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Representation of Hamiltonian Formalism in Dissipative Mechanical System

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Abstract

Hamiltonian mechanics is the root of classical mechanics. Hamiltonian function is the modified version of Lagrangian function which is of the first order differential equations with generalized coordinates, generalized momentum and time. So, Hamiltonian formulations play an important role in classical mechanics as well as in mechanical systems. In this study, we have established Hamiltonian formalism for dissipative system. We have demonstrated that, whether the class of dissipative mechanical system has an analytical solution or not, it can be represented as a Hamiltonian formalism. Dissipative system deals with the Damping force, Mechanical energy, Principle of least action, First integral, Jacobian matrix and the Non-conservative system deals with Fractional derivatives, Hamiltonian systems, non-conservative systems and Laplace transform.

Mathematics Subject Classification: 29

Keywords: Classical mechanics, Lagrangian function, Hamiltonian function, Non-conservative system, Jacobian matrix, Laplace transform.

Introduction

Hamiltonian mechanics, first introduced by William Rowan Hamilton in 1833, is one of the most essential tools of classical mechanics [1]. It is treated as the reformulation of classical mechanics which is developed form of Lagrangian mechanics in classical mechanics. It is observed that the Lagrangian and the Hamiltonian formulations can be constructed for different kinds of Dissipative and Non-conservative systems [2]. Hamilton originated Hamilton equations of motion and Hamiltonian formulation. In 1960s, Hori and Brouwer utilized the classical Hamiltonian formalism and a

Perturbation theory to solve non-conservative problem [6]. They did not attempt to derive the Hamiltonian formalism of non-conservative problem [9]. In 1990s Tveter made such an attempt and, obtained a so-called general Hamilton equation:

$$\dot{P}_i = -\left(\frac{\partial k}{\partial Q_i}\right)_{Qpt} + F\left(\frac{\partial r}{\partial Q_i}\right)_{Qpt} \quad , \quad \dot{Q}_i = -\left(\frac{\partial k}{\partial P_i}\right)_{Qpt} + F\left(\frac{\partial r}{\partial P_i}\right)_{Qpt} \qquad (1)$$

where $\{Q, P\}$ are canonical variable and r is the position vector depends on the canonical variable set $\{Q, P\}$ and t i.e., $r\{Q, P, t\}$. K is the transformed Hamiltonian; the subscript in the partial derivative expressions indicates the functional dependency of K and r. If the variable set $\{Q, P\}$ is transformed to the variable set $\{q, p\}$, where the position vector r depends on q and does not depend explicitly on t, i.e., r (q) Equation (1) can be reduces to

$$\dot{p}_i = -\left(\frac{\partial H}{\partial Q_i}\right)_{qp} + F\left(\frac{\partial r}{\partial Q_i}\right)_{qp} \quad , \quad \dot{q}_i = \left(\frac{\partial H}{\partial p_i}\right)_{qp} \qquad (2)$$

Where $F\left(\frac{\partial r}{\partial q_i}\right)_q$ denotes generalized force in direction i, H is the Hamiltonian. Yet both equation (1) and equation (2) are not Hamiltonian formalisms, because there is not a conservative Hamiltonian quantity (first integral). The resulting Hamiltonian is un-physical: it is unbounded from below and under time reversal the oscillator is transformed into its "mirrorimage". By this arbitrary trick dissipative systems can be handled as though they were conservative. Vujanovic, B.D. investigated dissipative systems from the vision of variational methods. Tarasov suggested a generalization of canonical quantization that maps a dynamical operator to a dynamical superoperator. Tarasov claimed that this approach allows defining consistent quantization procedure for non-Hamiltonian and dissipative systems. Kiehn considered that dissipation effects may be included by considering the dissipative systems for which the closed integral of action is a parameterdependent, conformal invariant of the motion [5]. He applied this idea to hydrodynamics. S.G. Rajeev considered that a large class of dissipative systems can be brought to a canonical form by introducing complex coordinates in phase space and a complex valued Hamiltonian [10]. In this study, the energy drained from the dissipative system is considered. In general an example of a damped oscillator was given to demonstrate their approach.

