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Implementation of 2-Point 2-Step Methods for the Solution of First Order ODEs

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Abstract

In this paper the 2 point 2 step methods (**2PG, M2PG, M2PF**) for solving system of first order ordinary differential equations are proposed. These methods at each step will approximate the solutions of initial value problems at two points simultaneously using variable step size. In addition, the stability of the proposed method are discussed. Examples are presented to illustrate the computational aspect of these methods.

Mathematics Subject Classification: 65L05, 65L06

Keywords: Block methods, Ordinary Differential Equation, Numerical results

1 Introduction

This paper considers a system of first order ordinary differential equations in the following form

$$Y' = F(x,Y) \quad Y(a) = Y_0 \ , \ \ a \leq x \leq b \tag{1}$$

where a and b are finite and $Y' = [y_1', y_2', \ldots, y_n']^T$, $Y = [y_1, y_2, \ldots, y_n]$ and $F = [f_1, f_2, \ldots, f_n]^T$. Block methods for numerical solution of first order ODEs have been proposed by several researcher [1, 2, 6, 7, 8]. These methods are one of the efficient methods for solving ordinary differential equations. The advantage of block method compare to single and multistep methods, is that, at each application of a block method, the solution will be approximated in more than one point. The number of points is depend on the structure of the block method.

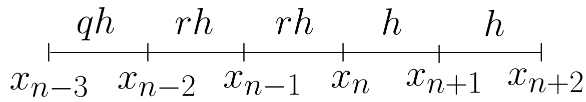

Figure 1: 2-point 2-step method

In Fig 1, at each step, these methods will estimate the solution at two points with step size h concurrently using three back approximated values of the previous block with setp size rh. The methods are based on predictor-corrector scheme $PE(CE)^m$ of Adams type methods with variable step size.

Majid et al. [5] introduced the 2-point fully implicit block method (**2PF**). The value of y_{n+1} and y_{n+2} were approximated by integrating (1) over the interval $[x_n, x_{n+1}]$ and $[x_n, x_{n+2}]$ respectively. In this paper, we try to derive three possible type of 2-point 2-step methods (**2PG, M2PG, M2PF**), using Lagrange interpolation polynomial. During the implementation of **2PG** method, the iteration Gauss Siedel style will be involved i.e. for obtaining corrector formula, the closest point in the interval for integrating (1) is considered. Therefore the approximated values of y_{n+1} and y_{n+2} are obtained by integrating (1), over the interval $[x_n, x_{n+1}]$ and $[x_{n+1}, x_{n+2}]$ respectively.

In **M2PG** method (modified **2PG** method), our aim is to decrease the number of function called without losing desired accuracy. This may be done by involving the first approximated point, in the set of interpolation points for obtaining predictor formula for second point. Similar way can be applied for modifying **2PF** method [5] and obtaining **M2PF**.

2 The 2PG method

In Figure 1, the solution of y_{n+1} and y_{n+2} at the points x_{n+1} and x_{n+2} respectively with step size h, will be approximated simultaneously using three back values at the points x_n,x_{n-1},x_{n-2} of the previous two step with step size rh. The method will compute two points concurrently using two earlier steps.

The formula of the **2PG** method are derived using Lagrange interpolation polynomial. The involved interpolation points for obtaining the corrector formula to approximate the solutions for the first and second point i.e. x_{n+1} and x_{n+2} are $\{(x_{n-2}, f_{n-2}), \ldots, (x_{n+2}, f_{n+2})\}$. The interval of integration for the first and second point are $[x_n, x_{n+1}]$, $[x_{n+1}, x_{n+2}]$ respectively. by integrating (1) over the corresponding interval, using **MATHEMATICA**, we may obtain the corrector formula for first and second point respectively, The 1^{st} point,

$$y_{n+1} = y_n + h\Big[-\frac{3+15r+20r^2}{240(1+r)(2+r)} f_{n+2} + \frac{18+75r+80r^2}{60(1+r)(1+2r)} f_{n+1} + \frac{7+45r+100r^2}{240r^2} f_n - \frac{7+30r}{60r^2(1+r)(2+r)} f_{n-1} + \frac{7+15r}{240r^2(1+r)(1+2r)} f_{n-2}\Big] \tag{2}$$

