f_i(X) + \beta_i x_i}{\beta_i} - \overline{x}_i + \overline{x}_i , \qquad i = 1,2,\ldots,m \tag{18}$$

Here, β_i 's are unknown real constant and it will be determined such that the convergence conditions (10) will be held, and speed convergence of ADM. By (6) and (9), for $i = 1,2,\ldots,m$, we have,

$x_{i,0} = \overline{x}_i$,

$$x_{i,1} = \frac{f_i(x_{1,0}, x_{2,0}, \ldots, x_{m,0}) + \beta_i x_{i,0}}{\beta_i} - \overline{x}_i = \frac{f_i(\overline{X})}{\beta_i},$$

and

$$x_{i,2} = \frac{x_{1,1} \frac{\partial}{\partial x_1} f_i(\overline{X}) + \cdots + x_{m,1} \frac{\partial}{\partial x_m} f_i(\overline{X})}{\beta_i} + x_{i,1} . \tag{19}$$

Now, choose an arbitrary number α, $0 < \alpha < 1$, and by attention to (10), we consider equation $x_{i,2} = \alpha\, x_{i,1}$. For $i = 1,2,\ldots,m$, we have:

$$x_{1,1} \frac{\partial}{\partial x_1} f_i(\overline{X}) + x_{2,1} \frac{\partial}{\partial x_2} f_i(\overline{X}) + \cdots + x_{m,1} \frac{\partial}{\partial x_m} f_i(\overline{X}) = \beta_i (\alpha - 1)\, x_{i,1} , \tag{20}$$

and (19.b) implies that,

$$\beta_i = \frac{f_i(\overline{X})}{x_{i,1}} . \tag{21}$$

Substituting (21) into (20) yields a $m \times m$ system of linear equations, as below:

$$\begin{cases} x_{1,1}\dfrac{\partial}{\partial x_1}f_1(\overline{X})+x_{2,1}\dfrac{\partial}{\partial x_2}f_1(\overline{X})+\cdots+x_{m,1}\dfrac{\partial}{\partial x_m}f_1(\overline{X})=f_1(\overline{X})(\alpha-1) \\ \vdots \\ x_{1,1}\dfrac{\partial}{\partial x_1}f_i(\overline{X})+x_{2,1}\dfrac{\partial}{\partial x_2}f_i(\overline{X})+\cdots+x_{m,1}\dfrac{\partial}{\partial x_m}f_i(\overline{X})=f_i(\overline{X})(\alpha-1) \\ \vdots \\ x_{1,1}\dfrac{\partial}{\partial x_1}f_m(\overline{X})+x_{2,1}\dfrac{\partial}{\partial x_2}f_m(\overline{X})+\cdots+x_{m,1}\dfrac{\partial}{\partial x_m}f_m(\overline{X})=f_m(\overline{X})(\alpha-1) \end{cases} \tag{22}$$

Then $x_{1,1}, x_{2,1}, \ldots, x_{m,1}$ are obtained with solving the above linear system of equations, and substituting these values into (21) yields β_i 's. Thus, we are able to compute an approximate solution of the system of nonlinear equations (1) by using part (II) of the algorithm (2.1).

To compute approximate solution :

Algorithm (2.1): Part (II) :

Input: System of nonlinear equations $f(x)=0$, Initial approximation $\overline{X}=(\overline{x}_1,\overline{x}_2,\ldots,\overline{x}_m)$, $\alpha(0<\alpha<1)$ and arbitrary numbers $\varepsilon_1,\ldots,\varepsilon_m>0$.
Output : Approximate solution $\widetilde{X}=(\widetilde{x}_1,\widetilde{x}_2,\ldots,\widetilde{x}_m)$.

Step 1:

If $\left|f_i(\overline{X})\right|<\varepsilon_i$, $i=1,2,\ldots,m$, then go to step 4.

Step 2:

Solve the system of linear equations (22) and substitute $x_{1,1}, x_{2,1}, \ldots, x_{m,1}$ into (21) to yield β_i 's.