Derivation Hamiltonian Formalism for Dissipative System

Generally we consider F is a damping force which depends on the generalized coordinates where variable $q_1, q_2, ...q_n$ coordinate and $\dot{q}_1, \dot{q}_2, ...\dot{q}_n$ is generalized velocity [11].
Again we consider F_i is the components of the generalized force

$$\because F_i\left(q_1, q_2, \ldots\ldots\ldots\ldots q_n, \dot{q}_1, \dot{q}_2, \ldots\ldots\ldots\ldots\ldots \dot{q}_n\right) = F\left(\frac{\partial r}{\partial q_i}\right) \tag{3}$$

Where r is the position vector which depends on the q and $\dot{q}$.

From the non conservative system the general form of Hamiltonian equation is,

$$\dot{P}_i = -\left(\frac{\partial k}{\partial Q_i}\right)_{Qpt} + F\left(\frac{\partial r}{\partial Q_i}\right)_{Qpt}, \quad \dot{Q}_i = -\left(\frac{\partial k}{\partial P_i}\right)_{Qpt} + F\left(\frac{\partial r}{\partial P_i}\right)_{Qpt} \tag{4}$$

Where $\{Q, P\}$ are canonical variable and r is the position vector depends on the canonical variable set $\{Q, P\}$ and t i.e., $r\{Q, P, t\}$ and K is the transformed Hamiltonian. The subscript in the partial derivative expression indicates the *k* and *r* are dependency. If the variable set $\{Q, P\}$ is transformed to the variable set $\{q, p\}$ where the position vector *r* depends on the *q* does not depends on *t*. Then the equation (4) becomes,

$$\dot{p}_i = -\left(\frac{\partial H}{\partial q_i}\right)_{qp} + F\left(\frac{\partial r}{\partial q_i}\right)_{q}, \quad \dot{q}_i = \left(\frac{\partial H}{\partial p_i}\right)_{qp} \tag{5}$$

From equation (5) by the help of (3) we get,

$$\dot{p}_i = -\left(\frac{\partial H}{\partial q_i}\right)_{qp} + F_i\left(q_1, q_2, \ldots\ldots\ldots\ldots q_n, \dot{q}_1, \dot{q}_2, \ldots\ldots\ldots\ldots\ldots\ldots \dot{q}_n\right), \ \dot{q}_i = \left(\frac{\partial H}{\partial p_i}\right)_{qp} \tag{6}$$

Let the new Hamiltonian quantity $\hat{H}$ and we do not change the definition of canonical momenta, hence the Hamiltonian equation becomes

$$\dot{p}_i = -\left(\frac{\partial \hat{H}}{\partial q_i}\right)_{qp}, \quad \dot{q}_i = \left(\frac{\partial \hat{H}}{\partial p_i}\right)_{qp} \tag{7}$$

Comparing the equations (6) and (7), we have

$$\left(\frac{\partial \hat{H}}{\partial q_i}\right)_{qp} = \left(\frac{\partial H}{\partial q_i}\right)_{qp} - F_i\left(q_1, q_2, \ldots\ldots\ldots\ldots q_n, \dot{q}_1, \dot{q}_2, \ldots\ldots\ldots\ldots\ldots\ldots \dot{q}_n\right)$$

$$\therefore \left(\frac{\partial \hat{H}}{\partial p_i}\right)_{qp} = \left(\frac{\partial H}{\partial p_i}\right)_{qp} \tag{8}$$

In classical mechanics the Hamiltonian $\hat{H}$ is mechanical energy.

$$\therefore \hat{H} = \int_{\gamma}\left(\frac{\partial \hat{H}}{\partial q_i}\right)_{qp} dq_i + \int_{\gamma}\left(\frac{\partial \hat{H}}{\partial q_i}\right)_{qp} dp_i + c \tag{9}$$

Where γ denotes the phase flow presented by the equation (6) and c is the constant which depends on the initial condition.

