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Confidence Interval for the Difference of Two Normal Population Means with a Known Ratio of Variances

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Abstract

Motivated by the recent work of Schechtman and Sherman [The two-sample t-test with a known ratio of variances. Statistical Methodology 4, 508-514, (2007).], we investigate, in this paper, a new exact confidence interval for the difference between two normal population means when the ratio of their variances is known. This is an extension of the case of equal variances where a confidence interval is constructed using an exact t-distribution, as opposed to the case of unequal variances with an *approximate* confidence interval. We derived analytic expressions to find the coverage probabilities and expected lengths of two confidence intervals, the Schechtman-Sherman confidence interval and the Welch-Satterthwaite confidence interval, in comparison with each other. Monte Carlo simulation results indicate that the new confidence interval for the difference between two normal means gives a better coverage probability (and a shorter expected length) than that of the well-known Welch-Satterthewaite confidence interval when a known ratio of their variances is large.

Mathematics Subject Classification: 62F25

Keywords: Coverage probability; Expected length; Welch-Satterthewaite confidence interval

1 Introduction

In this paper, we re-examine confidence intervals for the difference of two normal population means. Classically, when population variances are equal, a confidence interval based on the t-distribution with pooled sample variances is appropriate; otherwise, the Welch-Satterthwaite (WS, hereafter) confidence interval is preferable, see e.g. Satterthwaite (1941, 1946), Welch (1938). Miao and Chiou (2008) mentioned that the WS confidence interval also performs well, based on its coverage probability, in case of two population variances are equal. A confidence interval following the result of a preliminary F-test, that the population variances are equal, has been also examined recently, see e.g. Gans (1981) and Kabaila (2005) and references therein. This confidence interval is constructed using the preliminary F-test that is calculated first to motivate the prior belief (thought not certain) that the population variance are equal, followed by the choice between a confidence interval based on the pooled estimate of the common variance and the WS confidence interval. Confidence intervals for the difference of two means when both normality and equal variances assumptions may be violated are also considered in Miao and Chiou (2008) and references therein. These authors considered three confidence intervals; WS confidence interval and two adaptive confidence intervals. They used two pretests; Shapiro-Wilk test (Shapiro and Wilk (1965)) for normality test and a t-test for symmetry of distributions based on Miao et al. (2006). For normality pretest they suggested to use the WS confidence interval for the difference of two means when data is from normal distribution; otherwise, they suggested to transform data into the scale of logarithm, then apply the WS confidence interval to the log-transformed data and finally transform the interval back to the original scale. A confidence interval following the resulting of the symmetric pretest statistics t-test is constructed similarly. In addition, they suggested that the confidence interval following the resulting of the preliminary t-test outperforms other confidence intervals when data are from non-normal distributions.

Schechtman and Sherman (2007) described " a situation of a known ratio of variances arises in practice when two instruments reports (averaged) response of the same object based on a difference number of replicates. If the two instruments have the same precision for a single measurement, then the ratio of the variance of the responses is known, and it is simply the ratio of the number of replicates going into each response." They proposed a t-test statistic, which has an exact t-distribution with $n+m-2$ degrees of freedom, compared to the Satterthwaite's t-test statistic. They found that their proposed test has more power than an existing Satterthwaite's test. However, they did not investigate the coverage probability and the expected length of the confidence interval for the difference of two normal population means when the ratio of variances is known.

Our aim in this short paper is therefore to propose a new confidence interval for the difference of two normal population means when we know the ratio of two population variances. As in Niwitpong and Niwitpong (2008), we derive the coverage probability and its expected length of the new confidence interval compared to the well known WS confidence interval. Typically, confidence interval with a minimum coverage probability $1-\alpha$ and a shorter expected length is preferable.

The paper is organized as follows. Section 2 presents confidence intervals for difference of two normal population means. Coverage probabilities and expected lengths of confidence intervals in Section 2 are derived in Section 3. Section 4 gives simulation results of coverage probabilities and ratio of expected lengths of confidence intervals for difference of two normal population means for selected sample sizes with a range of values of known ratio of variances. Section 5 contains a discussion of the results and conclusions.