The 2^{nd} point,

$$y_{n+2} = y_{n+1} + h\Big[\frac{147+255r+100r^2}{240(1+r)(2+r)} f_{n+2} + \frac{78+165r+80r^2}{60(1+r)(1+2r)} f_{n+1} - \frac{23+45r+20r^2}{240r^2} f_n + \frac{23+30r}{60r^2(1+r)(2+r)} f_{n-1} - \frac{23+15r}{240r^2(1+r)(1+2r)} f_{n-2}\Big] \tag{3}$$

The predictor formula are derived similarly, but the involved interpolation points are $\{(x_{n-3}, f_{n-3}), \ldots, (x_n, f_n)\}$, so the predictor formula for first and second point in terms of r and q are respectively,

The 1^{st} point,

$$y_{n+1} = y_n + h\Big[-\frac{(1+2r)^2}{4q(q+r)(q+2r)} f_{n-3} + \frac{3+4q+12r+6qr+12r^2}{24qr^2} f_{n-2} - \frac{3+4q+16r+12qr+24r^2}{12r^2(q+r)} f_{n-1} + \frac{3+4q+20r+18qr+48r^2+24qr+48r^3}{24r^2(q+2r)} f_n\Big] \tag{4}$$

The 2^{nd} point,

$$y_{n+2} = y_{n+1} + h\Big[-\frac{(3+2r)(5+6r)}{4q(q+r)(q+2r)} f_{n-3} + \frac{45+28q+84r+18qr+36r^2}{24qr^2} f_{n-2} - \frac{45+28q+112r+36qr+72r^2}{12r^2(q+r)} f_{n-1} + \frac{45+28q+140r+54qr+144r^2+24qr^2+48r^3}{24r^2(q+2r)} f_n\Big] \tag{5}$$

2.1 The M2PG method

In this approach, the predictor formula for second point is improved. Therefore after prediction of the solution value at first point i.e. x_{n+1}, this point will be involved in the set of interpolation points for obtaining predictor formula for second point. The advantage of this approach is that, the order of predictor formula for second point is one more than the order of predictor formula for first point, Hence we may obtain better predicted value for the second point. Since subsequent corrector formula will use these two predicted values, so this approach will affect in number of iteration for obtaining desired accuracy, consequently we expect the decrease of the number of function called. Using **MATHEMATICA**, we may obtain the predictor formula for second point as follow,

$$y_{n+2} = y_{n+1} + h\Big[\frac{147+255r+100r^2}{60q(q+r)(q+2r)(1+q+2r)}f_{n-3} - \frac{147+85q+255r+50qr+100r^2}{120qr^2(1+2r)}f_{n-2}$$
$$+ \frac{147+85q+340r+100qr+200r^2}{60r^2(1+r)(q+r)}f_{n-1}$$
$$- \frac{147+85q+425r+150qr+400r^2+60qr^2+120r^3}{120r^2(q+2r)}f_n$$
$$+ \frac{372+225q+1125r+420qr+1120r^2+180qr^2+360r^3}{60(1+r)(1+2r)(1+q+2r)}f_{n+1}\Big] \quad (6)$$

The remainder of the formula are the same as the **2PG** method.

2.2 The M2PF method

The idea for deriving this method, is exactly the same idea for deriving **M2PG** method. Predictor formula for second point will stands for predicted value of first point. Therefore the predictor formula for second point is as follow,

$$y_{n+2} = y_n + h\Big[\frac{4(9+15r+5r^2)}{15q(q+r)(q+2r)(1+q+2r)}f_{n-3} - \frac{18+10q+30r+5qr+10r^2}{15qr^2(1+2r)}f_{n-2}$$
$$+ \frac{4(9+5q+20r+5qr+10r^2)}{15r^2(1+r)(q+r)}f_{n-1} - \frac{18+10q+50r+15qr+40r^2}{15r^2(q+2r)}f_n$$
$$+ \frac{4(24+15q+75r+30qr+80r^2+15qr^2+30r^3)}{15(1+r)(1+2r)(1+q+2r)}f_{n+1}\Big] \quad (7)$$

The remainder of the formula are the same as the **2PF** method.