Now find the $\hat{H}$ through line integral along the phase flow curve γ using by equation (6).

$$\int_\gamma \left(\frac{\partial \hat{H}}{\partial q_i}\right) dq_i = \int_\gamma \left[\left(\frac{\partial \hat{H}}{\partial q_i}\right)_{qp} - F_i\left(q_1, q_2, \ldots\ldots\ldots. q_n, \dot{q}_1, \dot{q}_2, \ldots\ldots\ldots. \dot{q}_n\right)\right] dq_i$$

$$\int_\gamma \left(\frac{\partial \hat{H}}{\partial p_i}\right)_{qp} dp_i = \int_\gamma \left(\frac{\partial H}{\partial p_i}\right)_{qp} dp_i \tag{10}$$

In equation (9), only use Hamiltonian H replace by the new Hamiltonian $\hat{H}$ we get,

$$H = \int_\gamma \left(\frac{\partial \hat{H}}{\partial q_i}\right)_{qp} dq_i + \int_\gamma \left(\frac{\partial \hat{H}}{\partial q_i}\right)_{qp} dp_i + c \tag{11}$$

Substituting the value of equations (10) and (11) in equation (9), we have

$$\hat{H} = \int_\gamma \left[\left(\frac{\partial \hat{H}}{\partial q_i}\right)_{qp} - F_i\left(q_1, q_2, \ldots\ldots\ldots. q_n, \dot{q}_1, \dot{q}_2, \ldots\ldots\ldots. \dot{q}_n\right)\right] dq_i + \int_\gamma \left(\frac{\partial \hat{H}}{\partial q_i}\right)_{qp} dp_i + c$$

$$= H - \int_\gamma F_i\left(q_1, q_2, \ldots\ldots\ldots. q_n, \dot{q}_1, \dot{q}_2, \ldots\ldots\ldots. \dot{q}_n\right) dq_i$$

$$\therefore \hat{H} = H - \int_\gamma F_i dq_i \tag{12}$$

The equation (7) is known as phase flow curve. Thus according to the Newton-Laplace principle of determinacy we can assume $q_i = q_i(t)$, $\dot{q}_i = \dot{q}_i(t)$

Hence we can reasonable assume F_i as

$$F_i\left(q_1(t(q_i)), q_2(t(q_i)), \ldots\ldots\ldots. q_n(t(q_i)), \dot{q}_1(t(q_i)), \dot{q}_2(t(q_i)), \ldots\ldots\ldots. \dot{q}_n(t(q_i))\right)$$

i.e., $F_i(q_i)$ is a function of q_i alone, thus we have

$$\int_\gamma F_i dq_i = \int_0^{q_i} F_i dq_i = w_i(q_i) \tag{13}$$

Now substituting the equation (13) into the equation (12), we obtain

$$\hat{H} = H - w_i(q_i) \tag{14}$$

where $-\sum_{i=1}^n w_i(q_i)$ denotes the negative work done by the damping force F. Then we must show that the Hamiltonian presented by the equation (14) satisfies the equation (8). In other words, the equation (7) is equivalent to a Newtonian motion equation. Substituting the equation (14) into the equation (7), we get

$$\frac{\partial \hat{H}}{\partial q_i} = \frac{\partial H}{\partial q_i} - \frac{\partial w_j(q_j)}{\partial q_i}, \frac{\partial \hat{H}}{\partial p_i} = \frac{\partial H}{\partial p_i} - \frac{\partial w_j(q_j)}{\partial p_i} \tag{15}$$

Where q_i and p_i are considered as distinct variable in Hamiltonian mechanics and we consider q_i and p_i as dependent variable in the process of construction of H.

