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Exponential Stability of Stochastic Fuzzy Hopfield Neural Networks with Time-Varying Delays and Impulses

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Abstract

In this paper, the model of stochastic fuzzy Hopfield neural networks with time-varying delays and impulses (ISFVDHNNs) is established as a modified Takagi-Sugeno (TS) fuzzy model in which the consequent parts are composed of a set of stochastic Hopfield neural networks with time-varying delays and impulses. Then, the global exponential stability in the mean square for ISFVDHNNs is studied by establishing an impulse fuzzy delay differential inequality. The sufficient condition, which is easily checked in practice by simple algebra methods, has a wider adaptive range and it also extends and improves some results in earlier publications.

Keywords: Stochastic; Fuzzy; Hopfield neural networks; Time-varying delays;Impulses; Exponential stability.

1 Introduction

Hopfield neural networks were first introduced by Hopfield [1]. For a few decades, Hopfield neural networks have been extensively investigated. Many applications have been found in different fields such as combinatorial optimization, signal processing and pattern recognition. These applications are built upon the stability analysis of the equilibrium of neural networks. Thus, the stability analysis is a necessary step for the design and applications of neural networks. Sometimes, neural

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networks have to be designed such that there is only equilibrium and this equilibrium is globally stable. As is well known, the exponential stable enjoys such nice properties.

In biological and artificial neural networks, the interactions between neurons are generally asynchronous which inevitably result in time delays. In electronic implementation of analog neural networks, nevertheless, the delays are usually time-varying due to the finite switching speed of amplifiers. It is known that time delays are often a source of instability of neural networks [2].

Besides delay effect, impulsive effect likewise exists in a wide variety of evolutionary processes in which states are changed abruptly at certain moments of time. The impulsive differential equations are adequate apparatus for mathematical simulation of many processes and phenomena in nature which are characterized by the fact that the system parameters are subject to short-term perturbations in time. Their study is assuming a greater importance [3]. In [4], [5], some results on this topic have been reported.

When performing the computation, there are many stochastic perturbations that affect the stability of neural networks. It was pointed out [6], [7] that a neural networks could be stabilized or destabilized by certain stochastic inputs. It implies that the stability analysis of stochastic neural networks also has primary significance in the research of neural networks. Recently, although the stability analysis of neural networks has received much attention, the stability of stochastic neural networks has not been widely studied. In [6], [8], some results related to this issue have been reported.

Fuzzy logic theory has shown to be an appealing and efficient approach to dealing with the analysis and synthesis problems for complex nonlinear systems. In [9], Takagi and Sugeno proposed an effective way to transform a nonlinear dynamic system to a set of linear sub-models via some fuzzy models by defining a linear input/output relationship as its consequence of individual plant rule. In [10], the standard TS fuzzy model was extended to one with time delays, and some stability conditions were presented in terms of linear matrix inequalities (LMIs). In [11], the TS fuzzy model with time delays was further extended to stochastic fuzzy Hopfield neural networks with time-varying delays (SFVDHNNs), and an exponential stability condition is given by constructing some appropriate Lyapunov-Krasovskii functionals and using the LMIs method.

In this paper, first, we further extend SFVDHNNs to describe the stochastic fuzzy Hopfield neural networks with time-varying delays and impulses (ISFVDHNNs). The system dynamics is captured by a set of fuzzy implications which characterize local relations in the state space. The local dynamics of each fuzzy rule is expressed by a stochastic Hopfield neural network with time-varying delays and impulses. The overall fuzzy model can be achieved by fuzzy "blending" of these nonlinear neural networks. Then, the stability of ISFVDHNNs is discussed by establishing an impulse fuzzy delay differential inequality. One criterion is given to guarantee the global exponential stability in the mean square for ISFVDHNNs. The sufficient condition, which is easily checked in practice by simple algebra methods, has a wider adaptive range and it also extends and improves some results in earlier publications.

2 Preliminaries

For $A, B \in R^{m\times n}$ or $A, B \in R^n$, $A \geq B$ $(A > B)$ means that each pair of corresponding elements of A and B satisfies the inequality " $\geq (>)$". Especially, A is called a nonnegative matrix if $A \geq 0$, and z is called a positive vector if $z > 0$.

$PC[I, R^n] \triangleq \{\varphi : I \to R^n \,|\varphi(t^+) = \varphi(t)$ for $t \in I$, $\varphi(t^-)$ exists for $t \in I$, $\varphi(t^-) = \varphi(t)$ for all but points $t_k \in I\}$, where $I \subset R$ is an interval, $\varphi(t^+)$ and $\varphi(t^-)$ denote the left limit and right limit of scalar function $\varphi(t)$, respectively. Especially, let $PC = PC([-\tau, 0], R^n)$.

