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Supply Chain Model for the Retailer's Ordering Policy Under Two Levels of Delay Payments in Fuzzy Environment

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Abstract

In this paper we discuss the retailer's ordering policy under two levels of delay payments by considering the selling price per item to be higher than the purchasing cost per item. The demand and selling price per item are taken as triangular fuzzy numbers. Function principle is used to calculate optimum cycle time and economic order quantity. Graded mean integration representation method is used for defuzzification.

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1. Introduction

In most business transactions, the supplier will allow a specified credit period to the retailer for payment without penalty to stimulate the demand of his products.

Before the end of credit period, the retailer can sell all the goods and accumulate revenue and earn interest. A higher rate of interest is charged if the payment is not settled by the end of the trade credit period. Goyal [8] established a single item inventory model under permissible delay in payments. Aggarwal and Jaggi [1] considered the inventory model with an exponential deterioration rate under the conditions of permissible delay in payments. Jamal et al [12] and Chang and Dye [8] extended this issue with allowable shortage Huang and Shinn [12] modeled an inventory system for retailer's pricing and lot sizing policy for exponentially deteriorating products under the conditions of permissible delay in payment. Teng [5] assumed that the selling price not equal to the purchasing price to modify Goyal's model [15]. Chung and Huang [1] developed a procedure to determine retailer's ordering policy. In Huang's model [2], he modified the assumption that the retailer will adopt the delay payments policy to stimulate the customers demand, taking unit selling price and purchasing price to be equal. Yung-Fu Huang's [16] developed easy to use procedure to find the optional ordering policy for the retailer. In this paper, the fuzzy inventory model is discussed by taking demand and unit selling price to be triangular fuzzy numbers we use Chen's function principle [17] to calculate the fuzzy total inventory cost and we use graded mean integration representation method [18] to defuzzify the triangular fuzzy total inventory cost

2. Methodology

2.1. Fuzzy Numbers

Any fuzzy subset of the real line R, whose membership function μ_A satisfied the following conditions is a generalized fuzzy number $\tilde{A}$.

(i) μ_A is a continuous mapping from R to the closed interval [0,1],

(ii) $\mu_A = 0, -\infty < x \le a_1$,

(iii) $\mu_A = L(x)$ is strictly increasing on $[a_1,a_2]$

(iv) $\mu_A = w_A, a_2 \le x \le a_3$

(v) $\mu_A = R(x)$ is strictly decreasing on $[a_3, a_4]$

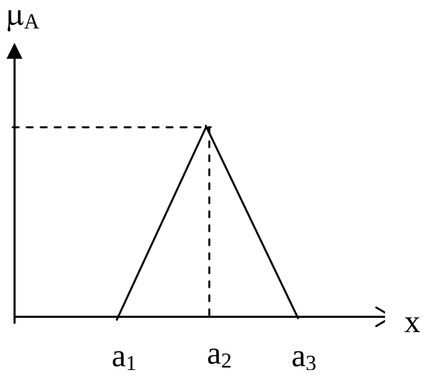
(vi) $\mu_A = 0, a_4 \le x < \infty$

where $0 < w_A \le 1$ and a_1, a_2, a_3 and a_4 are real numbers. Also this type of generalized fuzzy number be denoted as $\tilde{A} = (a_1, a_2, a_3, a_4; w_A)_{LR}$; When $w_A=1$, it can be simplified as $\tilde{A} = (a_1, a_2, a_3, a_4)_{LR}$.

2.2. Triangular Fuzzy Number

The fuzzy set $\breve{A} = (a_1, a_2, a_3)$ where $a_1 < a_2 < a_3$ and defined on R, is called the triangular fuzzy number, if the membership function of $\tilde{A}$ is given by

$$\mu_A = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, a_1 \le x \le a_2 \\ \dfrac{a_3-x}{a_3-a_2}, a_2 \le x \le a_3 \\ 0, \text{other wise} \end{cases}$$



2.3. The Function Principle

The function principle was introduced by Chen [5] to treat fuzzy arithmetical operations. This principle is used for the operation of addition, subtraction, multiplication and division of fuzzy numbers.

