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Some Solutions for a Class of Singular Equations of Even Order

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Abstract

We obtain all solutions which depend only on r for a class of iterated elliptic or ultrahyperbolic partial differential equations of even order with singular coefficient. Here, the essential operators include Laplace, wave, EPD (Euler-Poisson-Darboux) and GASPT (Generalized Axially Symmetric Potential Theory) operators.

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1 Introduction

In this paper, we consider a class of singular partial differential equations of the form

$$\left(\prod_{j=1}^{p} L_j^{q_j}\right) u = (L_1^{q_1} \dots L_p^{q_p})u = 0 \tag{1}$$

where p and $q_1, \dots, q_p$ are positive integers and

$$L_j = \sum_{i=1}^{n}\left(a_i^2 \frac{\partial^2}{\partial x_i^2} + \frac{\alpha_i^{(j)}}{x_i - x_i^0}\frac{\partial}{\partial x_i}\right) \pm \sum_{i=1}^{s}\left(b_i^2 \frac{\partial^2}{\partial y_i^2} + \frac{\beta_i^{(j)}}{y_i - y_i^0}\frac{\partial}{\partial y_i}\right) + \frac{\gamma_j}{r^2}. \tag{2}$$

The iterated operators $L_j^{q_j}$ are defined by the relations

$$L_j^k(u) = L_j\left[L_j^{k-1}(u)\right], \quad k = 1, \dots, q_j.$$

In (2), $a_i \neq 0$ $(i = 1, \dots, n)$, x_i^0 $(i = 1, \dots, n)$, $b_i \neq 0$ $(i = 1, \dots, s)$, y_i^0 $(i = 1, \dots, s)$ are real constants and $\alpha_i^{(j)}$ $(i = 1, \dots, n)$, $\beta_i^{(j)}$ $(i = 1, \dots, s)$, γ_j are real parameters and r is defined by

$$r^2 = \sum_{i=1}^{n}\left(\frac{x_i - x_i^0}{a_i}\right)^2 \pm \sum_{i=1}^{s}\left(\frac{y_i - y_i^0}{b_i}\right)^2. \tag{3}$$

The domain of each of the operator L_j is the set of all real valued functions $u(x, y)$ of class $C^2(\Omega)$ where $x = (x_1, \dots, x_n)$ and $y = (y_1, \dots, y_s)$ denote points in $\mathbb{R}^n$ and $\mathbb{R}^s$, respectively, and Ω is the regularity domain of u in $\mathbb{R}^{n+s}$. The operators L_j are elliptic or ultrahyperbolic with the sign positive or negative, respectively. Equation (1) includes iterated forms of some well known classical equations such as Laplace equation, wave equation, Euler-Poisson-Darboux (EPD) equation and Generalized Axially Symmetric Potential Theory (GASPT) equation as special cases.

Recently, r^m type solutions for various types of partial differential equations are studied by several authors [2-5]. We remark that Altın [1] obtained r^m type solutions for a class of partial differential equations which is a special case of (1) when $a_i = 1$ $(i = 1, \dots, n)$, $b_i = 1$ $(i = 1, \dots, s)$, $x_i^0 = 0$ $(i = 1, \dots, n)$, $y_i^0 = 0$ $(i = 1, \dots, s)$. The main object of this work is to extend the results derived by Altın [1] to solutions of the more general iterated equation (1).

2 r^m Type Solutions

Firstly, we will give the following lemmas.

Lemma 2.1 *For any real or complex parameter m,*

$$L_j(r^m) = F_j(m)\, r^{m-2} \tag{4}$$

where

$$2\psi_j = n + s - 2 + \sum_{i=1}^{n}\frac{\alpha_i^{(j)}}{a_i^2} + \sum_{i=1}^{s}\frac{\beta_i^{(j)}}{b_i^2} \tag{5}$$

and

$$F_j(m) = m(m + 2\psi_j) + \gamma_j. \tag{6}$$

Proof. The proof of this lemma can be done easily by applying the operator L_j to r^m.

Lemma 2.2 *For any real or complex parameter m,*

$$L_j^q\left(r^m\right)=\left\{\prod_{k=0}^{q-1}F_j\left(m-2k\right)\right\}r^{m-2q} \tag{7}$$

where the positive integer q is the iteration number.