3 stepsize control

The step size strategy in the code is the same as in [5], the choice for next step size will be limited to half, double or the same as the current step size. If the approximated solution at step k, has desired accuracy, i.e. it is acceptable, therefore the choice for next step will be double or the same as current step size which may be specified by step size controller. Otherwise the step size controller will allow the step size to become half.

Generally because of two reason, we need to have an estimation of local truncation error (LTE) at each step. Firstly one step is acceptable if the truncation error is less or equal to the given tolerance provided by user. Secondly step size controller needs to have local traction error at current step, for approximating new step size for next step. In our code an estimation of local truncation error is obtained by comparing the derived corrector formula of order p for second

point, and the same corrector formula for that point of order $p-1$.
The first predicted step size for h_{n+1} is given by,

$$h_{n+1} = \tau \times h_n \times \left(\frac{TOL}{LTE}\right)^{\frac{1}{p}}$$

where τ is a safety factor. The aim of utilizing this safety factor is to reduce the risk of the failure step. In the developed code, when the next step size is double, the ratio r is 0.5 and q can be 0.5 or 0.25, but if the next step size remain constant, r is 1 And q can be 1 or 2 or 0.5. In case of step size failure, r is 2, and q is 2. In order to reducing cost of time, all the coefficients of the formula are stored in the developed code.

4 Absolute Stability

Here we will discuss the absolute stability of **2PG** method using a linear first order test problem

$$y' = f = \lambda y \tag{8}$$

The stability region is plotted when the step size ratio is constant, doubled and halved for the method. The test equation (8) is substituted into the corrector formula of the **2PG** method. Setting the determinant of the corrector formula written in matrix form to zero will give the stability polynomial. The stability polynomials of **2PG** method at $r = 1, 0.5, 2$ are as follow,

For $r = 1$ we have,

$$t^4(1 - \frac{289}{360}\bar{h} + \frac{413}{2160}\bar{h}^2) + t^3(-1 - \frac{191}{180}\bar{h} - \frac{559}{720}\bar{h}^2) + t^2(-\frac{49}{360}\bar{h} - \frac{59}{720}\bar{h}^2) + \frac{t}{2160}\bar{h}^2 = 0 \tag{9}$$

For $r = 2$ we have,

$$t^4(1 - \frac{87}{100}\bar{h} + \frac{623}{2700}\bar{h}^2) + t^3(-1 - \frac{5291}{4800}\bar{h} - \frac{289}{540}\bar{h}^2) + t^2(-\frac{133}{4800}\bar{h} - \frac{1237}{86400}\bar{h}^2) + \frac{t}{86400}\bar{h}^2 = 0 \tag{10}$$

For $r = 0.5$ we have,

$$t^4(1 - \frac{147}{200}\bar{h} + \frac{847}{5400}\bar{h}^2) + t^3(-1 - \frac{407}{600}\bar{h} - \frac{1493}{1080}\bar{h}^2) + t^2(-\frac{44}{75}\bar{h} - \frac{589}{1350}\bar{h}^2) + \frac{8t}{675}\bar{h}^2 = 0 \tag{11}$$

where $\bar{h} = h\lambda$ and the stability regions are plotted in Figure 2.

The stability region is inside the boundary of the dotted points. This is expectable that the region should get larger with smaller step sizes. This can be seen easily in Fig 2, the stability region is larger when the step size is half ($r = 2$) compare to the step size being double ($r = 0.5$) or constant ($r = 1$). Since **2PG**, **M2PG** methods have the same corrector formula, therefore they have the same stability regions.