Suppose $\tilde{A}$ = (a_1, a_2, a_3,) and $\tilde{B}$ = (b_1, b_2, b_3) are two triangular fuzzy numbers. Then

(i) The addition of $\tilde{A}$ and $\tilde{B}$ is
$\tilde{A} + \tilde{B}$ = (a_1+b_1, a_2+b_2, a_3+b_3) where a_1, a_2, a_3, b_1, b_2, b_3 are any real numbers.

(ii) The multiplication of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A}$ x $\tilde{B}$ = (c_1, c_2, c_3)
where T = { a_1b_1, a_1b_3, a_3b_1, a_3b_3}
c_1 = min T, $c_2 = a_2b_2$, c_3 = Max T
If a_1, a_2, a_3, b_1, b_2, b_3 are all non zero positive real numbers, then
$\tilde{A}$ x $\tilde{B}$ = (a_1b_1, a_2b_2, a_3b_3)

(iii) - $\tilde{B}$ = ($-b_3$, $-b_2$, $-b_1$) then the subtraction of $\tilde{A}$ and $\tilde{B}$ is
$\tilde{A} - \tilde{B}$ = (a_1-b_3, a_1-b_2, a_3-b_1)
where a_1,a_2,a_3, b_1, b_2, b_3 are any real numbers

(iv) $\dfrac{1}{\tilde{B}} = \tilde{B}^{-1} = (1/b_3, 1/b_2, 1/b_1)$ where b_1, b_2, b_3 are all non zero positive real number, then the division of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A}/\tilde{B} = (a_1/b_3, a_2/b_2, a_3/b_1)$

(v) For any real number K, K $\tilde{A}$ = (Ka_1, Ka_2, Ka_3) if K > 0
K $\tilde{A}$ = (Ka_3, Ka_2, Ka_1) if K < 0

2.4. Graded Mean Integration Representation Method

If $\tilde{A}$ = (a_1, a_2, a_3, a_4, $w_A)_{LR}$ is a generalized fuzzy number then the defuzzified Value $P(\tilde{A})$ by graded mean integration representation method is given by

$$p(\tilde{A}) = \int_0^{w_a} h\left[\frac{L^{-1}(h)+R^{-1}(h)}{2}\right]dh \Big/ \int_0^{w_{A1}} hdh, \text{ with } 0 < h \le w_A \text{ and } 0 < w_A \le 1.$$

If $\tilde{A}$ = (a_1, a_2, a_3) is a triangular number then the graded mean integration representation of $\tilde{A}$ by above formula is

$$p(A) = 1/2\frac{\int_0^1 h[a_1 + h(a_2 - a_1) + a_3 - h(a_3 - a_2)]dh}{\int_0^1 hdh} = \frac{a_1 + 4a_2 + a_3}{6}$$

2.5. Notations

$\tilde{D}$	-	Fuzzy annual demand.
$\tilde{A}$	-	Fuzzy cost of placing an order.
C	-	Unit purchasing price.
$\tilde{s}$	-	Fuzzy unit selling price.
h	-	Unit holding cost per year excluding interest charges.
I_e	-	Interest which can be earned per rupee per year.
I_k	-	Interest payable per rupee in investment in inventory per year.
N	-	The first permissible delay period in setting the accounts.
M	-	The second permissible delay period in settling the accounts.
T	-	Time interval between successive orders.
TRC1 (T)	-	The total relevant cost per unit time when $T \geq M$.
TRC2 (T)	-	The total relevant cost per unit time when $N \leq T \leq M$.
TRC3 (T)	-	The total relevant cost per unit time when $0 \leq T \leq N$
$T\tilde{R}C1$ (T)	-	Fuzzy total relevant cost per unit time when $T \geq M$
$T\tilde{R}C2$ (T)	-	Fuzzy total relevant cost per unit time when $N \leq T \leq M$
$T\tilde{R}C3$ (T)	-	Fuzzy total relevant cost per unit time when $O \leq T \leq N$
$P(T\tilde{R}C1\ (T))$	-	defuzzified value of fuzzy total cost $T\tilde{R}C1$ (T)
$P(T\tilde{R}C2\ (T))$	-	defuzzified value of fuzzy total cost $T\tilde{R}C2$ (T)
$P(T\tilde{R}C3\ (T))$	-	defuzzified value of fuzzy total cost $T\tilde{R}C3$ (T)
T_1^0	-	optimal value of T_1
T_2^0	-	optimal value of T_2
T_3^0	-	optimal value of T_3

