

A Study of Firing Sidewise from an Airplane

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Abstract

The present study investigates the effect of the aerodynamic jump phenomenon on flat-fire free flight trajectory motion for projectiles launched horizontally from high-speed aircraft. The ammunition will be used has caliber .50 API M8 bullet type firing from M2 machine gun. The 6-DOF flight simulation modeling is applied for the free-trajectory prediction of spin-stabilized projectiles. This analysis includes constant coefficients of the most significant aerodynamic forces, moments and Magnus effects, which have been taken from official tabulated database, in addition to gravity acceleration.

Mathematics Subject Classification: 65Z05, 76G25

Keywords: Aerodynamic Jump, Trajectory Simulate, Magnus Effect

1 Introduction

External ballistics [9] deals with the behaviour of a non-powered projectile in flight. Many forces act upon the projectile during this phase including gravity, air resistance and air density. Pioneering English ballisticians Fowler, Gallop, Lock and Richmond [6] constructed the first rigid six-degree-of-freedom projectile exterior ballistic model. The trajectory deflection caused by aerodynamic jump on a .30 caliber machine gun bullet was first addressed in 1943 by

T. E. Sterne [10]. This reference was written from mathematicians who were tasked with generating firing tables for the machine guns, used sidewise fire from high speed airplanes. Various authors have simulated the aerodynamic jump phenomenon caused by aerodynamic asymmetry [3] and lateral force impulses [2]. The present work address a full six degree of freedom (6-DOF) projectile flight dynamics analysis for the accurate prediction of short and long trajectories of high spin-stabilized bullets firing sidewise from an aircraft. The proposed flight model takes into consideration the influence of the most significant forces and moments, based on appropriate constant mean values of the aerodynamic coefficients, in addition to wind and Magnus effects.

2 Projectile Model

The M8 API (armor piercing incendiary) was put into service in 1943 to replace the M1 Incendiary, and is still in service today. The M8 is built nearly identical to the M2 Armor Piercing except the M8 has 12 grains of incendiary mix in the nose instead of lead filler, and a lead caulking disc in the base acting as a seal. Having the same hardened steel core as the M2, the M8 matches the armor piercing capability of the M2 with the added advantage of incendiary effect. While it has considerably less incendiary mix than the M1, the performance of the M8 was greatly superior to the M1 because of it's ability to penetrate the target and ignite the material inside rather than just flash on the surface like the M1 often did, making for a greater first shot effect Pyrotechnic performance of these projectiles is only slightly less than the M1 Incendiary. The present analysis considers the type of representative flight bullet vehicle. Physical and geometric characteristics data of the above mentioned 12.7mm M8 API mm bullet are illustrated in Table 1:

Table 1. Physical and geometrical data of 12.7mm API M8 small bullet

Characteristics	12.7 mm API M8 Bullet
Reference diameter, mm	12.7
Total length, mm	13.84
Weight, kg	0.0419
Axial moment of inertia, kg/m^2	$0.784 \cdot 10^{-6}$
Transverse moment of inertia, kg/m^2	$0.738 \cdot 10^{-5}$

3 Trajectory Flight Simulation Model

A six degree of freedom rigid-bullet model [1], [4], [5], [8] has three rotations and three translations. The three translation components (x, y, z) describing

the position of the projectile's center of mass and the three Euler angles (ϕ , θ , ψ) describing the orientation of the projectile body with respect to Figure 1.

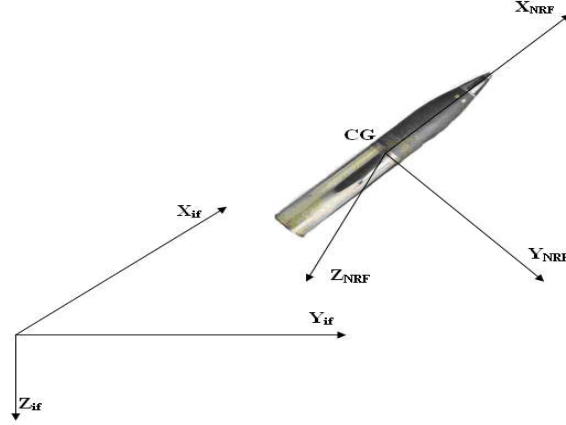


Figure 1: No-roll (moving) and fixed (inertial) coordinate systems for the projectile trajectory analysis