2 Confidence intervals for the difference of two normal population means

Let $X_1,\ldots,X_n$ and $Y_1,\ldots,Y_m$ be random samples form two independent normal distributions with means μ_x, μ_y and standard deviations σ_x and σ_y, respectively. The sample means and variances for X and Y are also denoted as $\bar{X}$, $\bar{Y}$, S_x^2 and S_y^2, respectively. We are interested in 100(1-α)% confidence interval for $\mu=\mu_x-\mu_y$ when we know the ratio of variances, say, $\sigma_y^2/\sigma_x^2=c$, where $c\geq 1$.

2.1 The Confidence interval for μ based on pooled estimate of variances and Welch-Satterthwaite methods

When it is assumed that $\sigma_x^2=\sigma_y^2$, it is well-known that, by using the pivotal quantity T_1 which is

$$T_1=\frac{(\bar{X}-\bar{Y})-(\mu_x-\mu_y)}{S_p\sqrt{\frac{1}{n}+\frac{1}{m}}},$$

the 100(1-α)% confidence interval for μ is

$$CI_1=\left[(\bar{X}-\bar{Y})-t_{1-\alpha/2,n+m-2}S_p\sqrt{\frac{1}{n}+\frac{1}{m}},(\bar{X}-\bar{Y})+t_{1-\alpha/2,n+m-2}S_p\sqrt{\frac{1}{n}+\frac{1}{m}}\right]$$

where

$$S_p^2=\frac{(n-1)S_x^2+(m-1)S_y^2}{n+m-2},$$

$S_x^2 = (n-1)^{-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$, $S_y^2 = (m-1)^{-1}\sum_{i=1}^{m}(Y_i - \bar{Y})^2$ and $t_{1-\alpha/2}$ is the $(1-\alpha/2)th$ percentile of t-distribution with $n+m-2$ degrees of freedom. Note that T_1 has an exact t-distribution with $n+m-2$ degrees of freedom .

In the case where the two variances differ, i.e. $\sigma_x^2 \neq \sigma_y^2$, the confidence interval for μ is constructed using the pivotal quantity T_2,

$$T_2 = \frac{(\bar{X}-\bar{Y}) - (\mu_x - \mu_y)}{\sqrt{\frac{S_x^2}{n} + \frac{S_y^2}{m}}}.$$

It is well-known that T_2 is *approximately* distributed as a t-distribution with degrees of freedom equal

$$df = \frac{(A+B)^2}{\frac{A^2}{n-1} + \frac{B^2}{m-1}}, A = \frac{S_x^2}{n}, B = \frac{S_y^2}{m}.$$

An approximate 100(1-α)% confidence interval for μ is therefore

$$CI_2 = \left[(\bar{X}-\bar{Y}) - t_{1-\alpha/2,df}\sqrt{\frac{S_x^2}{n} + \frac{S_y^2}{m}}, (\bar{X}-\bar{Y}) + t_{1-\alpha/2,df}\sqrt{\frac{S_x^2}{n} + \frac{S_y^2}{m}}\right]$$

where $t_{1-\alpha/2,df}$ is the $(1-\alpha/2)th$ percentile of T_2 distribution with degrees of freedom df.

The confidence interval CI_2 is known as the WS confidence interval, see e.g. Miao and Chiou (2008).

2.2 Confidence interval for μ with a known ratio of variances

Schechtman and Sherman (2007) proposed the test statistic T_3 (below), based on Sprott and Farewell (1993), of the hypothesis $H_0 : \mu_x - \mu_y = \Delta_0$ against the alternative hypothesis $H_A : \mu_x - \mu_y \neq (<,>)\Delta_0$. By means of Monte Carlo simulation, they found that the test statistic T_3 has better power than the test statistics T_2. However, they have not studied the confidence interval for μ using the pivotal test statistic T_3. Although the correspondence between hypothesis testing, $H_0 : \mu_x - \mu_y = \Delta_0$ against the alternative hypothesis $H_A : \mu_x - \mu_y \neq (<,>)\Delta_0$, and the confidence interval estimation had been well documented, it is worth to derive explicit expressions for the coverage probability and the expected length of the confidence interval for μ with a known ratio of variances.