Proof. We give the proof by induction on q. It is clear by (4) that the equality (7) is true for $q=1$. Now, let us assume that the equality is valid for $q-1$, that is,

$$L_j^{q-1}\left(r^m\right)=\left\{\prod_{k=0}^{q-2}F_j\left(m-2k\right)\right\}r^{m-2(q-1)}.$$

By applying the operator L_j to both sides of the above equality, we obtain

$$L_j^q\left(r^m\right)=\left\{\prod_{k=0}^{q-2}F_j\left(m-2k\right)\right\}L_j\big(r^{m-2(q-1)}\big).$$

Hence, by replacing m by $m-2\left(q-1\right)$ in (4), we get

$$\begin{aligned} L_j^q\left(r^m\right) &= \left\{\prod_{k=0}^{q-2}F_j\left(m-2k\right)\right\}F_j\big(m-2\left(q-1\right)\big)r^{m-2q} \\ &= \left\{\prod_{k=0}^{q-1}F_j\left(m-2k\right)\right\}r^{m-2q} \end{aligned}$$

which completes the proof.

Lemma 2.3 *For any positive integers $p, q_1, \ldots, q_p$*

$$\left(\prod_{j=1}^{p}L_j^{q_j}\right)\left(r^m\right)=\left\{\prod_{j=1}^{p}\prod_{k=0}^{q_j-1}F_j\big(m-2[Q\left(p\right)-Q\left(j\right)]-2k\big)\right\}r^{m-2Q(p)} \tag{8}$$

where $Q\left(j\right)=q_1+\cdots+q_j,\ j=1,\ldots,p$.

Proof. We give the proof induction on p. For any positive integer q_j, from (7) we have

$$L_j^{q_j}\left(r^m\right)=\left\{\prod_{k=0}^{q_j-1}F_j\left(m-2k\right)\right\}r^{m-2q_j}. \tag{9}$$

For $p=1$, (8) is reduced to

$$L_1^{q_1}\left(r^m\right)=\left\{\prod_{k=0}^{q_1-1}F_1\left(m-2k\right)\right\}r^{m-2q_1}$$

which gives (9) for $j=1$. Now assume that (8) holds for $p-1$, that is,

$$\left(\prod_{j=1}^{p-1}L_j^{q_j}\right)(r^m)=\left\{\prod_{j=1}^{p-1}\prod_{k=0}^{q_j-1}F_j\left(m-2[Q(p-1)-Q(j)]-2k\right)\right\}r^{m-2Q(p-1)}. \tag{10}$$

On the other hand, from (9) for $j=p$ we have

$$L_p^{q_p}\left(r^m\right)=\left\{\prod_{k=0}^{q_p-1}F_p\left(m-2k\right)\right\}r^{m-2q_p}.$$

Thus,

$$\begin{aligned}\left(\prod_{j=1}^{p}L_j^{q_j}\right)(r^m)&=\left(\prod_{j=1}^{p-1}L_j^{q_j}\right)\left(L_p^{q_p}\left(r^m\right)\right)\\&=\left(\prod_{j=1}^{p-1}L_j^{q_j}\right)\left(\left\{\prod_{k=0}^{q_p-1}F_p\left(m-2k\right)\right\}r^{m-2q_p}\right)\\&=\left\{\prod_{k=0}^{q_p-1}F_p\left(m-2k\right)\right\}\left(\prod_{j=1}^{p-1}L_j^{q_j}\right)\left(r^{m-2q_p}\right).\end{aligned}$$

Hence, by replacing m by $m-2q_p$ in (10), we obtain

$$\begin{aligned}\left(\prod_{j=1}^{p}L_j^{q_j}\right)(r^m)=&\left\{\prod_{k=0}^{q_p-1}F_p\left(m-2k\right)\right\}\\&\times\left\{\prod_{j=1}^{p-1}\prod_{k=0}^{q_j-1}F_j\left(m-2q_p-2[Q(p-1)-Q(j)]-2k\right)\right\}r^{m-2q_p-2Q(p-1)}\\=&\left\{\prod_{j=1}^{p}\prod_{k=0}^{q_j-1}F_j\left(m-2[Q\left(p\right)-Q\left(j\right)]-2k\right)\right\}r^{m-2Q(p)}\end{aligned}$$

where $Q(p-1)+q_p=Q(p)$ .Thus, the proof is complete.