Two mean coordinate systems are used for the computational approach of an atmospheric flight motion. The one is a plane fixed (inertial frame, if) at the firing site. The other is a no-roll rotating coordinate system on the projectile body (no-roll-frame, NRF, $\phi = 0$) with X_{NRF} axis along the projectile axis of symmetry and Y_{NRF} , Z_{NRF} axes oriented so as to complete a right-hand orthogonal system. Newton's laws of the motion state that rate of change of linear momentum must equal the sum of all the externally applied forces and the rate of change of angular momentum must equal the sum of all the externally applied moments, respectively. The force acting on the projectile comprises the weight, the aerodynamic force and the Magnus force. The moment acting on the projectile comprises the moment due to the standard aerodynamic force, the Magnus aerodynamic moment and the unsteady aerodynamic moment. The twelve state variables x , y , z , ϕ , θ , ψ , \bar{u} , \bar{v} , \bar{w} , \bar{p} , \bar{q} and \bar{r} are necessary to describe position, flight direction and velocity at every point of the projectile's atmospheric trajectory. Introducing the components of the acting forces and moments expressed in the no-roll-frame ($\bar{*}$) with the dimensionless arc length l measured in calibers of travel, as an independent variable

$$l = \frac{1}{D}s = \frac{1}{D} \int_0^t V dt$$

we derive the following equations of motion for six-dimensional flight:

$$\bar{x}'_{if} = \frac{D}{V_T} \cos\psi \cos\theta \bar{u}_{NRF} - \frac{D}{V_T} \sin\psi \bar{v}_{NRF} + \frac{D}{V_T} \cos\psi \sin\theta \bar{w}_{NRF} \quad (1)$$

$$\bar{y}'_{if} = \frac{D}{V_T} \cos\theta \sin\psi \bar{u}_{NRF} + \frac{D}{V_T} \cos\psi \bar{v}_{NRF} + \frac{D}{V_T} \sin\psi \sin\theta \bar{w}_{NRF} \quad (2)$$

$$\bar{z}'_{if} = -\frac{D}{V_T} \sin\theta \bar{u}_{NRF} + \frac{D}{V_T} \cos\theta \bar{w}_{NRF} \quad (3)$$

$$\bar{\phi}' = \frac{D}{V_T} \bar{p}_{NRF} + \frac{D}{V_T} \tan\theta \bar{r}_{NRF} \quad (4)$$

$$\bar{\theta}' = \frac{D}{V_T} \bar{q}_{NRF} \quad (5)$$

$$\bar{\psi}' = \frac{D}{V_T \cos\theta} \bar{r}_{NRF} \quad (6)$$

$$\bar{u}'_{NRF} = -\frac{D}{V_T} g \sin\theta - L_1 V_T C_D + \bar{v}_{NRF} \frac{D}{V_T} \bar{r}_{NRF} - \bar{q}_{NRF} \frac{D}{V_T} \bar{w}_{NRF} \quad (7)$$

$$\bar{v}'_{NRF} = -L_1 (C_{L\alpha} + C_D) \bar{v}_{NRF} - \frac{D}{V_T} \bar{r}_{NRF} \bar{w}_{NRF} \tan\theta - \frac{D}{V_T} \bar{u}_{NRF} \bar{r}_{NRF} \quad (8)$$

$$\bar{w}'_{NRF} = \frac{D}{V_T} g \cos\theta - L_1 (C_{L\alpha} + C_D) \bar{w}_{NRF} + \bar{u}_{NRF} \frac{D}{V_T} \bar{q}_{NRF} + \tan\theta \frac{D}{V_T} \bar{r}_{NRF} \bar{v}_{NRF} \quad (9)$$

$$\bar{p}'_{NRF} = D^5 \frac{\pi}{16 I_{XX}} \bar{p}_{NRF} \rho C_{LP} \quad (10)$$