Thus we have $\dfrac{\partial w_j(q_j)}{\partial q_i} = \dfrac{\partial\left(\int_0^{q_j} F_j dq_j\right)}{\partial q_i} = \dfrac{F_j \partial\left[q_j\right]_0^{q_j}}{\partial q_i} = F_j \dfrac{\partial q_j}{\partial q_i} = F_i$

Again $\dfrac{\partial w_j(q_j)}{\partial p_i} = \dfrac{\partial\left(\int_0^{q_j} F_j dq_j\right)}{\partial p_i} = \dfrac{F_j \partial\left[q_j\right]_0^{q_j}}{\partial p_i} = F_j \dfrac{\partial q_j}{\partial p_i} = F_j \cdot 0 = 0$

$$\therefore \frac{\partial w_j(q_j)}{\partial q_i} = F_i \ , \quad \frac{\partial w_j(q_j)}{\partial p_i} = 0 \tag{16}$$

Substituting the equation (16) into the equation (15), we get

$$\frac{\partial \hat{H}}{\partial q_i} = \frac{\partial H}{\partial q_i} - F_i \, , \frac{\partial \hat{H}}{\partial p_i} = \frac{\partial H}{\partial p_i} \tag{17}$$

The equations (17) and (8) are identical. Here we can consider that the Hamiltonian quantity $\hat{H}$ satisfy the Hamilton's equation (8). So the Hamiltonian quantity $\hat{H}$ in equation (14) is represented as $\hat{H} = H - \int_\gamma F.dr$

Proof of the first integral

According to the law of conservation of energy described by the expanded Hamiltonian quantity is an invariant of the phase flow. The proof is as follows:

In classical mechanics the Hamiltonian $\hat{H}$ is mechanical energy

$$\hat{H} = \int\left(\frac{\partial \hat{H}}{\partial q_i}\right)dq_i + \int\left(\frac{\partial \hat{H}}{\partial p_i}\right)dp_i + c \tag{18}$$

Where c is a constant, which depends on the initial condition. The equation (18) derivative of t is

$$\frac{d\hat{H}}{dt} = \frac{\partial \hat{H}}{\partial t} + \frac{\partial \hat{H}}{\partial p_i} p_i + \frac{\partial \hat{H}}{\partial q_i} q_i \tag{19}$$

According to the Newton-Laplace determinacy we get, $\hat{H} = H - w_i(q_i)$ (20)

Suppose that $\hat{H}$ is a function of p and q alone, therefore $\frac{d\hat{H}}{dt}=0$
So the equation (18) becomes

$$\frac{d\hat{H}}{dt}=\frac{\partial \hat{H}}{\partial p_i}p_i+\frac{\partial \hat{H}}{\partial q_i}q_i \tag{21}$$

In the Hamiltonian general equation the canonical set $\{Q,P\}$ is transformed to the new set $\{q,p\}$ and F is generalized force then

$$\dot{p}_i=-\left(\frac{\partial H}{\partial q_i}\right)+F_i\left(q_1,q_2,\ldots\ldots\ldots\ldots q_n,\dot{q}_1,\dot{q}_2,\ldots\ldots\ldots\ldots\ldots\ldots\dot{q}_n\right)$$

$$\therefore\ \dot{p}_i=-\left(\frac{\partial H}{\partial q_i}\right)+F_i\left(q_i,\dot{q}_i\right)\ \text{and}\ \dot{q}_i=\left(\frac{\partial H}{\partial p_i}\right) \tag{22}$$

Suppose the new Hamiltonian $\hat{H}$ and we do not change the definition of the canonical momenta so we can write the new Hamiltonian equation

$$\dot{p}_i=-\left(\frac{\partial H}{\partial q_i}\right),\dot{q}_i=-\left(\frac{\partial H}{\partial p_i}\right) \tag{23}$$

Comparing the equations (22) and (23), we get

$$\frac{\partial \hat{H}}{\partial q_i}=\frac{\partial H}{\partial q_i}-F_i\left(q_1,q_2,\ldots\ldots\ldots\ldots\ldots q_n,\dot{q}_1,\dot{q}_2,\ldots\ldots\ldots\ldots\dot{q}_n\right)$$

$$\therefore\left(\frac{\partial \hat{H}}{\partial p_i}\right)=\left(\frac{\partial H}{\partial p_i}\right) \tag{24}$$