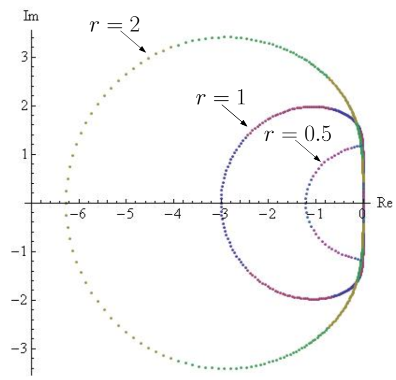

Figure 2: Stability region for 2PG method at r=2, 1 and 0.5

5 Numerical Results

In order to show the efficiency and applicability of our presented methods, we consider four given problems to compare our computed solutions with the solutions obtained by method in [5]. The following notation are used in the tables:

TOL	Tolerance
MTD	Method Employed
TS	Total Successful Steps
FS	Total Failure Steps
MAXE	Absolute value of the maximum error of the computed solution
AVERR	Average error
FN	Total Function Calls
TIME	The execution time taken in microsecond
2PF	Implementation of the two point block method in [5]
2PG	Implementation of the two point block method using Gauss Seidel iteration
M2PG	Implementation of the modified two point block method using Gauss Seidel iteration
M2PF	Implementation of the modified two point block method in [5]

Problem 1: $y' = -0.5y, \quad y(0) = 1, \quad [0, 20]$
Exact solution: $y(x) = e^{-0.5x}$
Source: Artificial problem

Problem 2: Nonlinear non stiff Krogh's problem
$y_i' = -\beta_i y_i + y_i^2, \quad y_i(0) = -1, \quad [0, 20], \quad i = 1, 2, 3, 4$

$\beta_1 = \beta_2 = 0.2$, $\beta_3 = 0.3$, $\beta_4 = 0.4$
Exact solution: $y_i(x) = \frac{\beta_i}{1+c_i e^{\beta_i x}}$, $c_i = -(1+\beta_i)$
Source: Johnson and Barney [4]

Problem 3: A two-body orbit problem (Mildly stiff)
$y_1' = y_3$, $y_2' = -y_4$, $y_3' = -\frac{y_1}{r^3}$, $y_4' = -\frac{y_2}{r^3}$, $r = \sqrt{y_1^2 + y_2^2}$
$y_1(0) = 1$, $y_2(0) = 0$, $y_3(x) = 0$, $y_4(x) = 1$, $[0, 20]$
Exact solution:
$y_1(x) = \cos(x)$, $y_2(x) = \sin(x)$
$y_3(x) = -\sin(x)$, $y_4(x) = \cos(x)$
Source: Hairer,et al. [3]

Problem 4: Linear nonstiff complex eigenvalues
$y_1' = -Ay_1 + By_2$, $y_2' = -By_1 - Ay_2$, $y_3' = -Cy_3 + Dy_4$, $y_4' = -Dy_3 - Cy_4$
$A = C = 1$, $B = D = \sqrt{3}$
$y_1(0) = 1$, $y_2(0) = 1$, $y_3(0) = 1$, $y_4(0) = 1$, $[0, 20]$
Exact solution:
$y_1(x) = e^{-Ax}(\cos Bx + \sin Bx)$, $y_2(x) = e^{-Ax}(\cos Bx - \sin Bx)$
$y_3(x) = e^{-Cx}(\cos Dx + \sin Dx)$, $y_4(x) = -e^{-Cx}(\cos Dx - \sin Dx)$
Source: Johnson and Barney [4]

The error calculated are defined as

$$(e_i)_t = \left| \frac{(y_i)_t - (y(x_i)_t)}{A + B(y(x_i))_t} \right|$$

Where $(y)_t$, is the t-th component of the approximate y. $A = 1$, $B = 0$ corresponds to the absolute error test, $A = 0$, $B = 1$ corresponds to the relative error test and finally $A = 1$, $B = 1$ corresponds to the mixed error test. The mixed error test is used for all the above problems. The maximum error and average error are defined as follows:

$$MAXE = \max_{1 \le i \le TS} \left(\max_{1 \le i \le N} (e_i)_t \right)$$

$$AVER = \frac{\sum_{i=1}^{TS} \sum_{t=1}^{N} (e_i)_t}{(P)(N)(TS)}$$

Where N is the number of equation in the system, TS is the number of successful steps and P is the number of points in the block. In the code, we iterate the corrector to convergence using the convergence criteria:

$$\left| y_{n+2}^{r+1} - y_{n+2}^{r} \right| < 0.1 \times TOL$$

Table 1: 2PF, 2PG, M2PG and M2PF methods for solving problem 1,τ=0.8

TOL	MTD	TS	FS	MAXE	AVER	FN	TIME
10^{-2}	2PF	18	0	1.27×10^{-4}	9.35×10^{-6}	107	305
	2PG	19	0	2.90×10^{-4}	2.06×10^{-5}	105	134
	M2PG	19	0	2.07×10^{-4}	1.36×10^{-5}	81	329
	M2PF	19	0	3.26×10^{-4}	2.14×10^{-5}	105	272
10^{-4}	2PF	29	0	2.06×10^{-6}	1.10×10^{-7}	173	410
	2PG	30	0	9.92×10^{-7}	1.13×10^{-7}	177	177
	M2PG	30	0	5.31×10^{-6}	3.90×10^{-7}	135	420
	M2PF	29	0	2.21×10^{-6}	1.56×10^{-7}	157	350
10^{-6}	2PF	51	0	1.24×10^{-8}	2.56×10^{-9}	311	534
	2PG	55	0	1.31×10^{-8}	2.24×10^{-9}	331	245
	M2PG	55	0	6.58×10^{-8}	4.97×10^{-9}	261	537
	M2PF	51	0	2.21×10^{-8}	2.69×10^{-9}	257	450
10^{-8}	2PF	104	0	1.50×10^{-10}	3.94×10^{-11}	633	1207
	2PG	115	0	1.32×10^{-10}	3.47×10^{-11}	677	471
	M2PG	115	0	2.19×10^{-10}	3.34×10^{-11}	583	1210
	M2PF	105	0	1.75×10^{-10}	4.56×10^{-11}	471	1017
10^{-10}	2PF	236	0	1.97×10^{-12}	4.18×10^{-13}	1417	2593
	2PG	261	0	1.29×10^{-12}	3.03×10^{-13}	1537	1066
	M2PG	261	0	1.40×10^{-12}	3.08×10^{-13}	1359	2769
	M2PF	236	0	2.17×10^{-12}	4.54×10^{-13}	1007	2055

and r is the number of iteration. The numerical results are tabulated in Tables 1-4. The results of Function called and execution time for the tested problems are also indicated in the histograms and graph lines in Fig 3 and 4, respectively.

Table 2: 2PF, 2PG, M2PG and M2PF methods for solving problem 2,τ=0.8

TOL	MTD	TS	FS	MAXE	AVER	FN	TIME
10^{-2}	2PF	20	0	3.02×10^{-4}	6.19×10^{-5}	121	864
	2PG	22	0	3.76×10^{-4}	9.16×10^{-5}	127	569
	M2PG	22	0	1.58×10^{-4}	4.45×10^{-5}	97	813
	M2PF	20	0	4.94×10^{-4}	5.25×10^{-5}	115	778
10^{-4}	2PF	34	0	2.12×10^{-6}	1.382×10^{-6}	217	918
	2PG	39	0	1.74×10^{-6}	1.00×10^{-6}	229	620
	M2PG	39	0	3.19×10^{-6}	2.04×10^{-6}	169	870
	M2PF	34	0	3.04×10^{-6}	1.21×10^{-6}	181	848
10^{-6}	2PF	66	0	2.81×10^{-8}	2.49×10^{-8}	419	1588
	2PG	76	0	1.87×10^{-8}	1.48×10^{-8}	449	1206
	M2PG	76	0	2.84×10^{-8}	1.05×10^{-8}	353	1554
	M2PF	66	0	2.81×10^{-8}	2.40×10^{-8}	325	1396
10^{-8}	2PF	143	0	3.47×10^{-10}	3.43×10^{-10}	875	3669
	2PG	167	0	1.70×10^{-10}	1.66×10^{-10}	987	2626
	M2PG	167	0	1.42×10^{-10}	6.93×10^{-11}	815	3725
	M2PF	143	0	3.44×10^{-10}	3.39×10^{-10}	659	3177
10^{-10}	2PF	331	0	3.46×10^{-12}	3.80×10^{-12}	1985	7512
	2PG	394	0	1.46×10^{-12}	1.60×10^{-12}	2335	6150
	M2PG	394	0	1.25×10^{-12}	6.76×10^{-13}	1949	8183
	M2PF	331	0	3.73×10^{-12}	3.96×10^{-12}	1413	6546