$$\begin{aligned} \bar{q}'_{NRF} &= 2L_2 (C_{L\alpha} + C_D) \bar{w}_{NRF} L_{CGCP} + D \frac{L_2}{V_T} C_{MPA} \bar{p}_{NRF} \bar{v}_{NRF} L_{CGCM} + \\ &+ D^2 L_2 C_{MQ} \bar{q}_{NRF} + 2DL_2 C_{MA} - \frac{D}{V_T} \bar{r}_{NRF} \frac{I_{XX}}{I_{YY}} \bar{p}_{NRF} \\ &- \frac{D}{V_T} \bar{r}_{NRF}^2 \tan\theta \end{aligned} \quad (11)$$

$$\begin{aligned} \bar{r}'_{NRF} &= -2L_2 (C_{L\alpha} + C_D) \bar{v}_{NRF} L_{CGCP} + D \frac{L_2}{V_T} C_{MPA} \bar{p}_{NRF} \bar{w}_{NRF} L_{CGCM} + \\ &+ D^2 L_2 C_{MQ} \bar{r}_{NRF} - 2DL_2 C_{MA} + \frac{D}{V_T} \bar{q}_{NRF} \frac{I_{XX}}{I_{YY}} \bar{p}_{NRF} \\ &+ \frac{D}{V_T} \bar{q}_{NRF} \bar{r}_{NRF} \tan\theta \end{aligned} \quad (12)$$

The projectile dynamics trajectory model consists of 12 first order non-linear ordinary differential equations, which are solved simultaneously by resorting to numerical integration using a 4th order Runge-Kutta method. In these equations, the following sets of simplifications are employed: the aerodynamic angles of attack α and sideslip β are small $\alpha \approx \bar{w}/V$, $\beta \approx \bar{v}/V$, the projectile is geometrically symmetrical $I_{XY} = I_{YZ} = I_{XZ} = 0$, $I_{YY} = I_{ZZ}$ and aerodynamically symmetric. With the afore-mentioned assumptions the expressions of the distance from the center of mass to both the standard aerodynamic and Magnus centers of pressure are simplified.

3.1 Aerodynamic model

For the projectile trajectory analysis, a constant flight dynamic model is proposed for the examined test cases. The above calculations are based on appropriate constant mean values of the experimental average aerodynamic coefficients variations taking from official tabulated database [9], as shown in Table 2.

Table 2. Aerodynamic parameters of atmospheric flight dynamic model

Aerodynamic Coefficients	12.7 mm Bullet
Drag C_D	0.63
Lift C_L	3.52
Magnus Moment C_{YPA}	0.27
Pitch Damping C_{MQ}	-6.6
Overturning Moment C_{MA}	2.6

3.2 Initial spin rate estimation

In order to have a statically stable flight projectile trajectory motion, the initial spin rate \bar{p}_0 prediction at the gun muzzle in the firing site is very important. According to McCoy definitions [9], the following form is used:

$$\bar{p}_0 = \frac{2\pi V_0}{\gamma D} (\text{rad/s}) \quad (13)$$

where V_0 is the initial firing velocity (m/s), γ the rifling twist rate at the $M2$ gun muzzle (calibers per turn), and D the reference diameter of the bullet type (m). Typical value of rifling γ is 29.41 calibers per turn for the 12.7 mm bullet.

4 Computational Simulation

The flight dynamic model of 12.7mm $M8$ API bullet type involves the solution of the set of the twelve nonlinear first order ordinary differential, Equations

(1 – 12), which are solved simultaneously by resorting to numerical integration using a 4th order Runge-Kutta method. Initial flight conditions for the dynamic trajectory bullet model with constant aerodynamic coefficients are illustrated in Table 3.