Substituting the equations (22) and (24) into the equation (21), we obtain

$$\frac{d\hat{H}}{dt}=\frac{\partial \hat{H}}{\partial p_i}p_i+\frac{\partial \hat{H}}{\partial q_i}q_i=\frac{\partial \hat{H}}{\partial p_i}\left(-\frac{\partial H}{\partial q_i}+F_i\left(q_i,\dot{q}_i\right)\right)+\frac{\partial \hat{H}}{\partial p_i}\left(\frac{\partial H}{\partial q_i}-F_i\left(q_i,\dot{q}_i\right)\right)=0$$

Thus the expanded Hamiltonian $\hat{H}$ is time independent.

The discussion about generalized momentum and mechanical momentum

The generalized momentum associated with the coordinate q shall be defined as

$$p = \frac{\partial L}{\partial \dot{q}} \tag{25}$$

In general the generalized momentum p is identical with the usual mechanical momentum. In derivation we see $\hat{H}$ differs from H, corresponding $\hat{L}$ differs from original L. According to the equation (25), the generalized momentum p shall vary corresponding. But in derivation about p does not vary correspondingly. We can explain the phenomenon from a derivation with another approach. In this part we denote the new Hamiltonian with $\tilde{H}$ and denote the new Lagrangian with $\tilde{L}$.

In an extra damping force F is added to the system one needs to solve the Lagrangian equation

$$\frac{d}{dt}\left(\frac{\partial_{q\dot{q}} L}{\partial \dot{q}_i}\right) - \frac{\partial_{q\dot{q}} L}{\partial q_i} = -F_i \tag{26}$$

Where the subscript indicates the functional dependence in the partial derivative operation. If we include the effects of the force in the Lagrangian, we may write

$$\tilde{L} = L + S \tag{27}$$

Where S is the contribution to the Lagrangian from F. In this case, we would have

$$\frac{d}{dt}\left(\frac{\partial_{q\dot{q}} \tilde{L}}{\partial \dot{q}_i}\right) - \frac{\partial_{q\dot{q}} \tilde{L}}{\partial q_i} = 0 \Rightarrow \frac{d}{dt}\left\{\frac{\partial_{q\dot{q}}}{\partial \dot{q}_i}(L+S)\right\} - \frac{\partial_{q\dot{q}}}{\partial q_i}(L+S) = 0$$

$$\therefore \frac{d}{dt}\left(\frac{\partial_{q\dot{q}} L}{\partial \dot{q}_i}\right) + \frac{d}{dt}\left(\frac{\partial_{q\dot{q}} S}{\partial \dot{q}_i}\right) - \frac{\partial_{q\dot{q}} L}{\partial q_i} - \frac{\partial_{q\dot{q}} S}{\partial q_i} = 0 \tag{28}$$

Subtracting the equation (26) from the equation (28), we get

$$\frac{d}{dt}\left(\frac{\partial_{q\dot{q}} S}{\partial \dot{q}_i}\right) - \frac{\partial_{q\dot{q}} S}{\partial q_i} = \frac{\partial_{q\dot{q}} L}{\partial q_i} - \frac{d}{dt}\left(\frac{\partial_{q\dot{q}} L}{\partial \dot{q}_i}\right) \Rightarrow \frac{d}{dt}\left(\frac{\partial_{q\dot{q}} S}{\partial \dot{q}_i}\right) - \frac{\partial_{q\dot{q}} S}{\partial q_i} = -(-F_i)$$

$$\therefore \frac{d}{dt}\left(\frac{\partial_{q\dot{q}} S}{\partial \dot{q}_i}\right) - \frac{\partial_{q\dot{q}} S}{\partial q_i} = F_i \tag{29}$$

The Hamiltonian is defined as

$$\tilde{H}(q,\hat{p},t)=\hat{p}_i\dot{q}_i-\tilde{L}(q,\dot{q},t) \tag{30}$$

Where $\tilde{p}$ is the canonical momentum and is defined as $\tilde{p}_i=\dfrac{\partial\tilde{L}}{\partial\dot{q}_i}$ (31)