Table 3: 2PF, 2PG, M2PG and M2PF methods for solving problem 3,τ=0.8

TOL	MTD	TS	FS	MAXE	AVER	FN	TIME
10^{-2}	2PF	30	0	1.02×10^{-1}	2.23×10^{-2}	261	1512
	2PG	30	0	6.99×10^{-2}	1.64×10^{-2}	273	1081
	M2PG	30	0	2.16×10^{-1}	5.04×10^{-2}	229	1306
	M2PF	30	0	8.26×10^{-2}	2.22×10^{-2}	247	1498
10^{-4}	2PF	61	0	1.47×10^{-3}	3.87×10^{-4}	443	2029
	2PG	61	0	1.45×10^{-3}	3.79×10^{-4}	453	1560
	M2PG	61	0	1.64×10^{-3}	4.32×10^{-4}	439	1858
	M2PF	61	0	1.70×10^{-3}	4.47×10^{-4}	415	1959
10^{-6}	2PF	137	0	2.01×10^{-5}	6.01×10^{-6}	1039	4681
	2PG	139	0	1.92×10^{-5}	5.59×10^{-6}	1047	3620
	M2PG	139	0	1.78×10^{-5}	4.89×10^{-6}	801	3714
	M2PF	137	0	1.87×10^{-5}	5.57×10^{-6}	795	4006
10^{-8}	2PF	322	0	2.09×10^{-7}	6.36×10^{-8}	1907	9825
	2PG	327	0	1.96×10^{-7}	5.68×10^{-8}	1937	7076
	M2PG	327	0	1.95×10^{-7}	5.73×10^{-8}	1913	8965
	M2PF	322	0	1.80×10^{-7}	5.23×10^{-8}	1295	8249
10^{-10}	2PF	781	0	2.21×10^{-9}	6.99×10^{-10}	4657	23027
	2PG	797	0	2.09×10^{-9}	6.33×10^{-10}	4753	17455
	M2PG	797	0	2.05×10^{-9}	6.19×10^{-10}	4687	21766
	M2PF	781	0	2.11×10^{-9}	6.55×10^{-10}	3133	19022

Table 4: 2PF, 2PG, M2PG and M2PF methods for solving problem 4,τ=0.5

TOL	MTD	TS	FS	MAXE	AVER	FN	TIME
10^{-2}	2PF	40	0	3.18×10^{-4}	1.18×10^{-4}	235	1541
	2PG	50	0	1.70×10^{-4}	1.03×10^{-4}	231	1435
	M2PG	44	0	3.95×10^{-4}	1.47×10^{-4}	215	1741
	M2PF	47	0	3.33×10^{-4}	1.72×10^{-4}	209	1618
10^{-4}	2PF	74	0	2.39×10^{-6}	8.55×10^{-7}	383	2048
	2PG	79	0	3.73×10^{-6}	1.11×10^{-6}	393	1825
	M2PG	79	0	4.46×10^{-6}	1.71×10^{-6}	333	2221
	M2PF	75	0	4.58×10^{-6}	1.34×10^{-6}	335	2022
10^{-6}	2PF	159	0	1.97×10^{-8}	7.33×10^{-9}	827	4463
	2PG	179	0	1.83×10^{-8}	8.95×10^{-9}	869	4070
	M2PG	177	0	2.71×10^{-8}	1.58×10^{-8}	719	5223
	M2PF	157	0	3.96×10^{-8}	8.51×10^{-9}	653	4092
10^{-8}	2PF	370	0	1.61×10^{-10}	6.81×10^{-11}	1953	10205
	2PG	422	0	2.14×10^{-10}	9.63×10^{-11}	2051	9599
	M2PG	420	0	2.56×10^{-10}	1.68×10^{-10}	1687	11554
	M2PF	368	0	2.80×10^{-10}	4.50×10^{-11}	1489	9353
10^{-10}	2PF	900	0	1.45×10^{-12}	6.03×10^{-13}	4795	24956
	2PG	1027	0	1.80×10^{-12}	9.12×10^{-13}	5047	23428
	M2PG	1027	0	2.62×10^{-12}	1.78×10^{-12}	4115	28209
	M2PF	900	0	2.95×10^{-12}	2.75×10^{-13}	3607	22921