As a result, it is of interest to construct the confidence interval for μ when we know the ratio of variances. The proposed confidence interval is constructed using the pivotal quantity,

$$T_3 = \frac{(\bar{X}-\bar{Y}) - (\mu_x - \mu_y)}{\tilde{S}_p\sqrt{\frac{1}{n} + \frac{c}{m}}}$$

where

$$\tilde{S}_p^2 = \frac{(n-1)S_X^2 + (m-1)S_{\tilde{y}}^2}{n+m-2}, S_{\tilde{y}}^2 = (m-1)^{-1}\sum_{i=1}^{m}\left(\tilde{Y}_i - \bar{Y}^*\right)^2,$$

$\tilde{Y}_i = Y_i/\sqrt{c}, (i = 1, 2, \ldots, m)$, $c \geq 1$ and $\bar{Y}^*$ is the sample mean of $\tilde{Y}_i$, $(i = 1, 2, \ldots, m)$. Schechtman and Sherman (2007) pointed out that the test statistic T_3 is an exact t-distribution with $n+m-2$ degrees of freedom. We now propose the new confidence interval for μ when a ratio of variances is known using the pivotal quantity T_3.

We choose the $t_{1-\alpha/2,n+m-2}$, which is the $(1-\alpha/2)th$ percentile of the t distribution with $n+m-2$ degrees of freedom, such that

$$1-\alpha = Pr\Big[-t_{1-\alpha/2,n+m-2} < T_3 < t_{1-\alpha/2,n+m-2}\Big].$$

It is easy to see that $100(1-\alpha/2)\%$ confidence interval for μ is

$$CI_3 = \left[(\bar{X}-\bar{Y}) - t_{1-\alpha/2,n+m-2}\tilde{S}_p\sqrt{\frac{1}{n}+\frac{c}{m}}, (\bar{X}-\bar{Y}) + t_{1-\alpha/2,n+m-2}\tilde{S}_p\sqrt{\frac{1}{n}+\frac{c}{m}}\right].$$

We evaluate these confidence intervals i.e. CI_2 and CI_3 using their coverage probabilities and expected lengths which are derived in the next section. We prefer a confidence interval with minimum coverage probability equal to a pre-specified value $1-\alpha$ and with a shorter expected length.

3 Coverage probabilities and expected lengths of confidence intervals for μ with a known ratio of variances

Theorems 1-2, below, show explicit expressions for the coverage probabilities and the expected lengths of confidence intervals CI_2 and CI_3 respectively.

Theorem 1. The coverage probability and the expected length of CI_2 when we know the ratio of variances, $\sigma_y^2/\sigma_x^2 = c$, are respectively

$$E[\Phi(W)-\Phi(-W)] \text{ and } \begin{cases} 2d\sigma_x(nm)^{-1/2}\delta\sqrt{r_1}F\left[\frac{-1}{2}, \frac{m-1}{2}, \frac{m+n-2}{2}; \frac{r_1-r_2}{r_1}\right] & \text{if } r_2 < 2r_1 \\ 2d\sigma_x(nm)^{-1/2}\delta\sqrt{r_2}F\left[\frac{-1}{2}, \frac{n-1}{2}, \frac{m+n-2}{2}; \frac{r_2-r_1}{r_2}\right] & \text{if } 2r_1 \leq r_2 \end{cases}$$

where $W = d\sigma_x^{-1}\sqrt{\frac{mS_x^2+nS_y^2}{m+cn}}$, $d = t_{1-\alpha/2,df}$, $\delta = \frac{\sqrt{2}\Gamma(\frac{m+n-1}{2})}{\Gamma(\frac{m+n-2}{2})}$, $r_1 = \frac{m}{n-1}$, $r_2 = \frac{cn}{m-1}$, $E(\cdot)$ is an expectation operator, $F(a, b, c; k)$ is the hypergeometric function defined by $F(a,b,c;k) = 1 + \frac{ab}{c}\frac{k}{1!} + \frac{a(a+1)b(b+1)}{c(c+1)}\frac{k^2}{2!} + ...$ where $|k| < 1$, see Press (1966), $\Gamma[\cdot]$ is the gamma function and $\Phi[\cdot]$ is the cumulative distribution function of $N(0,1)$.