Now using Lemma 2.3, we can prove the following theorem.

Theorem 2.4 *The function defined by*

$$
\begin{aligned}
u = & \sum_{j\in I_1}\sum_{k=0}^{q_j-1} r^{2[Q(p)-Q(j)]+2k-\psi_j}\left[c_{jk}^{(1)} r^{\sqrt{\psi_j^2-\gamma_j}} + c_{jk}^{(2)} r^{-\sqrt{\psi_j^2-\gamma_j}}\right] \\
& + \sum_{j\in I_2}\sum_{k=0}^{q_j-1} r^{2[Q(p)-Q(j)]+2k-\psi_j}\left[c_{jk}^{(1)} \cos\!\left(\sqrt{\gamma_j-\psi_j^2}\ln r\right) + c_{jk}^{(2)} \sin\!\left(\sqrt{\gamma_j-\psi_j^2}\ln r\right)\right] \\
& + \sum_{j\in I_3}\sum_{k=0}^{q_j-1} r^{2[Q(p)-Q(j)]+2k-\psi_j}\left[c_{jk}^{(1)} + c_{jk}^{(2)}\ln r\right]
\end{aligned}
\tag{11}
$$

is r^m type solution of the iterated equation (1). Here, $c_{jk}^{(1)}$ and $c_{jk}^{(2)}$ are arbitrary constants, ψ_j is as given in (5) and we divide the index set $I = \{j = 1, \ldots, p\}$ into three parts:

$$
\begin{aligned}
I_1 &= \left\{j \in I, \psi_j^2 - \gamma_j > 0\right\}, \\
I_2 &= \left\{j \in I, \psi_j^2 - \gamma_j < 0\right\}, \\
I_3 &= \left\{j \in I, \psi_j^2 - \gamma_j = 0\right\}.
\end{aligned}
$$

Proof. Let $m - 2[Q(p) - Q(j)] - 2k = M$. Then, since the roots of the quadratic equation

$$F_j\big(m - 2[Q(p) - Q(j)] - 2k\big) = M(M + 2\psi_j) + \gamma_j = 0 \tag{12}$$

are

$$
\begin{cases}
m_{jk}^{(1)} = 2[Q(p) - Q(j)] + 2k - \psi_j + \sqrt{\psi_j^2 - \gamma_j} \\
m_{jk}^{(2)} = 2[Q(p) - Q(j)] + 2k - \psi_j - \sqrt{\psi_j^2 - \gamma_j}
\end{cases}
\tag{13}
$$

we can rewrite (8) as

$$\left(\prod_{j=1}^{p} L_j^{q_j}\right)(r^m) = \left\{\prod_{j=1}^{p}\prod_{k=0}^{q_j-1}\big(m - m_{jk}^{(1)}\big)\big(m - m_{jk}^{(2)}\big)\right\} r^{m-2Q(p)}. \tag{14}$$

From (14), we conclude that for $j = 1, \ldots, p$ and $k = 0, 1, \ldots, q_j - 1$, the functions $r^{m_{jk}^{(1)}}$ and $r^{m_{jk}^{(2)}}$ are solutions of equation (1). Thus, since equation (1) is linear, by the superposition principle, the function

$$\sum_{j=1}^{p}\sum_{k=0}^{q_j-1}\left[c_{jk}^{(1)} r^{m_{jk}^{(1)}} + c_{jk}^{(2)} r^{m_{jk}^{(2)}}\right] \tag{15}$$

also satisfies equation (1).

We have three cases for the roots:

Case 1. If $j \in I_1$, then $m_{jk}^{(1)}$ and $m_{jk}^{(2)}$ are two different real roots. In this case, from (15), the function

$$\sum_{j\in I_1}\sum_{k=0}^{q_j-1}\left[c_{jk}^{(1)} r^{m_{jk}^{(1)}} + c_{jk}^{(2)} r^{m_{jk}^{(2)}}\right]$$
$$=\sum_{j\in I_1}\sum_{k=0}^{q_j-1} r^{2[Q(p)-Q(j)]+2k-\psi_j}\left[c_{jk}^{(1)} r^{\sqrt{\psi_j^2-\gamma_j}} + c_{jk}^{(2)} r^{-\sqrt{\psi_j^2-\gamma_j}}\right]$$

satisfies (1).