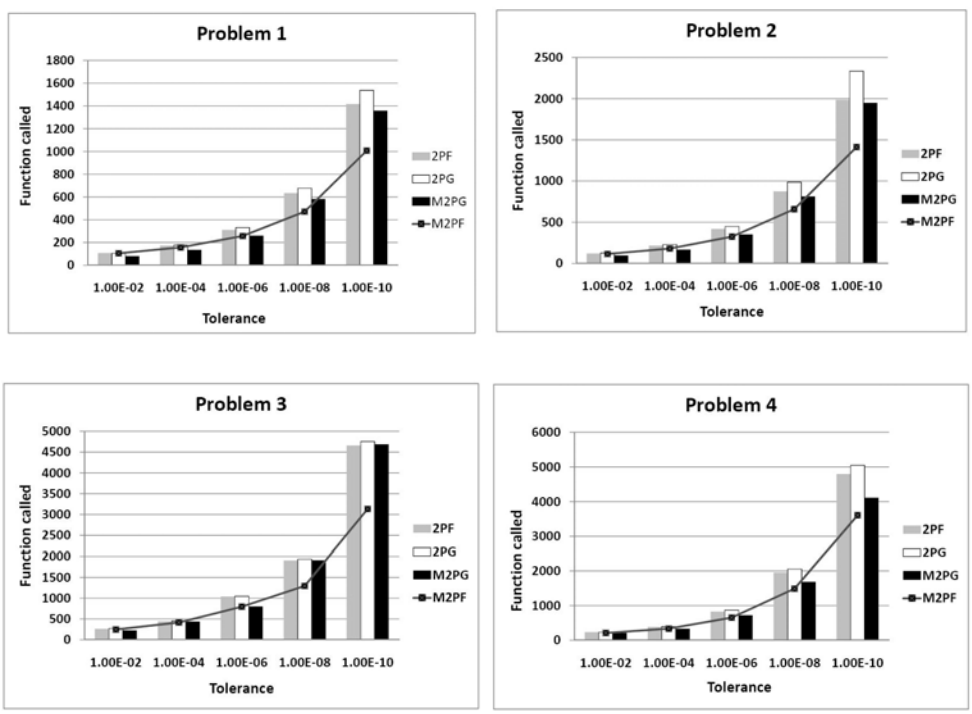

Figure 3: Results of function called for Problem 1-4

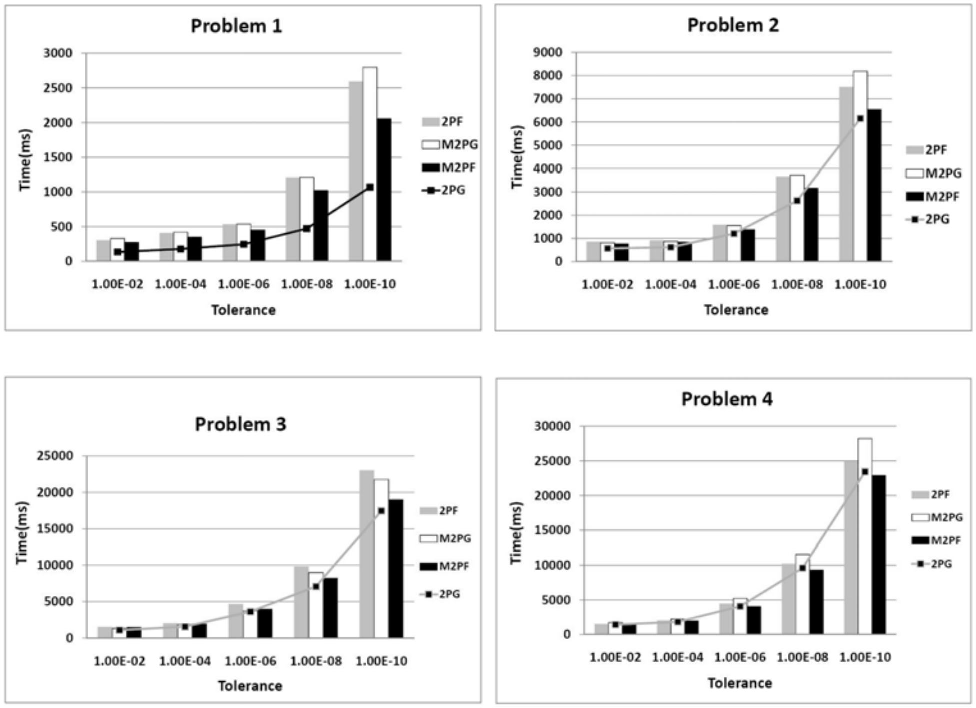

Figure 4: Results of execution time for Problem 1-4

From Tables (1-4), it can be observed that in all tested problems, the total number of steps and the maximum errors obtained by M2PF and M2PG methods, are comparable to those, obtained by 2PF and 2PG methods respectively. However, in Fig 3, it is obvious that, the number of function called taken by M2PF is less than that in other methods, specially for finer tolerance. This could be justified by the fact that, M2PF method needs less iteration for the solution to be convergent. Since in M2PF the order of predictor formula for second point is greater than that in 2PF and 2PG, so the convergence criteria will be satisfied and M2PF doesn't need more iterations. Also in M2PF method the value of y_n which has obtained its sufficient accuracy in previous block is involved for obtaining the predicted value for the second point in the current block. But in M2PG method the value of y_{n+1} which is in current block and it has not obtained its desired accuracy is involved. Therefore, the M2PG method needs more iterations.

In Fig 4, it can be seen that, the execution time of the 2PG method is faster than the other methods. Although M2PF method has less number of function called compared to other methods but the execution time is still expensive. The extra term in the predictor formula for the second point in M2PF method has affected the timing. In problem 4, the execution time provided by M2PF is faster than 2PG because of the big difference between the number of function called between M2PF and 2PG methods (especially for finer tolerance).