Table 3. Initial flight parameters of the bullet examined test cases

Initial flight data	12.7 mm M8 API Bullet
x,m	0.0
y,m	0.0
z,m	300,200,100
ϕ , deg	0.0
θ , deg	0.0
ψ , deg	0.0
u, m/s	928.0
v, m/s	0.0
w, m/s	0.0
p, rad/s	0.27
q, rad/s	0.0
r, rad/s	0.0

5 Aerodynamic Jump Deflection

For modern high-velocity, small yaw, flat-fire trajectories the tangent of the deflection angle due to aerodynamic jump phenomenon [2], [3], [6], [7] is given by the following expression:

$$A_J = -K_Y^2 \left(\frac{C_{LA}}{C_{MA}} \right) \left(iP\lambda_0 - \lambda_0' \right) \quad (14)$$

where λ_0 is the initial complex yaw (or sideslip) angle

$$\lambda_0 = i \sin \beta_0$$

and λ_0' the initial complex yaw rate or tip-off rate (rad/caliber). If a spinning projectile like the 12.7mm API M8 has an initial yaw λ_0 , but no significant λ_0' , equation (14) reduces to the simpler form:

$$A_J = -K_X^2 \left(\frac{C_{LA}}{C_{MA}} \right) \left(\frac{2\pi}{\gamma} \right) \sin \beta_0 \quad (15)$$

K_X^2 is the non-dimension axial moment of inertia, C_{LA} and C_{MA} are lift and overturning aerodynamic coefficients of 12.7mm bullet at the firing point, depending on the initial launch Mach number. Firing sidewise with V_{fir} relative to the helicopters flight path motion V_{hel} , the total initial muzzle velocity of the projectile V_0 is

$$V_0 = \sqrt{V_{fir}^2 + V_{hel}^2}$$

The directed angle β_0 , from the shifted velocity vector V_0 to the projectile's spin axis (firing line) is a pure sideslip (yaw) angle (zero pitch component) expressed in the form:

$$\sin\beta_0 = \left(\frac{V_{hel}}{V_0}\right) \sin\omega \quad (16)$$

Equation (15) combining with equation (16) gives the following proposed engineering correlation formula for the aerodynamic jump performance for flat-firing sidewise from low-speed helicopters:

$$A_J = -K_X^2 \left(\frac{C_{LA}}{C_{MA}}\right) \left(\frac{2\pi}{\gamma}\right) \left(\frac{V_{hel}}{V_0}\right) \sin\omega \quad (17)$$

The above equation (17) states, that if the physical, geometric and aerodynamic characteristics of the projectile and the initial muzzle firing and helicopter velocities remain constant, maximum A_J occurs when the projectile firing perpendicular ($\omega = 90^\circ$) relative to V_{hel} .

6 Results and Discussion

The flight path trajectory motions with constant aerodynamic coefficients of the 12.7mm bullet with initial firing velocity of 928 m/s, rifling twist 29.41 caliber per turn, are indicated in Figures 2 and 3. In Figure 2, the atmospheric model with constant aerodynamic and Magnus coefficients is applied for .50 caliber bullet firing at 300m, 200m, 100m height while the wind effects are neglected. The target impact point predictions of the above trajectories are 2,060m, 1,895m and 1,625m, respectively. Also the same are depicted in figure 3 for firing sidewise at 300m, 200m, 100m height with 10m/s crosswind where the corresponding impact points are 2,045m, 1,884m and 1,614m to helicopter's flight path direction. The bullet of 12.7mm diameter (Figure 4) is also examined the angle of attack for its atmospheric constant flight trajectory firing at different heights. The values for the angle of attack at 300m, 200m, 100m height are 2.67o, 2.1o, 1.04o, respectively. Many projectiles show significant nonlinear aerodynamic behavior, even at relatively small amplitude pitching and yawing motion. In the present analysis this synthesized projectile atmospheric motion is plotted in Figure 5 for the first 100m range, the .50 caliber projectile is firing with 899.16m/sec at 90 deg relative to airplane's flight speed of 231m/sec and the combined total muzzle velocity is almost 928m/s. The produced initial yaw (or sideslip) angle is approximately 14.41 degrees. The first maximum yaw angle is 17 degrees and at the 100m range has damped

to 9 degrees, and it is still damping. The aerodynamic jump at range of 100m produces a deflection of almost 82cm, as indicated in Figure 6.