Case 2. If $j \in I_2$, then $m_{jk}^{(1)}$ and $m_{jk}^{(2)}$ are both complex and conjugate as

$$m_{jk}^{(1)}, m_{jk}^{(2)} = 2\left[Q\left(p\right) - Q\left(j\right)\right] + 2k - \psi_j \pm i\sqrt{\gamma_j - \psi_j^2}.$$

In this case, from (15), the function

$$\sum_{j\in I_2}\sum_{k=0}^{q_j-1}\left[a_{jk}^{(1)} r^{m_{jk}^{(1)}} + a_{jk}^{(2)} r^{m_{jk}^{(2)}}\right]$$
$$=\sum_{j\in I_2}\sum_{k=0}^{q_j-1} r^{2[Q(p)-Q(j)]+2k-\psi_j}\left[c_{jk}^{(1)}\cos\left(\sqrt{\gamma_j-\psi_j^2}\ln r\right) + c_{jk}^{(2)}\sin\left(\sqrt{\gamma_j-\psi_j^2}\ln r\right)\right]$$

satisfies (1). Here, we use Euler formula

$$r^{\pm i\sqrt{\gamma_j-\psi_j^2}} = e^{\pm i\sqrt{\gamma_j-\psi_j^2}\ln r} = \cos\left(\sqrt{\gamma_j-\psi_j^2}\ln r\right) \pm i\sin\left(\sqrt{\gamma_j-\psi_j^2}\ln r\right)$$

and $a_{jk}^{(1)} + a_{jk}^{(2)} = c_{jk}^{(1)}$, $i\left(a_{jk}^{(1)} - a_{jk}^{(2)}\right) = c_{jk}^{(2)}$, $i = \sqrt{-1}$ as usual.

Case 3. Finally, if $j \in I_3$, then $m_{jk}^{(1)} = m_{jk}^{(2)}$ is a multiple root, that is,

$$m_{jk}^{(1)} = m_{jk}^{(2)} = 2\left[Q\left(p\right) - Q\left(j\right)\right] + 2k - \psi_j = m_{jk}^{(0)}.$$

In this case, (14) can be written as

$$\left(\prod_{j=1}^{p} L_j^{q_j}\right)\left(r^m\right) = G_1^2\left(m\right) G_2\left(m\right) r^{m-2Q(p)} \tag{16}$$

where

$$\prod_{j\in I_3}\prod_{k=0}^{q_j-1}\left(m-m_{jk}^{(0)}\right)=G_1\left(m\right)$$

and

$$\prod_{j\in I\setminus I_3}\prod_{k=0}^{q_j-1}\left(m-m_{jk}^{(1)}\right)\left(m-m_{jk}^{(2)}\right)=G_2\left(m\right).$$

Now, by taking the derivative with respect to m both sides of (16), we obtain

$$\left(\prod_{j=1}^{p}L_j^{q_j}\right)(r^m\ln r)=G_1(m)\left\{2G_1^{'}(m)G_2(m)r^{m-2Q(p)}+G_1(m)\frac{\partial}{\partial m}\left[G_2(m)r^{m-2Q(p)}\right]\right\}. \tag{17}$$

Since $G_1\left(m_{jk}^{(0)}\right)=0$ for $j\in I_3$ and $k=0,\dots,q_j-1$, taking $m=m_{jk}^{(0)}$ in (16) and (17), we get

$$\left(\prod_{j=1}^{p}L_j^{q_j}\right)\left(r^{m_{jk}^{(0)}}\right)=0 \text{ and } \left(\prod_{j=1}^{p}L_j^{q_j}\right)\left(r^{m_{jk}^{(0)}}\ln r\right)=0.$$

Hence, for $j\in I_3$ and $k=0,\dots,q_j-1$, each of the functions $r^{m_{jk}^{(0)}}$ and $r^{m_{jk}^{(0)}}\ln r$ and their superposition

$$\sum_{j\in I_3}\sum_{k=0}^{q_j-1}r^{2[Q(p)-Q(j)]+2k-\psi_j}\left[c_{jk}^{(1)}+c_{jk}^{(2)}\ln r\right]$$

satisfy (1).

Summing up the above three cases with the superposition principle we get (11), which proves the theorem.