For $x \in R^n$, $A \in R^{n\times n}$, we define $[x]^+ = (|x_1|, \cdots, |x_n|)^T$, $[A]^+ = (|a_{ij}|)_{n\times n}$ and introduce the corresponding norm as $||x|| = \max\limits_{1\leq i\leq n}\{|x_i|\}$, $||A|| = \max\limits_{1\leq i\leq n} \sum_{j=1}^{n} |a_{ij}|$.

Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq t_0}, P)$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq t_0}$ satisfying the usual conditions (i.e. it is right continuous and F_0 contains all P-null sets). $\omega(t) = (\omega_1(t), \cdots, \omega_n(t))^T$ is an n-dimensional Brownian motion defined on $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq t_0}, P)$. Let $C[[-\tau, 0], R^n]$ denote the family of all continuous R^n-valued functions ϕ on $[-\tau, 0]$ with the norm $||\phi|| = \sup_{-\tau\leq t\leq 0} |\phi(t)|$, where $|\cdot|$ is Euclidean norm of R^n and $[\phi(t)]_\tau = ([\phi_1(t)]_\tau, \cdots, [\phi_n(t)]_\tau)^T$, where $[\phi_i(t)]_\tau = \sup_{-\tau\leq s\leq 0}\{\phi_i(t+s)\}$. Denote by $C^b_{F_0}([-\tau, 0], R^n)$ the family of all bounded F_0-measurable, $C([-\tau, 0], R^n)$-valued random variables ϕ satisfying $||\phi||^2_{L_2} = \sup\limits_{-\tau\leq t\leq 0} E|\phi(t)|^2 < \infty$, where E denotes the expectation of stochastic process.

In this paper, a general class of ISFVDHNNs is discussed. As in [9] and [11], the model of ISFVDHNNs is composed of r plant rules that can be described as follows:

Plant Rule l:

IF $\theta_1(t)$ is η^l_1 and $\cdots$ and $\theta_p(t)$ is η^l_p,

THEN

$$\begin{cases} dx(t) &= [-A_l x(t) + B_l f(x(t-\tau(t)))]dt + \sigma_l(x(t), x(t-\tau(t)), t)d\omega(t),\ t \neq t_k, \\ x(t) &= H(t^-, x(t^-)),\ t = t_k,\ k = 1, 2, \cdots, \\ x(t) &= \phi(t),\ t \in [t_0 - \tau, t_0], \end{cases} \tag{1}$$

where $x(t) = (x_1(t), \cdots, x_n(t))^T$ is the state vector associated with the neurons, $l = 1, \cdots, r$, $\eta^l_s(s = 1, \cdots, p)$ are the fuzzy sets, $\theta(t) = (\theta_1(t), \cdots, \theta_n(t))^T$ is the premise variable vector, r is the number of fuzzy IF-THEN rules, $A_l = \text{diag}\{a^l_i\}$ with $a^l_i > 0$ $(i = 1, \cdots, n, l = 1, \cdots, r)$, $B_l = (b^l_{ij})_{n\times n}$ is the interconnection matrix, $\sigma_l =: R^n \times R^n \times R^+ \to R^{n\times n}$, that is $\sigma_l(x, y, t) = (\sigma^l_{ij}(x, y, t))_{n\times n}$, $f = (f_1, \cdots, f_n)^T : R^n \to R^n$ is the neuron activation function, $H = (H_1, \cdots, H_n)^T : R^+ \times R^n \to R^n$ is the impulse function, $0 \leq \tau(t) \leq \tau$, where τ is a positive constant, $t_k(k = 1, 2, \cdots,)$

is the monotonically increasing sequence, which satisfies that $\lim_{k\to\infty} t_k = \infty$. The initial condition $\phi \in C^b_{F_0}([-\tau, 0], R^n)$.

In this paper, we always assume that f, $\sigma_l\,(l = 1, \cdots, r)$ are continuous, and system (1) has a solution on the entire $t \geq t_0 - \tau$, which is denoted by $x(t)$ and all solutions of system (1) are continuous on the right and limitable on the left.