Step 3:

Consider Adomian's polynomials $B_{i,j}$,

$$\frac{f_i(x_1,x_2,\ldots,x_m)+\beta_i x_i}{\beta_i}=\sum_{j=0}^{+\infty}B_{i,j}\ , \qquad i=1,2,\ldots,m$$

set:

$$\begin{cases} x_{1,0}=\bar{x}_1 \\ x_{1,1}=B_{1,0}-\bar{x}_1 \\ x_{1,2}=B_{1,1} \\ \vdots \\ x_{1,5}=B_{1,4} \end{cases}, \quad \begin{cases} x_{2,0}=\bar{x}_2 \\ x_{2,1}=B_{2,0}-\bar{x}_2 \\ x_{2,2}=B_{2,1} \\ \vdots \\ x_{2,5}=B_{2,4} \end{cases}, \quad \ldots, \quad \begin{cases} x_{m,0}=\bar{x}_m \\ x_{m,1}=B_{m,0}-\bar{x}_m \\ x_{m,2}=B_{m,1} \\ \vdots \\ x_{m,5}=B_{m,4} \end{cases}$$

For $1 \le i \le m$, choose the largest p_i ($1 \le p_i \le 5$) such that the conditions $|x_{i,1}| < |x_{i,2}| < \cdots < |x_{i,p_i}|$ are held.

Now, set

$\tilde{x}_1 = x_{1,0} + x_{1,1} + \cdots + x_{1,p_1}$,

$\tilde{x}_2 = x_{2,0} + x_{2,1} + \cdots + x_{2,p_2}$,

$\vdots$

$\tilde{x}_m = x_{m,0} + x_{m,1} + \cdots + x_{m,p_m}$

and go to step 1.

Step 4:

Set $\tilde{X} = \bar{X}$ and stop.

□

3. Test Problems

In this section, some examples are solved by the proposed algorithm of this paper. The obtained results show that the proposed algorithm can appropriately solve the system of nonlinear equations. In addition, by the proposed algorithm we can obtain all real solutions of system (1). Here, the algorithm is performed by maple 8 with 8 digits precision for examples (1) and (3) and with 16 digits precision for example (2), and we let $\alpha = 0.1$.

Example 1, [3,6,7]: Consider the system of nonlinear equations,

$$\begin{cases} x^2 - 10x + y^2 + 8 = 0 \\ xy^2 + x - 10y + 8 = 0 \end{cases}, \tag{23}$$

which has two solutions,

$$X^1 = (1,1)^t, \tag{24.a}$$

and

$$X^2 = (2.1934394, 3.0204665)^t. \tag{24.b}$$

In [3,6] and [7] the first solution, (24.a), was obtained by a modification of ADM. Here, we obtain all solutions (24.a) and (24.b) by algorithm(2.1). For this reason, we rewrite (23) as below,

$$\begin{cases} x = \dfrac{\overbrace{x^2 - 10x + y^2 + \beta x}^{F}}{\beta} + \dfrac{\overbrace{8}^{a}}{\beta} \\ \\ y = \dfrac{\overbrace{xy^2 + x - 10y + \gamma y}^{G}}{\gamma} + \dfrac{\overbrace{8}^{b}}{\gamma} \end{cases}.$$

Now, by using part(I) of the algorithm(2.1): we obtain, a linear system of equation:

$$\begin{cases} x_0 F_x(0,0) + y_0 F_y(0,0) = -0.9a - F(0,0) \\ x_0 G_x(0,0) + y_0 G_y(0,0) = -0.9b - G(0,0) \end{cases},$$

then $x_0 = 0.72$ and $y_0 = 0.792$ and substituting these values into $\beta = \dfrac{a}{x_0}$ and $\gamma = \dfrac{b}{y_0}$ yields $\beta = 11.111111$ and $\gamma = 10.101010$. Table 1 reports the obtained results.

To compute initial approximation(algorithm (2.1), part(I)) for example 1(24.a)

i	x_i	y_i
0	0.72000000	0.79200000
1	0.17510976	0.12391138
2	0.057870005	0.043439671
3	0.023621284	0.019158913
4	0.010947670	0.0095110140
5	0.0055125683	0.0050594794
Initial Approximation	$\overline{x} = \sum_{i=0}^{5} x_i = 0.99306128$	$\overline{x} = \sum_{i=0}^{5} x_i = 0.99308045$

Table 1

So, initial approximation $\overline{X} = (0.99306128, 0.99308045)^T$ is obtained. Now, by using part(II) of the algorithm (2.1), we have, $\beta = 6.7050724$ and $\gamma = 6.7050435$ where table 2 shows the obtained results.