If the damping force F was not presented, we would have

$$H(L,p,t)=p_i\dot{q}_i-L(q,\dot{q},t) \tag{32}$$

Where the momentum is defined as equation (25). By using equation (27) and comparing the equations (30) and (32), we obtain

$$\tilde{p}_i\dot{q}_i-\tilde{L}(q,\dot{q},t)=p_i\dot{q}_i-L(q,p,t)\Rightarrow\tilde{p}_i\dot{q}_i-(L+S)(q,\dot{q},t)=p_i\dot{q}_i-L(q,p,t)$$
$$\Rightarrow\tilde{p}_i\dot{q}_i-L(q,\dot{q},t)-S(q,\dot{q},t)=p_i\dot{q}_i-L(q,p,t)\Rightarrow\tilde{p}_i\dot{q}_i-S=p_i\dot{q}_i\Rightarrow\tilde{p}_i\dot{q}_i=p_i\dot{q}_i+S$$
$$\therefore\tilde{p}_i=p_i+\frac{\partial_{q\dot{q}}S}{\partial\dot{q}_i} \tag{33}$$

Again using equations (30) and (32), we get

$$\tilde{H}(q,\tilde{q},t)=p_i\dot{q}_i+\left(\frac{\partial_{q\dot{q}}S}{\partial\dot{q}_i}\right)\dot{q}_i-\tilde{L}(q,\dot{q},t)=p_i\dot{q}_i-L(q,\dot{q},t)-S+\left(\frac{\partial_{q\dot{q}}S}{\partial\dot{q}_i}\right)\dot{q}_i$$

$$\therefore\tilde{H}(q,p,t)=H(q,p,t)+\left(\frac{\partial_{q\dot{q}}S}{\partial\dot{q}_i}\right)\dot{q}_i-S \tag{34}$$

The contribution to Hamiltonian form the damping force F is given by,

$$\tilde{H}-H=\left(\frac{\partial_{q\dot{q}}S}{\partial\dot{q}_i}\right)\dot{q}_i-S \tag{35}$$

If we take a time derivative of ΔH and integrating we have,

$$\tilde{H}-H=\int\left(\frac{d(\tilde{H}-H)}{dt}\right)dt=\int\dot{q}_i\left(-\left(\frac{\partial_{q\dot{q}}S}{\partial\dot{q}_i}\right)+\frac{d}{dt}\left(\frac{\partial_{q\dot{q}}S}{\partial\dot{q}_i}\right)\right)dt \qquad \text{Help of (29)}$$

$$=-\int F_i dq_i \tag{36}$$

We observe that the increment in the value of increment ΔH from the damping force F is equal to the negative work done by the force on the system. Accordingly ΔH can be represented as function of q, without loss of generality. We can consider S as a function independent of $\dot{q}$.

So the equation (35) becomes, $\Delta H=S$ (37)

Again we know from (27), $\tilde{L} = L + S = L + \Delta H$ [help of (37)] (38)

Now we can derive equation (33), we get

$$\tilde{p}_i = p_i + \frac{\partial_{q\dot{q}} S}{\partial \dot{q}_i} = p_i \tag{39}$$

We see in this part the generalized momentum p is equivalent to mechanical momentum.

The relation between the two Hamiltonian systems:

If we consider a new Hamiltonian system from above part,

$$\dot{\tilde{p}}_i = -\frac{\partial \tilde{H}}{\partial \tilde{q}_i}, \dot{\tilde{q}}_i = -\frac{\partial \tilde{H}}{\partial \tilde{p}_i} \tag{40}$$

Where $\tilde{q}_i = q_i$. Again we convert the damping force in to a function of q alone. If we transformation a set $\{\tilde{p}_i, \tilde{q}_i\}$ in to a set $\{p_i, q_i\}$. So the equation (25) becomes

$$\dot{p}_i = -\left(\frac{\partial \hat{H}}{\partial q_i}\right), \dot{q}_i = -\left(\frac{\partial \hat{H}}{\partial p_i}\right) \tag{41}$$

This transformation can not affect the value of the Hamiltonian.