The defuzzified output of system (1) is represented as follows:

$$\begin{cases} dx(t) = \sum\limits_{l=1}^{r} h_l(\theta(t)) \times [(-A_l x(t) + B_l f(x(t-\tau(t))))dt \\ \qquad\qquad +\sigma_l(x(t), x(t-\tau(t)), t)d\omega(t)],\ t \neq t_k, \\ x(t) = H(t^-, x(t^-)),\ t = t_k,\ k = 1, 2, \cdots, \end{cases} \tag{2}$$

where $h_l(\theta(t)) = \frac{\nu_l(\theta(t))}{\sum\limits_{l=1}^{r}\nu_l(\theta(t))}$, $\nu_l(\theta(t)) = \prod\limits_{s=1}^{p} \eta^l_s(\theta_s(t))$. According to the theory of fuzzy sets, it is obvious that $\nu_l(\theta(t)) \geq 0$, $l = 1, \cdots, r$, $\sum\limits_{l=1}^{r} \nu_l(\theta(t)) > 0$ for all t. Therefore, it implies

$$h_l(\theta(t)) \geq 0,\ l = 1, \cdots, r,\ \sum_{l=1}^{r} h_l(\theta(t)) = 1,\ \text{for all } t. \tag{3}$$

Definition 1 *For ISFVDHNN (2) and every $\phi \in C^b_{F_0}([-\tau, 0], R^n)$, the trivial solution is globally exponentially stable in the mean square if there exist a positive scalar $\lambda > 0$ and a positive vector $z > 0$ such that*

$$Ex^2(t) \leq ze^{-\lambda(t-t_0)}, \quad t \geq t_0, \tag{4}$$

where $Ex^2(t) = (Ex_1^2(t), \cdots, Ex_n^2(t))^T$. The positive scalar λ is called to be the exponential convergent rate.

3 Stability Criterion for ISFVDHNNs

In this section, we first establish an impulse fuzzy delay differential inequality and then give some criteria about the exponential stability of system (1).

Lemma 1 *Let $u(t) = (u_1(t), \cdots, u_n(t))^T \in C[[t_0, \infty], R^n]$ be a solution of the following fuzzy delay differential inequality with the initial condition $u(s) \in PC$, $-\tau \leq s \leq t_0$,*

$$D^+u(t) \leq \sum_{l=1}^{r} h_l(\theta(t)) \times [P_l u(t) + Q_l[u(t)]_\tau],\ t \geq t_0, \tag{5}$$

where $P_l = (p_{ij}^l)_{n\times n}$ *with* $p_{ij}^l \geq 0$ *for* $i \neq j$, $l = 1, \cdots, r$, $Q_l = (q_{ij}^l)_{n\times n} \geq 0$ *for* $l = 1, \cdots, r$, *and* $h_l(\theta(t))$, $l = 1, \cdots, r$, *satisfy (3) . If there exists a positive vector* $z = (z_1, \cdots, z_n)^n$ *such that*

$$(P_l + Q_l)z < 0, \quad l = 1, \cdots, r, \tag{6}$$

then we have

$$u(t) \leq ze^{-\lambda(t-t_0)}, \quad t \geq t_0, \tag{7}$$

where the positive constant λ *is determined by the following inequality*

$$[\lambda E + (P_l + Q_l e^{\lambda\tau})]z < 0,\ l = 1, \cdots, r, \tag{8}$$

for the given z.

Proof Since $(P_l + Q_l)z < 0$ holds for $l = 1, \cdots, r$, by continuity, there exists at least a constant $\lambda > 0$ such that (8) holds, i.e.,

$$\lambda z_i + \sum_{j=1}^{n}(p_{ij}^l + q_{ij}^l e^{\lambda\tau})z_j < 0,\ i = 1, \cdots, n,\ l = 1, \cdots, r. \tag{9}$$

For the initial condition $u(s) \in PC$, $-\tau \leq s \leq t_0$, by (6), we always can choose a z such that

$$u(t) \leq ze^{-\lambda(t-t_0)},\ -\tau \leq t \leq t_0. \tag{10}$$

In order to prove (7), we first prove that for any $\varepsilon > 0$

$$u_i(t) < (1+\varepsilon)z_i e^{-\lambda(t-t_0)} \triangleq v_i(t),\ t \geq t_0. \tag{11}$$

If (11) is not true, using the continuity of $u(t)$, there must exist a $t^* > t_0$ and some integer m such that

$$u_m(t^*) = v_m(t^*), \quad D^+u_m(t^*) \geq v\prime_m(t^*), \tag{12}$$

$$u_i(t) \leq v_i(t),\ t_0 - \tau \leq t \leq t^*,\ i = 1, \cdots, n. \tag{13}$$

Then we have

$$\begin{aligned} D^+u_m(t^*) &\leq \sum_{l=1}^{r} h_l(\theta(t^*)) \sum_{j=1}^{n}(p_{mj}^l u_j(t^*) + q_{mj}^l u_j(t^*-\tau)) \\ &\leq \sum_{l=1}^{r} h_l(\theta(t^*)) \sum_{j=1}^{n}(p_{mj}^l + q_{mj}^l e^{\lambda\tau})(1+\varepsilon)z_j e^{-\lambda(t^*-t_0)} \\ &< -\sum_{l=1}^{r} h_l(\theta(t^*))\lambda z_m(1+\varepsilon)e^{-\lambda(t^*-t_0)} \\ &= -\lambda z_m(1+\varepsilon)e^{-\lambda(t^*-t_0)} \\ &= v\prime_m(t^*), \end{aligned} \tag{14}$$

the first inequality is because (5); the second inequality is because (13) and $p_{ij}^l \geq 0$ for $i \neq j$, $q_{ij}^l \geq 0$, $i, j = 1, \cdots, n$, $l = 1, \cdots, r$; the third inequality is because (3) and (9).