To compute approximate solution(algorithm (2.1), part(II)) for example 1(24.a)

i	x_i	y_i
0	0.99306128	0.99308045
1	0.00622912	0.00620762
2	0.0006229059	0.00062076813
3	0.00007382763	0.000079235223
4	0.000011366801	0.000009708530
5	0.0000010563089	0.0000020621349
Approximate solution	$\bar{x}=\sum_{i=0}^{5} x_i = 0.99999957$	$\bar{x}=\sum_{i=0}^{5} x_i = 0.99999985$

Table 2

Again, by using part(II) of the algorithm (2.1), we have, $\beta = 8.1364842$ and $\gamma = 2.3255823$ where table 3 shows the obtained results.

To compute approximate solution(algorithm (2.1), part(II)) for example 1(24.a)

i	x_i	y_i
0	0.99999957	0.99999985
1	$4.\times 10^{-7}$	1.5×10^{-7}
2	4.3580657×10^{-8}	-2.200007×10^{-8}
3	-4.6767016×10^{-9}	9.1159560×10^{-8}
4	2.2329157×10^{-8}	-2.2645121×10^{-7}
5	-5.5288594×10^{-8}	5.7174389×10^{-7}
Approximate solution	$\bar{x}=\sum_{i=0}^{3} x_i = 1.0000000$	$\bar{x}=\sum_{i=0}^{2} x_i = 0.99999998$

Table 3

Table 1 , table 2 and table 3 yield,

$$\tilde{x} = \underbrace{{}^1x_0 + \cdots + {}^1x_5 + {}^2x_1 + \cdots + {}^2x_5 + {}^3x_1 + \cdots + {}^3x_3}_{14\ terms} = 1.0000000$$

$$\tilde{y} = \underbrace{{}^1y_0 + \cdots + {}^1y_5 + {}^2y_1 + \cdots + {}^2y_5 + {}^3y_1 + {}^3y_2}_{13\ terms} = 0.99999998 .$$

It should be noted that with the same number of terms, standard ADM[3, 7] gives the answer (0.99994803, 0.99989706) and revised ADM[6] gives the answer (0.99998402, 0.99996938). It is clear that the series given by proposed method converges faster than the series given by existing ADMs to the exact solution.

To obtain, second solution (24.b), part(II) of the algorithm (2.1) with initial approximation $\overline{\mathrm{X}} = (3, 4)^t$ is used and table 4 shows the results:

To compute approximate solution(algorithm (2.1), part(II)) for example 1,(24.b)

iteration	β	γ	$\bar{x}$	$\bar{y}$
1	−5.8181818	−31.916010	2.4328124	3.3451562
2	−4.2475289	−17.090370	2.2306774	3.0745330
3	−3.7556006	−12.137028	2.1933526	3.0207778
4	30.330850	−.47809693	2.1934389	3.0204794
5	182.20389	−3.1825843	2.1934393	3.0204667

Table 4

Table 4 yields,

$$\tilde{x} = \underbrace{{}^{1}x_0 + \cdots + {}^{1}x_2 + {}^{2}x_1 + {}^{2}x_2 + {}^{3}x_1 + \cdots + {}^{3}x_4 + {}^{4}x_1 + \cdots + {}^{4}x_3 + {}^{5}x_1 + {}^{5}x_2}_{14\ terms} = 2.1934393.$$

$$\tilde{y} = \underbrace{{}^{1}y_0 + \cdots + {}^{1}y_2 + {}^{2}y_1 + {}^{2}y_2 + {}^{3}y_1 + \cdots + {}^{3}y_5 + {}^{4}y_1 + \cdots + {}^{4}y_3 + {}^{5}y_1 + \cdots + {}^{5}y_3}_{16\ terms} = 3.0204667.$$

Clearly, the results show the advantage of using the proposed algorithm to obtain all solutions of this problem.