Again we know if $\tilde{p}_i$ and p_i are generalized momentum and mechanical momentum, then we can write $p_i = \tilde{p}_i - \frac{\partial_{q\dot{q}} S}{\partial \dot{q}_i}$ (42)

Let M be the Jacobian matrix of the transformation then,

$$M = \begin{bmatrix} \frac{\partial q_i}{\partial \tilde{q}_i} & \frac{\partial q_i}{\partial \tilde{p}_i} \\ \frac{\partial p_i}{\partial \tilde{q}_i} & \frac{\partial p_i}{\partial \tilde{p}_i} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{\partial_{q\dot{q}} S}{\partial q_i \partial \dot{q}_i} & 1 \end{bmatrix} = \begin{bmatrix} I_n & O_n \\ -\frac{\partial_{q\dot{q}} S}{\partial q_i \partial \dot{q}_i} & I_n \end{bmatrix} \tag{43}$$

Where I_n is an identity matrix and O_n is a null matrix.

Suppose, $J = \begin{bmatrix} O_n & I_n \\ -I_n & O_n \end{bmatrix}$ (44)

Substituting (43) in the equation $M^T JM$, we get $M^T JM = J$ (45)

where the subscript "T" denotes transpose matrix. Thus the transformation is unitary canonical.

It can be shown by another way that the value of the new Hamiltonian is not affected by the proposed transformation,

Let, $\varsigma = \{p_i, q_i\}^T$ and $Z = \{\tilde{p}_i, \tilde{q}_i\}^T$

Then we have, $\frac{d\varsigma}{dt} = J\hat{H}_\xi$ and $\frac{dZ}{dt} = J\tilde{H}_Z$

$$\frac{d\varsigma}{dt} = \frac{\partial\varsigma}{dZ}\frac{dZ}{\partial t} + \frac{\partial\varsigma}{\partial t} = MJ\tilde{H}_Z^T + \frac{\partial\varsigma}{\partial t} = MJ\tilde{H}_Z^T + \frac{\partial\varsigma}{\partial t} \quad (46)$$

According to the equation (42) we know that S does not dependent explicitly on t, Thus $\frac{\partial\varsigma}{\partial t} = 0$. Then we have, $\frac{\partial\varsigma}{\partial t} = MJ\tilde{H}_Z^T$ (47)

Because, $\tilde{H}_Z^T = \left(\tilde{H}_\xi \frac{\partial\varsigma}{\partial Z}\right)^T = \left(\tilde{H}_\xi M\right)^T = M^T \tilde{H}_\varsigma^T$ (48)

Again according to the equation (45), we have $M^T JM = J$, thus equation (47) can be represented as $\frac{\partial\varsigma}{\partial t} = J\tilde{H}_\varsigma^T$ (49)

Comparing equations (47) and (49) help of (48), we obtain

$$MJ\tilde{H}_Z^T = J\tilde{H}_\varsigma^T \quad \Rightarrow M\tilde{H}_Z^T = \tilde{H}_\varsigma^T \quad \Rightarrow \hat{H} = \tilde{H}$$

Hence the transformation does not affect the value of the Hamiltonian.

Conclusion

Finally we may conclude that Hamiltonian formulation or mechanics is a developing form of Lagrangian mechanical system. Above all, we believe that this study will play pioneer role in exploring further study in the field of Mathematical Physics. This study deals with the discussion of Hamiltonian formulation for Dissipative mechanical system. The first integral in general is not analytically integrable, with the exception of the original mechanical system, which are integrable. The reason is that the work done by damping force depends on the phase flow. If the system is integrable, then the phase flow can be explicit written out, the system has an analytical solution, and therefore the work done by damping force can be explicitly integrated, consequently H can be explicitly represented. However, from the point of view of physics, the expanded Hamilton quantity is still a first integral, namely total energy. It can be concluded that whether the class of dissipative mechanical system has an analytical solution or not, it can be represented as a Hamiltonian formalism.

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