The contradiction between (14) and the inequality of (12) shows that (11) holds for any $t \geq t_0$. Letting $\varepsilon \to 0$, we obtain that (7) holds for any $t \geq t_0$. The proof is completed.

Remark 1 By the property of M-matrix ([12]), condition (6) can be replaced by $D_l = -(P_l + Q_l)$ ($l = 1, \cdots, r$) are nonsingular M-matrices and $\bigcap_{l=1}^r \Omega_M(D_l)$ is nonempty, where $\Omega_M(D) = \{z|\ Dz > 0,\ z > 0\}$.

Theorem 1 *Assume that*

(H1) For any $x \in R^n$, there exists a nonnegative diagonal matrix $L = diag\{L_1, \cdots, L_n\}$ such that

$$[f(x)]^+ \leq L[x]^+. \tag{15}$$

(H2) For any $x_j, y_j \in R$, there exist nonnegative matrices $C_l = (c_{ij}^l)_{n\times n}$, $D_l = (d_{ij}^l)_{n\times n}$, $l = 1, \cdots, r$, such that

$$|\sigma_{ij}^l(x_j, y_j, t)| \leq c_{ij}^l|x_j| + d_{ij}^l|y_j|,\ i, j = 1, \cdots, n,\ t \geq t_0. \tag{16}$$

(H3) There exists a positive vector $z = (z_1, \cdots, z_n)^T$ such that

$$(P_l + Q_l)z < 0, \quad l = 1, \cdots, r, \tag{17}$$

where $P_l = (p_{ij}^l)_{n\times n}$ with $p_{ii}^l = -2a_i^l + \sum\limits_{j=1}^n |b_{ij}^l|L_j + 2r(c_{ii}^l)^2$, $p_{ij}^l = 2r(c_{ij}^l)^2$, $i \neq j$, $Q_l = (q_{ij}^l)_{n\times n}$ with $q_{ij}^l = |b_{ij}^l|L_j + 2r(d_{ij}^l)^2$, $i, j = 1, \cdots, n$.

(H4) There exist nonnegative matrices $R_k = (R_{ij}^k)_{n\times n}$, $k = 1, 2, \cdots$, such that

$$[H(t_k^-, x(t_k^-))]^+ \leq R_k[x(t_k^-)]^+,\ k = 1, 2, \cdots. \tag{18}$$

(H5) Let

$$\gamma_k = \max\{1,\ ||R_k||^2\},\ k = 1, 2, \cdots, \tag{19}$$

and there exists a constant η such that

$$\frac{\ln \gamma_k}{t_k - t_{k-1}} \leq \eta < \lambda, \quad k = 1, 2, \cdots, \tag{20}$$

where the positive constant λ is determined by the following inequality

$$(\lambda E + P_l + Q_l e^{\lambda\tau})z < 0,\ l = 1, \cdots, r, \tag{21}$$

for the given z.

Then the trivial solution of ISFVDHNNs (2) is globally exponentially stable in the mean square and exponential convergent rate equals $\lambda - \eta$.

Proof By Condition (H3) and continuity, one can know there at least exists a positive constant λ such that (21) holds.

Calculating the derivative of $x_i^2(t),\ t \in [t_{k-1}, t_k),\ k = 1, 2, \cdots$, we have

$$\begin{aligned}
dx_i^2(t) &= 2x_i(t)dx_i(t) + (dx_i(t))^2 \\
&= \sum_{l=1}^{r} h_l(\theta(t))[-2a_i^l x_i^2(t) + 2\sum_{j=1}^{n} b_{ij}^l x_i(t) f_j(x_j(t-\tau(t)))]dt \\
&\quad + \sum_{l=1}^{r} h_l(\theta(t)) \sum_{j=1}^{n} 2x_i(t)\sigma_{ij}^l(x_j(t), x_j(t-\tau(t)), t)d\omega_j(t) \\
&\quad + \sum_{l=1}^{r} h_l^2(\theta(t)) \sum_{j=1}^{n} (\sigma_{ij}^l(x_j(t), x_j(t-\tau(t)), t))^2 dt \\
&\quad + 2\sum_{1\le l_1<l_2\le r}^{r} h_{l_1}(\theta(t))h_{l_2}(\theta(t)) \\
&\quad\quad \sum_{j=1}^{n} \sigma_{ij}^{l_1}(x_j(t), x_j(t-\tau(t)), t)\sigma_{ij}^{l_2}(x_j(t), x_j(t-\tau(t)), t)dt. \qquad (22)
\end{aligned}$$

The first equality is because Itô formula, the second equality is because $(dt)^2 = dt \cdot d\omega_j(t) = 0$, $(d\omega_j(t))^2 = 1$, $d\omega_i(t) \cdot d\omega_j(t) = 0\ (i \neq j)$.