Remark 2.5 *In the special case $\gamma_j=0$ for any $j\in I$, the quadratic equation (12) has the root $m-2\left[Q\left(p\right)-Q\left(j\right)\right]-2k=M=0$. In this case, since the values*

$$m_{jk}=2\left[Q\left(p\right)-Q\left(j\right)\right]+2k$$

are nonnegative integers for $k=0,1,\dots,q_j-1$, the functions $r^{2[Q(p)-Q(j)]+2k}$ are polynomial solutions of equation (1).

Remark 2.6 *For any* $j \in I$ *and* $k = 0, 1, \ldots, q_j - 1$, *if* $-\psi_j + \sqrt{\psi_j^2 - \gamma_j}$ *are even integers and*

$$m_{jk}^{(1)} = 2\left[Q\left(p\right) - Q\left(j\right)\right] + 2k - \psi_j + \sqrt{\psi_j^2 - \gamma_j} \geq 0$$

then $r^{m_{jk}^{(1)}}$ *are polynomial solutions of equation (1).*

Remark 2.7 *For any* $j \in I$ *and* $k = 0, 1, \ldots, q_j - 1$, *if* $-\psi_j - \sqrt{\psi_j^2 - \gamma_j}$ *are even integers and*

$$m_{jk}^{(2)} = 2\left[Q\left(p\right) - Q\left(j\right)\right] + 2k - \psi_j - \sqrt{\psi_j^2 - \gamma_j} \geq 0$$

then $r^{m_{jk}^{(2)}}$ *are polynomial solutions of equation (1).*

3 Solutions of Type $u = u\left(r\right)$

In this section, we will show that all solutions which depend on only r for the equation (1) can be expressed by formula (11).

Lemma 3.1 *For the function* $u = u\left(r\right)$,

$$L_j u = e^{-2t}\left(D^2 + 2\psi_j D + \gamma_j\right) u = e^{-2t} F_j\left(D\right) u \tag{18}$$

where ψ_j, F_j *are given by (5), (6), respectively, and* $D = \frac{d}{dt}$, $r = e^t$.

Proof. Taking into consideration L_j and r given by (2) and (3), respectively, if we apply the operator L_j to the function $u = u\left(r\right)$, we obtain

$$L_j u = r^{-2}\left\{r^2 \frac{d^2}{dr^2} + \left(1 + 2\psi_j\right) r \frac{d}{dr} + \gamma_j\right\} u$$

where the operator in the bracket is an Euler type operator. If we let $r = e^t$, then we can write as

$$L_j u = e^{-2t}\left(D^2 + 2\psi_j D + \gamma_j\right) u = e^{-2t} F_j\left(D\right) u$$

where $D = \frac{d}{dt}$. Thus, the proof is complete.

Lemma 3.2 *For any positive integer* q

$$L_j^q u = e^{-2qt}\left\{\prod_{k=0}^{q-1} F_j\left(D - 2k\right)\right\} u. \tag{19}$$

Proof. We give the proof by induction on q. It is clear by (18) that the equality (19) is true for $q = 1$. Now, let us assume that the equality is valid for $q - 1$, that is,

$$L_j^{q-1} u = e^{-2(q-1)t} \left\{ \prod_{k=0}^{q-2} F_j \left(D - 2k \right) \right\} u. \tag{20}$$

Applying the operator L_j on both sides of (20) and using the relation $L_j = e^{-2t} F_j \left(D \right)$ in (18), we obtain

$$\begin{aligned} L_j^q u &= L_j \left[e^{-2(q-1)t} \left\{ \prod_{k=0}^{q-2} F_j \left(D - 2k \right) \right\} u \right] \\ &= e^{-2t} F_j \left(D \right) \left[e^{-2(q-1)t} \left\{ \prod_{k=0}^{q-2} F_j \left(D - 2k \right) \right\} u \right]. \end{aligned}$$

From ordinary differential equations, we know that, for any polynomials of the operator D with constant coefficients G and H and for any constant α, the following relation is valid

$$G \left(D \right) \left\{ e^{-\alpha t} H \left(D \right) u \right\} = e^{-\alpha t} G \left(D - \alpha \right) H \left(D \right) u.$$

Considering this property, we get

$$L_j^q u = e^{-2t} e^{-2(q-1)t} F_j \big(D - 2(q-1) \big) \left\{ \prod_{k=0}^{q-2} F_j \left(D - 2k \right) \right\} u = e^{-2qt} \left\{ \prod_{k=0}^{q-1} F_j \left(D - 2k \right) \right\} u$$

which gives the desired result.