Example 2, [7]: Consider the system of nonlinear equations,

$$\begin{cases} x^3 + y^3 - 6x + 3 = 0 \\ x^3 - y^3 - 6y + 2 = 0 \end{cases}, \qquad (25)$$

which has three solutions,

$$X^1 = (0.532370372327903, 0.351257447590883)^T, \qquad (26.a)$$

$$X^2 = (1.882719112000601, 1.175129224067372)^T, \qquad (26.b)$$

and

$$X^3 = (-2.423800710235022, -1.489322078683482)^T. \qquad (26.c)$$

In [6] the first solution, (26.a), was obtained by standard ADM. Here, we obtain all solutions (26.a),(26.b) and (26.c) by algorithm(2.1). By using part(I) of algorithm(2.1) we obtain, $\beta = \gamma = 6.666666666666667$ and initial approximations: $\bar{x} = 0.5320515165748498$ and $\bar{y} = 0.3510661821519659$. By using part(II) of algorithm (2.1), we obtain,

To compute approximate solution(algorithm (2.1), part(II)) for example 2(26.a)

iteration	β	γ	$\bar{x}$	$\bar{y}$
1	5.476638070480274	5.504433323962262	0.532370368785013	0.351257445466074
2	5.475283031908003	5.502708640204112	0.532370372327868	0.351257447590862

Table 5

In table 5, the approximate solution converges to solution (26.a), as follows.

$$\underbrace{{}^{1}x_0 + \cdots + {}^{1}x_5 + {}^{2}x_1 + \cdots + {}^{2}x_5 + {}^{3}x_1 + \cdots + {}^{3}x_5}_{16\ terms} = 0.532370372327868$$

$$\underbrace{{}^{1}y_0 + \cdots + {}^{1}y_5 + {}^{2}y_1 + \cdots + {}^{2}y_5 + {}^{3}y_1 + \cdots + {}^{3}y_5}_{16\ terms} = 0.351257447590862.$$

With the same number of terms, standard ADM[7] gives the answer (0.53237037215376, 0.351257447485824).
Next, to obtain the second real solution (26.b), algorithm (2.1),part(II), is used with initial approximation $\overline{X} = (3, 2)^T$ and the obtained results are as below:

$$\underbrace{{}^1x_0 + {}^1x_1 + {}^1x_2 + {}^2x_1 + {}^2x_2 + {}^3x_1 + \cdots + {}^3x_3 + {}^4x_1 + \cdots + {}^4x_5 + {}^5x_1 + \cdots + {}^5x_5 + {}^6x_1 + \cdots + {}^6x_5}_{23\ terms} =$$
$$1.88271911511528127$$

$$\underbrace{{}^1y_0 + {}^1y_1 + {}^1y_2 + {}^2y_1 + {}^2y_2 + {}^3y_1 + \cdots + {}^3y_3 + {}^4y_1 + \cdots + {}^4y_4 + {}^5y_1 + \cdots + {}^5y_4 + {}^6y_1 + \cdots + {}^6y_4}_{20\ terms} =$$
$$1.175129225772663$$

Furthermore, to obtain the third real solution (26.c), algorithm (2.1),part(II), is used with initial approximation $\overline{X} = (-3, -1)^T$ and the obtained results are as below:

$$\underbrace{{}^1x_0 + {}^1x_1 + {}^1x_2 + {}^2x_1 + \cdots + {}^2x_4 + {}^3x_1 + \cdots + {}^3x_5 + {}^4x_1 + \cdots + {}^4x_5}_{17\ terms} = -2.423800710246008$$

$$\underbrace{{}^1y_0 + \cdots + {}^1y_3 + {}^2y_1 + {}^2y_2 + {}^3y_1 + \cdots + {}^3y_5 + {}^4y_1 + \cdots + {}^4y_5}_{16\ terms} = -1.489322078671146$$

Example 3, [3, 6]: Consider the system of nonlinear equations,

$$\begin{cases} 15x + y^2 - 4z - 13 = 0 \\ x^2 + 10y - e^{-z} - 11 = 0 \\ \quad y^3 - 25z + 22 = 0 \end{cases}, \qquad (27)$$

where $X = (1.0421496, 1.0310913, 0.92384815)^T$ is the solution of this system. By using part(I) of the algorithm(2.1), we obtain, $\beta_1 = -13.115416$, $\beta_2 = -10.882469$ and $\beta_3 = 27.777778$ and initial approximation, $\bar{x} = 1.0124180$, $\bar{y} = 1.0310941$, $\bar{z} = 0.92389968$. Then by using part(II) of the algorithm(2.1), these initial approximations are improved, as in table 6.