Proof. Since, for normal samples, $\bar{X}$, $\bar{Y}$, S_x^2 and S_y^2 are independent of one another. From CI_2, we have

$$\begin{aligned}
1-\alpha &= P\left[(\bar{X}-\bar{Y}) - d\sqrt{\frac{S_x^2}{n}+\frac{S_y^2}{m}} < \mu_x - \mu_y < (\bar{X}-\bar{Y}) + d\sqrt{\frac{S_x^2}{n}+\frac{S_y^2}{m}}\right] \\
&= P\left[\frac{-d\sqrt{\frac{S_x^2}{n}+\frac{S_y^2}{m}}}{\sigma_x\sqrt{n^{-1}+cm^{-1}}} < \frac{(\mu_x-\mu_y)-(\bar{X}-\bar{Y})}{\sigma_x\sqrt{n^{-1}+cm^{-1}}} < \frac{d\sqrt{\frac{S_x^2}{n}+\frac{S_y^2}{m}}}{\sigma_x\sqrt{n^{-1}+cm^{-1}}}\right] \\
&= P\left[\frac{-d\sqrt{\frac{S_x^2}{n}+\frac{S_y^2}{m}}}{\sigma_x\sqrt{n^{-1}+cm^{-1}}} < Z < \frac{d\sqrt{\frac{S_x^2}{n}+\frac{S_y^2}{m}}}{\sigma_x\sqrt{n^{-1}+cm^{-1}}}\right] \\
&= E[I_{\{-W<Z<W\}}(\xi)], I_{\{-W<Z<W\}}(\xi) = \begin{cases} 1, & \text{if } \xi \in \{-W<Z<W\} \\ 0, & \text{otherwise} \end{cases} \\
&= E[E[I_{\{-W<Z<W\}}(\xi)]|S], S = (S_x^2, S_y^2)' \\
&= E[\Phi(W) - \Phi(-W)]
\end{aligned}$$

where $Z \sim N(0,1)$.

The length of CI_2, L_{CI_2}, is $2d\sqrt{\frac{S_x^2}{n}+\frac{S_y^2}{m}}$ and the expected length of L_{CI_2} is

$$\begin{aligned}
2dE\left[\sqrt{\frac{mS_x^2+nS_y^2}{nm}}\right] &= 2d\sigma_x(nm)^{-1/2}E\left[\sqrt{\frac{mS_x^2+nS_y^2}{\sigma_x^2}}\right] \\
&= 2d\sigma_x(nm)^{-1/2}E\left[\sqrt{\frac{\left(\frac{m}{n-1}\right)(n-1)S_x^2}{\sigma_x^2} + \frac{c\left(\frac{n}{m-1}\right)(m-1)S_y^2}{c\sigma_x^2}}\right] \\
&= 2d\sigma_x(nm)^{-1/2}E[\sqrt{r_1Z_1+r_2Z_2}] \\
&= \begin{cases} 2d\sigma_x(nm)^{-1/2}\delta\sqrt{r_1}F\left[\frac{-1}{2}, \frac{m-1}{2}, \frac{m+n-2}{2}; \frac{r_1-r_2}{r_1}\right] & \text{if } r_2 < 2r_1 \\ 2d\sigma_x(nm)^{-1/2}\delta\sqrt{r_2}F\left[\frac{-1}{2}, \frac{n-1}{2}, \frac{m+n-2}{2}; \frac{r_2-r_1}{r_2}\right] & \text{if } 2r_1 \le r_2 \end{cases}
\end{aligned}$$

where $Z_1 = \frac{(n-1)S_x^2}{\sigma_x^2} \sim \chi^2_{n-1}, Z_2 = \frac{(m-1)S_y^2}{\sigma_y^2} \sim \chi^2_{m-1}$ and for more details of $E[\sqrt{r_1Z_1+r_2Z_2}]$, see Press (1966, pp. 456-458). Thus we complete the proof.