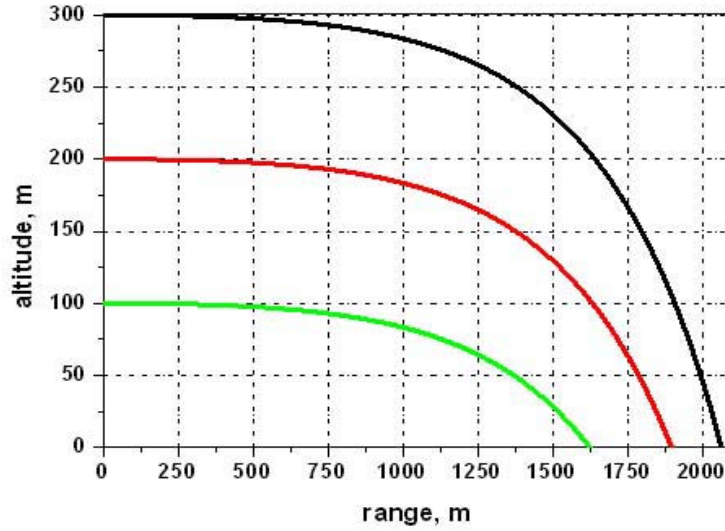


Figure 2: Impact points with constant aerodynamic coefficients for 12.7mm bullet

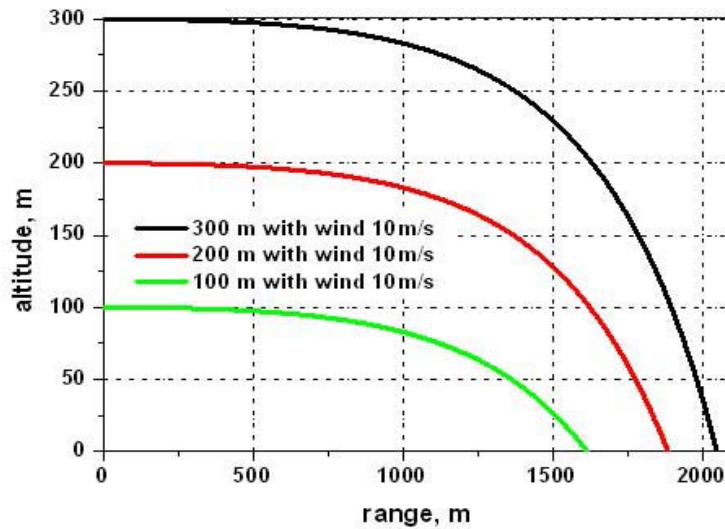


Figure 3: Flight path trajectories with constant aerodynamic coefficients with crosswind 10m/s for 12.7mm bullet

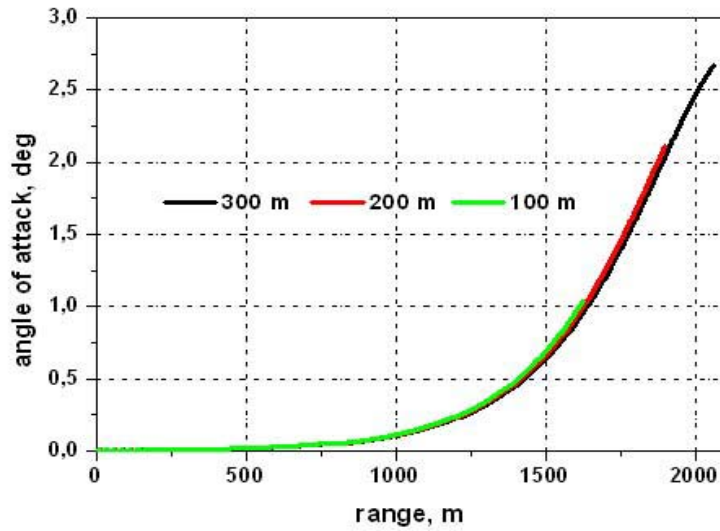


Figure 4: Angle of attack versus range with constant aerodynamic coefficients for 12.7mm bullet