Integrating both sides of (22) from t_{k-1} to t, $t \in [t_{k-1}, t_k)$, $k = 1, 2, \cdots$, and then taking expectations, yields

$$\begin{aligned}
Ex_i^2(t) &= Ex_i^2(t_{k-1}) + \int_{t_{k-1}}^{t} \sum_{l=1}^{r} h_l(\theta(\xi))[-2a_i^l Ex_i^2(\xi) \\
&\quad + 2\sum_{j=1}^{n} b_{ij}^l E(x_i(\xi) f_j(x_j(\xi-\tau(\xi))))]d\xi \\
&\quad + \int_{t_{k-1}}^{t} \sum_{l=1}^{r} h_l^2(\theta(\xi)) \sum_{j=1}^{n} E(\sigma_{ij}^l(x_j(\xi), x_j(t-\tau(\xi)), \xi))^2 d\xi \\
&\quad + \int_{t_{k-1}}^{t} 2\sum_{1\le l_1<l_2\le r}^{r} h_{l_1}(\theta(\xi))h_{l_2}(\theta(\xi)) \\
&\quad\quad \sum_{j=1}^{n} E(\sigma_{ij}^{l_1}(x_j(\xi), x_j(\xi-\tau(\xi)), t)\sigma_{ij}^{l_2}(x_j(\xi), x_j(\xi-\tau(\xi)), \xi))d\xi. (23)
\end{aligned}$$

By the property of Dini derivative, we obtain that

$$D^+ Ex_i^2(t) = \sum_{l=1}^{r} h_l(\theta(t))[-2a_i^l Ex_i^2(t) + 2\sum_{j=1}^{n} b_{ij}^l E(x_i(t) f_j(x_j(t-\tau(t))))]$$

$$
\begin{aligned}
&+E\sum_{l=1}^{r}h_l^2(\theta(t))\sum_{j=1}^{n}(\sigma_{ij}^{l}(x_j(t),x_j(t-\tau(t)),t))^2\\
&+E\sum_{1\le l_1<l_2\le r}2h_{l_1}(\theta(t))h_{l_2}(\theta(t))\\
&\quad\sum_{j=1}^{n}(\sigma_{ij}^{l_1}(x_j(t),x_j(t-\tau(t)),t)\sigma_{ij}^{l_2}(x_j(t),x_j(t-\tau(t)),t))\\
\le\ &\sum_{l=1}^{r}h_l(\theta(t))[-2a_i^lEx_i^2(t)+2\sum_{j=1}^{n}b_{ij}^{l}E(x_i(t)f_j(x_j(t-\tau(t))))]\\
&+E\sum_{l=1}^{r}h_l^2(\theta(t))\sum_{j=1}^{n}(\sigma_{ij}^{l}(x_j(t),x_j(t-\tau(t)),t))^2\\
&+E(r-1)\sum_{l=1}^{r}h_l^2(\theta(t))\sum_{j=1}^{n}(\sigma_{ij}^{l}(x_j(t),x_j(t-\tau(t)),t))^2\\
=\ &\sum_{l=1}^{r}h_l(\theta(t))[-2a_i^lEx_i^2(t)+2\sum_{j=1}^{n}b_{ij}^{l}E(x_i(t)f_j(x_j(t-\tau(t))))]\\
&+rE\sum_{l=1}^{r}h_l^2(\theta(t))\sum_{j=1}^{n}(\sigma_{ij}^{l}(x_j(t),x_j(t-\tau(t)),t))^2, t\in[t_{k-1},t_k). \qquad (24)
\end{aligned}
$$

Since $0\le h_l(\theta(t))\le 1$, $l=1,\cdots,r$, so $\sum\limits_{l=1}^{r}h_l^2(\theta(t))\le\sum\limits_{l=1}^{r}h_l(\theta(t))$, then by (24), for $t\in[t_{k-1},t_k),\ k=1,2,\cdots$, we have

$$
\begin{aligned}
D^+Ex_i^2(t)\ \le\ &\sum_{l=1}^{r}h_l(\theta(t))[-2a_i^lEx_i^2(t)+2\sum_{j=1}^{n}b_{ij}^{l}E(x_i(t)f_j(x_j(t-\tau(t))))\\
&+r\sum_{j=1}^{n}E(\sigma_{ij}^{l}(x_j(t),x_j(t-\tau(t)),t))^2]. \qquad (25)
\end{aligned}
$$