2.6. Assumptions

i) Inventory deals with single item only

ii) Replenishment rate is infinite and is instantaneous

iii) Shortages are not allowed

iv) Lead time is zero

v) Unit selling price is greater than unit purchasing price

vi) During the time the account is not settled, generated rules revenue is deposited in an interest bearing account. At the end of this period, the retailer pays off all units sold, keeps profits and starts paying for the interest charges on the items in stocks

vii) If the retailer pays by N, the suppler does not charge the retailer. If the retailer pays after N and before M, he can keep the surplus of the difference in an interest bearing account at the rate of I_c/unit per year. If the retailer pays after m, the supplier charges the retailer an interest rate of I_k.

3. Fuzzy Mathematical Model

Her $\tilde{A}$ = (a_1, a_2, a_3), $\tilde{s}$ = (d_1, d_2, d_3) and $\tilde{s}$ = (s_1, s_2, s_3) be triangular fuzzy numbers based on the above assumptions, fuzzy model can be expressed as

$$\text{TRC1 (T)} = \begin{cases} \text{T}\tilde{\text{R}}\text{C1 (T)}, & \text{if } T \geq M \quad \text{...... (3.1)} \\ \text{T}\tilde{\text{R}}\text{C2 (T)}, & \text{if } N \leq T \leq M \quad \text{...... (3.2)} \\ \text{T}\tilde{\text{R}}\text{C3 (T)}, & \text{if } 0 < T \leq M \quad \text{...... (3.3)} \end{cases}$$

Where

$$\text{TRC1 (T)} = \frac{\tilde{A}}{T} + \tilde{D} * \tilde{h}\ T/2 + CI_k\ \tilde{D}\ CT - M)^2/2T - \frac{I_e\ (M^2 - N^2)}{2T}\ \tilde{s} * \tilde{D} \text{...... (3.4)}$$

$$\text{TRC1 (T)} = \frac{1}{T}(a_1, a_2, a_3) + \frac{Th}{2}(d_1, d_2, d_3) + \frac{CI_k\ (T-M)^2}{2T}(d_1, d_2, d_3) - \frac{I_e(M^2-N^2)}{2T}(s_1d_1, s_2d_2, s_3d_3)$$

$$= \left(\frac{a_1}{T} + \frac{Th}{2}d_1 + \frac{CI_k}{2T}(T-M)^2 d_1 - \frac{I_e\ (M^2-N^2)}{2T}s_3d_3,\right.$$
$$\frac{a_2}{T} + \frac{Th}{2}d_2 + \frac{CI_k}{2T}(T-M)^2\ d_2 - \frac{I_e(M^2-N^2)}{2T}s_2d_2,$$
$$\left.\frac{a_3}{T} + \frac{Th}{2}d_3 + \frac{CI_k}{2T}(T-M)^2\ d_3 - \frac{I_e(M^2-N^2)}{2T}s_1d_1\right) \quad \text{...... (3.5)}$$