Lemma 3.3 *For any positive integers $p, q_1, \dots, q_p$*

$$\left(\prod_{j=1}^{p} L_j^{q_j} \right) u = e^{-2Q(p)t} \left\{ \prod_{j=1}^{p} \prod_{k=0}^{q_j - 1} F_j \left(D - 2 \left[Q \left(p \right) - Q \left(j \right) \right] - 2k \right) \right\} u \tag{21}$$

where $Q \left(j \right) = q_1 + \cdots + q_j$, $j = 1, \dots, p$.

Proof. By using induction argument on p, this is easily proved in a manner similar to the proof of Lemma 3.2.

Theorem 3.4 *All solutions of type $u = u \left(r \right)$ for the equation (1) can be expressed by the formula (11).*

Proof. Equating (21) expression to zero, we obtain an ordinary differential equation with constant coefficients and of order $2Q(p) = 2(q_1 + \cdots + q_p)$

$$\left\{\prod_{j=1}^{p}\prod_{k=0}^{q_j-1} F_j\left(D - 2\left[Q(p) - Q(j)\right] - 2k\right)\right\} u = 0. \tag{22}$$

The indicial equation for this equation

$$\begin{aligned}\prod_{j=1}^{p}\prod_{k=0}^{q_j-1} F_j\left(m - 2\left[Q(p) - Q(j)\right] - 2k\right) &= 0\\ \prod_{j=1}^{p}\prod_{k=0}^{q_j-1}\left(m - m_{jk}^{(1)}\right)\left(m - m_{jk}^{(2)}\right) &= 0\end{aligned}$$

where $m_{jk}^{(1)}$ and $m_{jk}^{(2)}$ are as defined by (13). Thus the solution of (22) is given by

$$\begin{aligned}u = &\sum_{j\in I_1}\sum_{k=0}^{q_j-1}\left[c_{jk}^{(1)} e^{\left(2[Q(p)-Q(j)]+2k-\psi_j+\sqrt{\psi_j^2-\gamma_j}\right)t} + c_{jk}^{(2)} e^{\left(2[Q(p)-Q(j)]+2k-\psi_j-\sqrt{\psi_j^2-\gamma_j}\right)t}\right]\\ &+\sum_{j\in I_2}\sum_{k=0}^{q_j-1} e^{(2[Q(p)-Q(j)]+2k-\psi_j)t}\left[c_{jk}^{(1)}\cos\left(\sqrt{\gamma_j-\psi_j^2}\,t\right) + c_{jk}^{(2)}\sin\left(\sqrt{\gamma_j-\psi_j^2}\,t\right)\right]\\ &+\sum_{j\in I_3}\sum_{k=0}^{q_j-1} e^{(2[Q(p)-Q(j)]+2k-\psi_j)t}\left[c_{jk}^{(1)} + c_{jk}^{(2)} t\right].\end{aligned}$$

If we set $t = \ln r$, the corresponding solution for (1) is given by (11). Thus, the proof is complete.

Remark 3.5 *Note that, substituting* $u = r^m$ *in (21) and considering* $r = e^t$, *we obtain*

$$\begin{aligned}\left(\prod_{j=1}^{p} L_j^{q_j}\right)(r^m) &= e^{-2Q(p)t}\left\{\prod_{j=1}^{p}\prod_{k=0}^{q_j-1} F_j\left(D - 2\left[Q(p) - Q(j)\right] - 2k\right)\right\} e^{mt}\\ &= e^{\left(m-2Q(p)\right)t}\left\{\prod_{j=1}^{p}\prod_{k=0}^{q_j-1} F_j\left(m - 2\left[Q(p) - Q(j)\right] - 2k\right)\right\}\\ &= r^{m-2Q(p)}\left\{\prod_{j=1}^{p}\prod_{k=0}^{q_j-1} F_j\left(m - 2\left[Q(p) - Q(j)\right] - 2k\right)\right\}\end{aligned}$$

which was given previously by (8). That is, (21) reduces to (8). Similarly, we can see that (18) and (19) reduces to (4) and (7), respectively.

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