From Conditions (H1), (H2) and (25), for $t\in[t_{k-1},t_k),\ k=1,2,\cdots$, we can derive that

$$
\begin{aligned}
D^+Ex_i^2(t)\ \le\ &\sum_{l=1}^{r}h_l(\theta(t))[-2a_i^lEx_i^2(t)+2\sum_{j=1}^{n}|b_{ij}^{l}|E(|x_i(t)|L_j|x_j(t-\tau(t)))|)\\
&+r\sum_{j=1}^{n}E(c_{ij}^{l}|x_j(t)|+d_{ij}^{l}|x_j(t-\tau(t))|)^2]\\
\le\ &\sum_{l=1}^{r}h_l(\theta(t))[-2a_i^lEx_i^2(t)+\sum_{j=1}^{n}|b_{ij}^{l}|L_jE(|x_i(t)|^2+|x_j(t-\tau(t)))|^2)
\end{aligned}
$$

$$
\begin{aligned}
&+2r\sum_{j=1}^{n}E((c_{ij}^{l})^{2}|x_{j}(t)|^{2}+(d_{ij}^{l})^{2}|x_{j}(t-\tau(t))|^{2})] \\
= &\sum_{l=1}^{r}h_{l}(\theta(t))[(-2a_{i}^{l}+\sum_{j=1}^{n}|b_{ij}^{l}|L_{j})Ex_{i}^{2}(t) \\
&+\sum_{j=1}^{n}2r(c_{ij}^{l})^{2}Ex_{j}^{2}(t)+\sum_{j=1}^{n}(|b_{ij}^{l}|L_{j}+2r(d_{ij}^{l})^{2})Ex_{j}^{2}(t-\tau(t))].
\end{aligned}
\tag{26}
$$

Let $V(t)=(V_1(t),\cdots,V_n(t))^T$, $V_i(t)=x_i^2(t)$, $i=1,\cdots,n$, from the definition of P_l, Q_l and (26), we obtain that

$$
D^{+}EV(t)\le\sum_{l=1}^{r}h_{l}(\theta(t))[P_{l}EV(t)+Q_{l}[EV(t)]_{\tau}],\ t\in[t_{k-1},t_{k}),\ k=1,2,\cdots, \tag{27}
$$

where $EV(t)=(EV_1(t),\cdots,EV_n(t))^T$.

Using the discrete part of (1), we obtain that

$$
\begin{aligned}
x_{i}^{2}(t_{k}) &= (H_{i}(t_{k}^{-},x(t_{k}^{-})))^{2}\le(\sum_{j=1}^{n}R_{ij}^{k}x_{j}(t_{k}^{-}))^{2}\le\sum_{j=1}^{n}R_{ij}^{k}\sum_{j=1}^{n}R_{ij}^{k}x_{j}^{2}(t_{k}^{-}) \\
&\le ||R_{k}||\sum_{j=1}^{n}R_{ij}^{k}x_{j}^{2}(t_{k}^{-}),\ k=1,2,\cdots,
\end{aligned}
\tag{28}
$$

the first inequality is because Condition (H4), the second inequality is because Hölder inequality, the third inequality is because the definition of norm $||\cdot||$.

Then taking expectations from both sides of (28), we have

$$
Ex_{i}^{2}(t_{k})\le||R_{k}||\sum_{j=1}^{n}R_{ij}^{k}Ex_{j}^{2}(t_{k}^{-}),\ k=1,2,\cdots, \tag{29}
$$

that is

$$
EV(t_{k})\le||R_{k}||R_{k}EV(t_{k}^{-}),\ k=1,2,\cdots. \tag{30}
$$

For the initial condition $\phi\in PC$, by (17), we always can choose a z such that

$$
u(t)=\phi(t)\le ze^{-\lambda(t-t_{0})},\ -\tau\le t\le t_{0}. \tag{31}
$$

From (17) and (27), we know that all the assumptions of Lemma 1 are true. So, by Lemma 1, we derive that

$$
EV(t)\le ze^{-\lambda(t-t_{0})},\ t_{0}\le t<t_{1}. \tag{32}
$$

By (30), we have

$$\begin{aligned} EV(t_1) &\le ||R_1||R_1EV(t_1^-) \le ||R_1||R_1ze^{-\lambda(t_1-t_0)} \le ||R_1||^2ze^{-\lambda(t_1-t_0)} \\ &\le \gamma_1 ze^{-\lambda(t_1-t_0)}, \end{aligned} \tag{33}$$

the second inequality is because (32), the third inequality is because $R_1z \le ||R_1||z$, the last inequality is because (19).