Theorem 2. The coverage probability and the expected length of CI_3 when we know the ratio of variances, $\sigma_y^2 = c\sigma_x^2$, are respectively

$$E[\Phi(W_1) - \Phi(-W_1)] \text{ and } 2^{3/2}d_1\sigma_x\sqrt{\frac{m+nc}{nm(n+m-2)}}\frac{\Gamma(\frac{n+m-1}{2})}{\Gamma(\frac{n+m-2}{2})}$$

where $W_1 = d_1\sigma_x^{-1}\tilde{S}_p$, $d_1 = t_{1-\alpha/2,n+m-2}$ and $\Phi[\cdot]$ is the cumulative distribution function of $N(0,1)$.

Proof. From CI_3, we have

$$\begin{aligned}
1-\alpha &= P\left[(\bar{X}-\bar{Y}) - d_1\tilde{S}_p\sqrt{\frac{1}{n}+\frac{c}{m}} < \mu_x - \mu_y < (\bar{X}-\bar{Y}) + d_1\tilde{S}_p\sqrt{\frac{1}{n}+\frac{c}{m}}\right] \\
&= P\left[\frac{-d_1\tilde{S}_p\sqrt{\frac{1}{n}+\frac{c}{m}}}{\sigma_x\sqrt{n^{-1}+cm^{-1}}} < \frac{(\mu_x-\mu_y)-(\bar{X}-\bar{Y})}{\sigma_x\sqrt{n^{-1}+cm^{-1}}} < \frac{d_1\tilde{S}_p\sqrt{\frac{1}{n}+\frac{c}{m}}}{\sigma_x\sqrt{n^{-1}+cm^{-1}}}\right] \\
&= E[I_{\{-W_1<Z<W_1\}}(\tau)], I_{\{-W_1<Z<W_1\}}(\tau) = \begin{cases} 1, & \text{if } \tau \in \{-W_1 < Z < W_1\} \\ 0, & \text{otherwise} \end{cases} \\
&= E[E[I_{\{-W_1<Z<W_1\}}(\tau)]|\tilde{S}_p^2] \\
&= E[\Phi(W_1) - \Phi(-W_1)]
\end{aligned}$$

where $Z \sim N(0,1)$.

The length of CI_3, L_{CI_3}, is $2d_1\tilde{S}_p\sqrt{\frac{1}{n}+\frac{c}{m}}$ and the expected length of L_{CI_3} is

$$\begin{aligned}
2d_1\sqrt{\frac{1}{n}+\frac{c}{m}}E[\tilde{S}_p] &= 2d_1\sigma_x\sqrt{\frac{1}{n}+\frac{c}{m}}\sqrt{\frac{1}{n+m-2}}E\left[\sqrt{\frac{(n-1)S_x^2+(m-1)S_{\tilde{y}}^2}{\sigma_x^2}}\right] \\
&= 2d_1\sigma_x\sqrt{\frac{1}{n}+\frac{c}{m}}\sqrt{\frac{1}{n+m-2}}E\left[\sqrt{\frac{(n-1)S_x^2}{\sigma_x^2}+\frac{(m-1)S_y^2}{c\sigma_x^2}}\right] \\
&= 2d_1\sigma_x\sqrt{\frac{1}{n}+\frac{c}{m}}\sqrt{\frac{1}{n+m-2}}E(\sqrt{V}) \\
&= 2^{3/2}d_1\sigma_x\sqrt{\frac{1}{n}+\frac{c}{m}}\sqrt{\frac{1}{n+m-2}}\frac{\Gamma(\frac{n+m-1}{2}))}{\Gamma(\frac{n+m-2}{2})}
\end{aligned}$$

where $V \sim \chi^2_{n+m-2}$ and $\mathrm{E}(\sqrt{V}) = \frac{2^{1/2}\Gamma(\frac{1}{2}+\frac{n+m-2}{2})}{\Gamma(\frac{n+m-2}{2})}$, see Casella and Berger (1990). Thus we complete the proof.