$$P(\text{T}\tilde{\text{R}}\text{C1 (T)}) = \frac{1}{6}\left\{\left(\frac{a_1}{T} + \frac{Th}{2}d_1 + \frac{CI_k}{2T}(T-M)^2 d_1 - \frac{I_e(M^2-N^2)}{2T}s_3d_3\right)\right.$$
$$+ 4\left(\frac{a_2}{T} + \frac{Th}{2}d_2 + \frac{CI_k}{2T}(T-M)^2 d_2 - \frac{I_e(M^2-N^2)}{2T}s_2d_2\right) +$$
$$\left.\left(\frac{a_3}{T} + \frac{Th}{2}d_3 + \frac{CI_k}{2T}(T-M)^2 d_3 - \frac{I_e\ (M^2-N^2)}{2T}s_1d_1\right)\right\} \quad \text{...... (3.6)}$$

$$\frac{d}{dT}P(\text{T}\tilde{\text{R}}\text{C1 (T)}) = 0$$

gives

$$T_1^0 = \sqrt{\frac{2(a_1 + 4a_2 + a_3) + M^2 CI_k (d_1 + 4d_2 + d_3) + (N^2 - M^2) I_e (s_3 d_3 + 4s_2 d_2 + s_1 d_1)}{\{(h + CI_k)(d_1 + 4d_2 + d_3)\}}}, \quad \text{..... (3.7)}$$

if $2(a_1 + 4a_2 + a_3) + M^2 CI_k (d_1 + 4d_2 + d_3) + (N^2 - M^2) I_e (s_3 d_3 + 4s_2 d_2 + s_1 d_1) > 0$

$$TRC2\,(T) = \frac{\tilde{A}}{T} + \frac{\tilde{D}\,Th}{2} - \tilde{s} * \tilde{D} I_e (2MT - N^2 - T^2) / 2T$$

$$= \frac{1}{T}(a_1, a_2, a_3) + \frac{Th}{2}(d_1, d_2, d_3) - \frac{I_e(2MT - N^2 - T^{21}}{2T}(s_1 d_1, s_2 d_2, s_3 d_3) \quad \text{..... (3.8)}$$

$$= \left(\frac{a_1}{T} + \frac{Thd_1}{2} - \frac{I_e(2MT - N^2 - T^2)}{2T} s_3 d_3,\ \frac{a_2}{T} + \frac{Thd_2}{2} - \frac{I_e(2MT - N^2 - T^2)}{2T} s_2 d_2,\ \frac{a_3}{T} + \frac{Thd_3}{2} - \frac{I_e(2MT - N^2 - T^2)}{2T} s_1 d_1\right) \quad \text{..... (3.9)}$$

$$P\,(T\tilde{R}C2\,(T)) = \frac{1}{6}\left\{\left(\frac{a_1}{T} + \frac{Thd_1}{2} - \frac{I_e(2MT - N^2 - T^2)}{2T} s_3 d_3\right) + 4\left(\frac{a_2}{T} + \frac{Thd_2}{2} - \frac{I_e(2MT - N^2 - T^2)}{2T} s_2 d_2\right) + \left(\frac{a_3}{T} + \frac{Thd_3}{2} - \frac{I_e(2MT - N^2 - T^2)}{2T} s_1 d_1\right)\right\} \quad \text{..... (3.10)}$$

$$\frac{d}{dT} P(T\tilde{R}C2\,(T)) = 0$$

gives

$$T_2^0 = \sqrt{\frac{2(a_1 + 4a_2 + a_3) + N^2 I_e (s_3 d_3 + 4s_2 d_2 + s_1 d_1)}{h\,(d_1 + 4d_2 + d_3) + I_e (s_3 d_3 + 4s_2 d_2 + s_1 d_1)}} \quad \text{..... (3.11)}$$