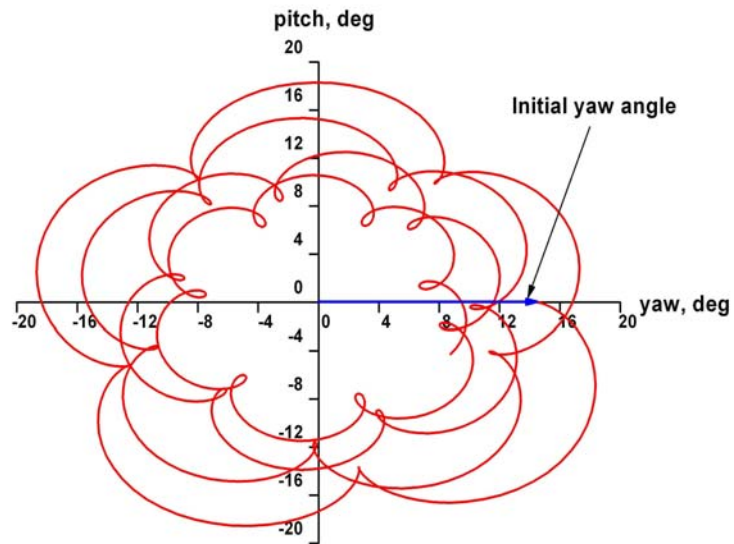


Figure 5: Coupled pitching and yawing motion for the .50 API M8 bullet fired sidewise (perpendicular, 90 degrees) from an aircraft path motion flying at 231 m/sec

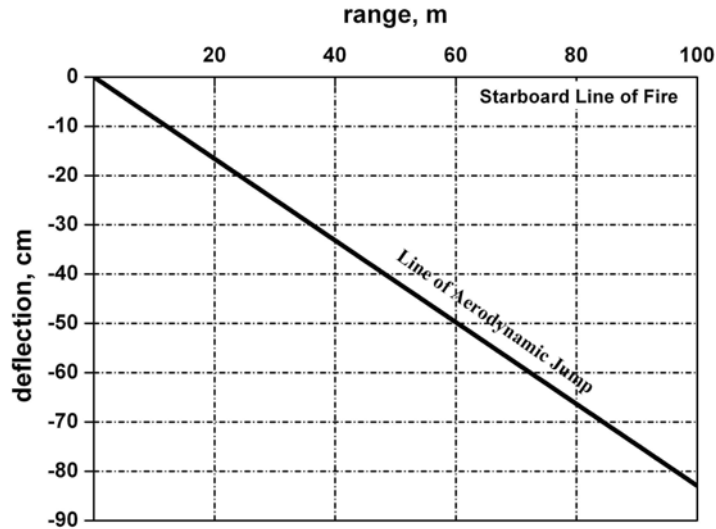


Figure 6: Deflection versus range with constant aerodynamic coefficients for 12.7mm bullet

7 Conclusions

The six degrees of freedom (6-DOF) simulation flight dynamics model is applied for the prediction of spin-stabilized projectiles firing sidewise from high-speed airplanes. Constant aerodynamic force and moment coefficients based on Mach number variations and total angle of attack effects are included for the 12.7mm bullet trajectory simulations. The aerodynamic jump has the most important effect caused by the pitching and yawing motion and investigated more closely.

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NOMENCLATURE

- A_J = aerodynamic jump
- C_D = drag force aerodynamic coefficient
- C_{La} = lift force aerodynamic coefficient
- C_{LP} = roll damping moment aerodynamic coefficient
- C_{MQ} = pitch damping moment aerodynamic coefficient
- C_{MA} = overturning moment aerodynamic coefficient
- C_{MPA} = Magnus moment aerodynamic coefficient
- D = projectile reference diameter, m
- g = sea – level acceleration gravity, $9.80665m/s^2$
- ρ = density of air, kg/m^3
- ϕ, θ, ψ = projectile roll, pitch and yaw angles, respectively
- α, β = aerodynamic angles of attack and sideslip
- L_{CGCM} = distance from the center of mass (CG) to the Magnus center of pressure (CM)

L_{CGCP} = distance from the center of mass (CG) to the aerodynamic center of pressure (CM)

L_1, L_2 = dimensional coefficients, $\frac{\pi\rho D^3}{8m}$ and $\frac{\pi\rho D^3}{16I_{YY}}$, respectively