6 Conclusion

In this paper, we proposed three methods (2PG, M2PG, M2PF) for solving system of initial value problems (ODEs). After comparing the results of these methods with 2PF method [5] and with them selves as well, we can conclude that, each of these methods will give comparable results in terms of maximum error and total number of steps. But M2PF method has less number of function called compare to other methods, whereas the execution time of 2PG method is faster than the other methods.

References

[1] K. Burrage, *Efficient Block Predictor-Corrector Methods with a Small Number of Corrections*, J. of Comp. and App. Math., 45 (1993), 139-150.

[2] Fatunla, *Block Methods for Second Order ODEs*, Intern. J. Comp. Math. 41(1990), 55-63.

[3] E. Hairer, S.P. Norsett, G. Wanner, *Solving Ordinary Differential Equations I*, Springer-Verlag, Berlin, 1993.

[4] A.I. Johnson,J.R. Barney, *Numerical solution of large systems of stiff ordinary differential equations*, in:L. Lapidus,W.E. Schiesser(Eds.), Modular Simulation Framework, Numerical Methods for Differential Systems, Academic Press Inc, New York, 1976,pp.97-124.

[5] Z.A. Majid, M. Suleiman, F. Ismail, M. Othman *2-point 1 block Diagonally and 2-point 1 block Fully Implicit Methods for solving First Order Ordinary Differential Equations*, Proceedings of the 12^th National Symposium on Mathematical Sciences, No 34.

[6] W.E. Milne, *Numerical Solution of Differential Equations*, Wiley, New York, 1953.

[7] J.B. Rosser, *A Runge-Kutta for all seasons*, SIAM Rev. 9 (1967), 417-452.

[8] L.F. Shampine and H. A. Watts, *Block implicit one-step methods*, Math. comp. 23(1969),731-740 (1991)243-254.

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