4 Simulation results

In this section, we compare confidence intervals for μ via Monte Carlo simulation, using functions written in R, in variety of situations to see how coverage probabilities and expected lengths of confidence intervals CI_2 and CI_3, see Theorems 1-2, may depend on sample sizes and on the ratio of the variances. Our simulation experiments are as follows:

Setup 1. $n = 20$ and $m = 5$,

Setup 2. $n = 20$ and $m = 10$,
Setup 3. $n = 20$ and $m = 20$,
Setup 4. $n = 40$ and $m = 20$,
Setup 5. $n = 40$ and $m = 40$.

Following Schechtman and Sherman (2007), we chose $\sigma_y^2/\sigma_x^2 = c = 1, 2, 4, 8$, $\mu_x - \mu_y = \triangle$, $\triangle = 0, 1, 2, 3$ and $\sigma_x^2 = 1$. We compare confidence intervals of CI_2 and CI_3 based on their coverage probabilities and expected lengths, with a nominal value of 0.95 throughout. Comparison of coverage probabilities of above intervals, using Theorems 1-2, for Setup 1 - Setup 5, based on 100,000 simulations, are given in Tables 1-3. The ratio of expected lengths for each intervals, using Theorems 1-2, are also given in Tables 1-3. Note that our proposed confidence interval CI_3 reduces to the pooled estimate of variances, confidence interval CI_1 when $c = 1$. From Tables 1-3, coverage probabilities of our proposed confidence interval, CI_3 are equal or above the nominal value of 0.95. In addition, for $n = m$, CI_2 is slightly better than CI_3 when $c \leq 2$, while CI_3 is better than when $c > 2$, in term of coverage probability. A ratio of expected lengths, $E($ lengtht of $CI_2)/E($ lengtht of $CI_3)$ given in Theorems 1-2, is estimated using Monte Carlo simulation. Results of this ratio of expected lengths of each intervals in Tables 1-3 show that our proposed confidence interval CI_3 has shorter expected lengths than that of WS confidence interval CI_2 for every case of $\triangle$ and the value $c \geq 1$, especially for case $m = 20, n = 5$ and for every case of $\triangle$ and the values of $c \geq 1$. Additionally, for large value of c, i.e. c=8, the expected length of the confidence interval CI_3 is far shorter than the expected length of the confidence interval CI_2 for every sample sizes of n and m except $n = m$. Furthermore, it is straightforward, from Theorems 1-2, to see that for $n = m$ and $c = 1$, both confidence intervals have the same length.

5 Conclusion

We proposed, in this paper, the confidence interval for the difference of two normal population means when a ratio of variances is known. As in Schechtman and Sherman (2007), our proposed confidence interval, constructed using the pivotal quantity T_3, has more data from the second sample, while the WS confidence interval did not use the fact that a ratio of variances is known. Therefore our proposed interval performs well in term of its coverage probability, for $c > 3$, and its expected length. In other words, for most cases, coverage probabilities of CI_3 are equal or above the nominal value of 0.95 and this confidence interval has also a shorter confidence interval compared to the WS confidence interval for $n > m$. These results are similar to those results of Niwitpong and Niwitpong (2008) who constructed prediction intervals for the difference of two normal sample means with a known ratio of variances. Based on our Monte Carlo results, we recommend our proposed confidence interval, CI_3, for the difference of two normal population means with a known ratio of variances, i.e. $c \geq 1$.

We also note here that there is no need to use the preliminary test F-test in comparing two sample means, see e.g. Gans (1981) and Kabaila (2005), for this case since we assume that the ratio of variances is known.