So in term of (32), (33) and (19), we derive that

$$EV(t) \le \gamma_1 ze^{-\lambda(t-t_0)},\ t_0 \le t \le t_1. \tag{34}$$

By Condition (H3), we have

$$(P_l + Q_l)\gamma_1 z < 0. \tag{35}$$

So by (27), (35) and using Lemma 1 again, we obtain that

$$EV(t) \le \gamma_1 ze^{-\lambda(t-t_0)},\ t_1 \le t < t_2. \tag{36}$$

Then by (30), we derive that

$$EV(t) \le \gamma_1\gamma_2 ze^{-\lambda(t-t_0)},\ t_1 \le t \le t_2. \tag{37}$$

Therefore, by simple mathematical induction, we conclude that

$$EV(t) \le \gamma_1 \cdots \gamma_{k-1} ze^{-\lambda(t-t_0)},\ t_{k-1} \le t < t_k,\ k = 1, 2, \cdots . \tag{38}$$

From (20), we have $\gamma_k \le e^{\eta(t_k-t_{k-1})},\ k = 1, 2, \cdots$, then, for $t_{k-1} \le t < t_k$

$$\gamma_1 \cdots \gamma_k \le e^{\eta(t_1-t_0)} \cdots e^{\eta(t_{k-1}-t_{k-2})} \le e^{\eta(t_{k-1}-t_0)} \le e^{\eta(t-t_0)}. \tag{39}$$

Combining (38) and (39), we can conclude that

$$EV(t) \le ze^{-(\lambda-\eta)(t-t_0)},\ t_{k-1} \le t < t_k,\ k = 1, 2, \cdots , \tag{40}$$

which implies that all the conclusions of Theorem 1 hold. The proof is completed.

Remark 2 When $H(t, x) = x$, that is, there is no impulses in system (2), ISFVDHNN (2) degenerates into the following stochastic fuzzy Hopfield neural networks with time-varying delays (SFVDHNN) [11].

$$dx(t) = \sum_{l=1}^{r} h_l(\theta(t)) \times [(-A_l x(t) + B_l f(x(t - \tau(t))))dt + \sigma_l(x(t), x(t - \tau(t)), t)d\omega(t)]. \tag{41}$$

By Theorem 1, we can easily obtain the following result.

Theorem 2 *Suppose that Conditions (H1), (H2) and (H3) hold, then the trivial solution of SFVDHNN (41) is globally exponentially stable in the mean square.*

When $r = 1$ and $H(t, x) = x$, (2) degenerates into the following stochastic Hopfield neural networks with time-varying delay.

$$dx(t) = -Ax(t) + Bf(x(t - \tau(t))))dt + \sigma(x(t), x(t - \tau(t)), t)d\omega(t). \tag{42}$$

If using the Lyapunov-Krasovskii functional method, such as in [6], [13], [14], to discuss the stability of (42), one can not drop the assumption on the differentiability of time-varying delay $\tau(t)$, but by the method in this paper, we need not the assumption on the differentiability of time-varying delay $\tau(t)$. And we can obtain a similar theorem (only let $r = 1$ in Condition (H3))with Theorem 2.

4 An Illustrative Example

In this section, we will give an example to illustrate the global exponential stability in the mean square for (2) further.

Example Let $r = 2$. Consider the following plant rules of a ISFVDHNN:
IF $\theta_1(t)$ is η_1^l and $\cdots$ and $\theta_p(t)$ is η_p^l,
THEN

$$\begin{cases} dx(t) & = & [-A_l x(t) + B_l f(x(t - \tau(t)))]dt \\ & & +\sigma_l(x(t), x(t - \tau(t)), t)d\omega(t),\ t \neq t_k,\ l = 1, 2, \\ x(t) & = & H(t^-, x(t^-)),\ t = t_k,\ k = 1, 2, \cdots, \end{cases} \tag{43}$$

where $t_1 = 0.1$, $t_k = t_{k-1} + 0.5k$, for $k = 2, 3, \cdots . \eta_s^l(s = 1, \cdots, p)$ are the fuzzy sets, $\theta(t) = (\theta_1(t), \cdots, \theta_n(t))^T$ is the premise variable vector, $\omega(t)$ is a 2×1 Brownian motion and $f_i(x_i(t)) = \tanh(x_i(t)),\ i = 1, 2,\ \tau(t) = |\cos(t)|$,

$$A_1 = \begin{pmatrix} 10 & 0 \\ 0 & 8 \end{pmatrix}, A_2 = \begin{pmatrix} 8 & 0 \\ 0 & 6 \end{pmatrix}, B_1 = \begin{pmatrix} -0.6 & 0.24 \\ 0.5 & -0.46 \end{pmatrix}, B_2 = \begin{pmatrix} 1 & 1 \\ 0.2 & 0.16 \end{pmatrix},$$

$$\sigma_1 = \begin{pmatrix} 0.2x_1(t) - 0.3x_1(t - \tau(t)) & 0.5x_2(t) \\ 0.4x_1(t - \tau(t)) & -0.1x_2(t) + 0.2x_2(t - \tau(t)) \end{pmatrix},$$

$$\sigma_2 = \begin{pmatrix} 0.5x_1(t) + 0.4x_1(t - \tau(t)) & -0.3x_2(t - \tau(t)) \\ 0.2x_1(t) & 0.4x_2(t) \end{pmatrix}.$$