To compute approximate solution(algorithm (2.1), part(II)) for example 3

iteration	β_1	β_2	β_3	$\bar{x}$	$\bar{y}$	$\bar{z}$
1	-16.679365	-779.39964	35.475592	1.0421403	1.0311801	0.92385953
2	-0.23871431	-10.924785	0.097727713	1.0421470	1.0310925	0.92385100
3	-20.522277	-7.2990077	26.255660	1.0421494	1.0310912	0.92384818

Table 6

where,

$$\underbrace{{}^{1}x_0+{}^{1}x_1+{}^{2}x_1+\cdots+{}^{2}x_5+{}^{3}x_1+{}^{4}x_1+\cdots+{}^{4}x_5}_{13\ terms}=1.0421496$$

$$\underbrace{{}^{1}y_0+\cdots+{}^{1}y_5+{}^{2}y_1+\cdots+{}^{2}y_3+{}^{3}y_1+{}^{3}y_2+{}^{4}y_1+\cdots+{}^{4}y_5}_{16\ terms}=1.0310912$$

$$\underbrace{{}^{1}z_0+\cdots+{}^{1}z_5+{}^{2}z_1+\cdots+{}^{2}z_4+{}^{3}z_1+{}^{4}z_1+\cdots+{}^{4}z_5}_{16\ terms}=0.92384816$$

The advantage of using the proposed algorithm with respect to the existing ADMs is clearly demonstrated for these examples.

Remarks: In this paper, a modified Adomian decomposition method to solve system of nonlinear equations was proposed. The merit of this method is that it is more efficient than the existing ADMs, due to the fact that,

i) The proposed method is able to find all solutions of a given system whereas the existing ADMs are able to find only one solution(see examples 1-3).
ii) The proposed method converges to a solution of given system faster than the existing ADMs(see examples 1-3).

References

[1] K. Abbaoui and Y. Cherruault, Convergence of Adomian's method applied to nonlinear equations, Mathl. Comput. Modelling, 20 (9) (1994) 69-73.

[2] E. Babolian and J. Biazar, Solution of system of Non-Linear Equations by modified Adomian decomposition method, Appl. Math. Comput. 132 (2002) 167-172.

[3] E. Babolian, J. Biazar and A. R. Javadi, Solution of a system of nonlinear equations by Adomian decomposition method, Appl. Math. Comput., 150 (2004) 847-854.

[4] Y. Cherruault, G. Adomian, K. Abbaoui and R. Rach, Further remarks on convergence of decomposition method, Bio-Medical Computing, 38 (1995) 89-93.

[5] M. M. Hosseini and H. Nasabzadeh, On the convergence of Adomian decomposition method, Appl. Math. Comput. 182 (2006) 536-543.

[6] H. Jafari and V. Daftardar-Gejji, Revised Adomian decomposition method for solving a system of nonlinear equations, Appl. Math. Comput. 175 (2006) 1-7.

[7] D. Kaya and S. M. El-Sayed, Adomian's decomposition method applied to systems of nonlinear algebraic equations, Appl. Math. Comput. 154 (2004) 487-493.

[8] D. Lesnic, Convergence of Adomian decomposition method: Preiodic temperatures, Computers Math. Applic., 44(2002) 13-24.

[9] V. Seng, K. Abbaoui and Y. Cherruault, Adomian's polynomials for nonlinear operators, Math. Comput. Modeling, 24 (1996) 59-65.

[10] A. M. Wazwaz, A new algorithm for calculating Adomian polynomials for nonlinear operators, Applied Mathematics and Computation, 111 (2000) 53-69.

Received: January, 2009