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Table 1: The coverage probabilities of CI_2, CI_3 and $E(CI_2)/E(CI_3)$ and the number of simulation runs = 100,000.

n	m	△	c	CI_2	CI_3	$E(CI_2)/E(CI_3)$
20	5	0	1	0.94411	0.94996	1.13853
			2	0.94293	0.94998	1.16563
			4	0.94407	0.95024	1.18158
			8	0.94689	0.95016	1.19026
		1	1	0.94376	0.94988	1.13853
			2	0.94284	0.94996	1.16563
			4	0.94403	0.95005	1.18158
			8	0.94699	0.95008	1.19026
		2	1	0.94419	0.95010	1.13853
			2	0.9427	0.94982	1.16563
			4	0.94387	0.94982	1.18158
			8	0.94667	0.95002	1.19026
		3	1	0.94398	0.95004	1.13853
			2	0.94307	0.95016	1.16563
			4	0.94429	0.94995	1.18158
			8	0.94626	0.94985	1.19026
20	10	0	1	0.94963	0.94993	1.01746
			2	0.94921	0.95007	1.03294
			4	0.94873	0.95002	1.04475
			8	0.94878	0.95003	1.05210
		1	1	0.95003	0.95014	1.01746
			2	0.94907	0.94995	1.03294
			4	0.94887	0.95011	1.04475
			8	0.94913	0.94990	1.05210
		2	1	0.94971	0.94994	1.01746
			2	0.94892	0.94997	1.03294
			4	0.94902	0.95016	1.04475
			8	0.94892	0.95000	1.05210
		3	1	0.94980	0.95011	1.01746
			2	0.94893	0.94996	1.03294
			4	0.94895	0.95025	1.04475
			8	0.94871	0.95014	1.05210

Table 2: The coverage probabilities of CI_2, CI_3 and $E(CI_2)/E(CI_3)$ and the number of simulation runs = 100,000.

n	m	△	c	CI_2	CI_3	$E(CI_2)/E(CI_3)$
20	20	0	1	0.95043	0.95008	1.00000
			2	0.95010	0.94987	1.00089
			4	0.94993	0.95001	1.00439
			8	0.94989	0.94998	1.00817
		1	1	0.95044	0.95009	1.00000
			2	0.95027	0.95009	1.00089
			4	0.94995	0.95005	1.00439
			8	0.94981	0.94994	1.00817
		2	1	0.95018	0.94983	1.00000
			2	0.95015	0.94994	1.00089
			4	0.94996	0.95002	1.00439
			8	0.94967	0.95000	1.00817
		3	1	0.95026	0.94991	1.00000
			2	0.95028	0.95010	1.00089
			4	0.95002	0.95012	1.00439
			8	0.94989	0.95003	1.00817
40	20	0	1	0.94992	0.95005	1.01689
			2	0.94979	0.94999	1.03209
			4	0.94996	0.94992	1.04372
			8	0.94997	0.94995	1.05097
		1	1	0.94994	0.95001	1.01689
			2	0.94986	0.94997	1.03209
			4	0.94978	0.95012	1.04372
			8	0.95009	0.95017	1.05097
		2	1	0.95004	0.95010	1.01689
			2	0.94990	0.95007	1.03209
			4	0.94970	0.95002	1.04372
			8	0.94965	0.94992	1.05097
		3	1	0.94992	0.95010	1.01689
			2	0.94981	0.95004	1.03209
			4	0.94974	0.94992	1.04372
			8	0.94991	0.94997	1.05097

Table 3: The coverage probabilities of CI_2, CI_3 and $E(CI_2)/E(CI_3)$ and the number of simulation runs = 100,000.

	m	△	c	CI_2	CI_3	$E(CI_2)/E(CI_3)$
40	40	0	1	0.95011	0.95002	1.00000
			2	0.95007	0.95003	1.00111
			4	0.95005	0.95001	1.00495
			8	0.94994	0.94994	1.00896
		1	1	0.95005	0.94997	1.00000
			2	0.94991	0.94990	1.00111
			4	0.94995	0.94998	1.00495
			8	0.94992	0.94993	1.00896
		2	1	0.95008	0.95000	1.00000
			2	0.95004	0.95001	1.00116
			4	0.94990	0.94999	1.00495
			8	094998	0.95002	1.00896
		3	1	0.95005	0.94996	1.00000
			2	0.95004	0.95001	1.00111
			4	0.94999	0.95000	1.00495
			8	0.94989	0.95000	1.00896