Obviously, $\tau = 1$, assumptions (H1) and (H2) are satisfied with $L = \text{diag}\{1, 1\}$ and

$$C_1 = \begin{pmatrix} 0.2 & 0.5 \\ 0 & 0.1 \end{pmatrix}, C_2 = \begin{pmatrix} 0.5 & 0 \\ 0.2 & 0.4 \end{pmatrix}, D_1 = \begin{pmatrix} 0.3 & 0 \\ 0.4 & 0.2 \end{pmatrix}, D_2 = \begin{pmatrix} 0.4 & 0.3 \\ 0 & 0 \end{pmatrix},$$

respectively. So, by the definition of P_l, Q_l, $l = 1, 2$, we obtain that

$$P_1 = \begin{pmatrix} -19 & 1 \\ 0 & -15 \end{pmatrix}, P_2 = \begin{pmatrix} -13 & 0 \\ 0.16 & -11 \end{pmatrix}, Q_1 = \begin{pmatrix} 0.96 & 0.24 \\ 1.14 & 0.62 \end{pmatrix}, Q_2 = \begin{pmatrix} 1.64 & 1.36 \\ 0.2 & 0.16 \end{pmatrix},$$

therefore,

$$P_1 + Q_1 = \begin{pmatrix} -18.04 & 1.24 \\ 1.14 & -14.38 \end{pmatrix}, P_2 + Q_2 = \begin{pmatrix} -11.36 & 1.36 \\ 0.36 & -10.84 \end{pmatrix},$$

thus assumption (H3) is satisfied with $z = (1, 1)^T$. So if $H(t, x) = x$ then system (43) becomes SFVDHNN. By Theorem 2 system (43) has exactly one globally exponentially stable trivial solution $(0, 0)^T$ in the mean square.

Remark 3 Clearly, the delays $\tau(t)$ do not satisfy the assumption on differentiability, so the stability criterion in [11] can not apply to this example.

Next we consider the case where

$$H(t_k, x(t_k)) = e^{0.15k} \begin{pmatrix} 0.3x_1(t_k) & -0.7x_2(t_k) \\ -0.4x_1(t_k) & 0.3x_2(t_k) \end{pmatrix}, \quad k = 1, 2, \cdots . \tag{44}$$

We can verify that point $(0, 0)^T$ is also the trivial solution for (43), and the parameter of assumption (H4) is as follows:

$$R_k = e^{0.15k} \begin{pmatrix} 0.3 & 0.7 \\ 0.4 & 0.3 \end{pmatrix}, \quad k = 1, 2, \cdots . \tag{45}$$

and $||R_k||^2 = (e^{0.15k})^2 = e^{0.3k}$, $k = 1, 2, \cdots$. So $\gamma_k = \max\{1, e^{0.3k}\} = e^{0.3k}$, $k = 1, 2, \cdots$. For $z = (1, 1)^T$, there exists a positive constant $\lambda = 0.8$ such that

$$(\lambda E + P_1 + Q_1 e^{\lambda\tau})z = (-14.5294, -10.2830)^T < (0, 0)^T, \tag{46}$$

and

$$(\lambda E + P_2 + Q_2 e^{\lambda\tau})z = (-5.5234, -9.2388)^T < (0, 0)^T. \tag{47}$$

So for $k = 1, 2, \cdots$, we have

$$\frac{\ln \gamma_k}{t_k - t_{k-1}} = \frac{\ln e^{0.3k}}{0.5k} = 0.6 < \lambda. \tag{48}$$

Clearly, all assumptions of Theorem 1 are satisfied, so the trivial solution $(0, 0)^T$ of system (43) is globally exponentially stable in the mean square and the exponentially convergent rate is equal to 0.2.

5 Conclusion

In this paper, we first establish the model of stochastic fuzzy Hopfield neural networks with time-varying delays and impulses (ISFVDHNNs), and then by establishing an impulsive fuzzy delay differential inequality, some simple criterions have been derived for the global exponential stability in the square mean of ISFVDHNNs. The simple criterions are also demonstrated by an example. We can see that Theorem 1 and Theorem 2 not only extend and improve some previous results, but also give some new criteria expressed in terms of system parameters.

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