Similarly

$$T\tilde{R}C3\,(T) = \frac{1}{T}(a_1, a_2, a_3) + \frac{Th}{2}(d_1, d_2, d_3) - I_e(M-N)(s_1 d_1, s_2 d_2, s_3 d_3) \quad \text{..... (3.12)}$$

$$P(T\tilde{R}C3\,(T)) = \frac{1}{6}\left\{\left(\frac{a_1}{T} + \frac{Thd_1}{2} - I_e(M-N) s_3 d_3\right) + 4\left(\frac{a_2}{T} + \frac{Thd_2}{2} - I_e(M-N) s_2 d_2\right) + \left(\frac{a_3}{T} + \frac{Thd_2}{2} - I_e(M-N)\, s_1\, d_1\right)\right\} \quad \text{..... (3.13)}$$

$$\frac{d}{dT}\ P(T\tilde{R}C3\ (T))\ =\ 0$$

gives

$$T_3^0\ =\ \sqrt{\frac{\{2(a_1+4a_2\ +a_3)}{h\ (d_1\ +4d_2\ +d_3)}} \qquad \text{..... (3.14)}$$

4. Decision Rule of the optional cycle time T

From the above expressions, equation (3.7) implies that the optimal value of T for the case $T \geq M$, that is $T_3^0 \geq m$ we substitute (3.7) into $T_1 \geq m$ if and only if

$$2(a_1 + 4a_2 + a_3) \geq (d_1 + 4d_2 + d_3)m^2\ (h + (s_1 + 4s_2 + s_3)I_e) - N^2 I_e(s_1 d_1 + 4s_2 d_2 + s_3 d_3) \qquad \text{..... (3.15)}$$

Similarly equation (3.11) implies that the optimal value of T for the case $N \leq T \leq M$, that is $N \leq T_2^0 \leq M$, the above inequality is true if and only if

$$(d_1 + 4d_2 + d_3)N^2 h \leq 2(a_1 + 4a_2 + a_3) \leq (d_1 + 4d_2 + d_3)M^2(h + I_e(s_1 + 4s_2 + s_3) - N^2 I_e(s_1 d_1 + 4s_2 d_2 + s_3 d_3) \qquad \text{..... (3.16)}$$

Finally equation (3.14) implies that the optimal value of T for the case $T \leq N$ that is $T_3^0 \leq N$ we substitute equation (3.14) into $T_3^0 \leq N$ we can obtain the inequality is true if and only if $2(a_1 + 4a_2 + a_3) \leq N^2 h(d_1 + 4d_2 + d_3)$ ..... (3.17)

Let $\Delta_3 = -2(a_1 + 4a_2 + a_3) + (d_1 + 4d_2 + d_3)\ m^2\ (h + I_e(s_1 + 4s_2 + s_3)) - N^2 I_e(s_1 d_1 + 4s_2 d_2 + s_3 d_3)$ ..... (3.18)

$\Delta_4 = -2(a_1 + 4a_2 + a_3) + (d_1 + 4d_2 + d_3)N^2 h$ ..... (3.19)

We observe that

(i) $T^0 = T_3^0$, if $\Delta_4 > 0$

(ii) $T^0 = T_2^0$, if $\Delta_3 \geq 0$ and $\Delta_4 \leq 0$

(iii) $T^0 = T_1^0$, if $\Delta_3 \leq 0$

5. Numerical Example

Let $\tilde{A} = (48, 50, 52)$, $\tilde{D} = (4800, 5000, 5200)$ be triangular fuzzy numbers, h = \$5/unit/year, l_k = \$0.15/\$/year, I_e = \$0.12/\$/year and M = 0.1 year.

$\tilde{s}$	N = 0.02 year			N = 0.05 year			N = 0.08 year	
	Δ_3	Δ_4	T^0	Δ_3	Δ_4	T^0	Δ_4	T^0
(118, 120, 122)	> 0	< 0	T_2^0 = 0.036	> 0	< 0	T_2^0 = 0.054	> 0	T_3^0 = 0.063
(158, 160, 162)	> 0	< 0	T_2^0 = 0.034	> 0	< 0	T_2^0 = 0.053	> 0	T_3^0 = 0.063
(180, 200, 220)	> 0	< 0	T_2^0 = 0.032	> 0	< 0	T_2^0 = 0.053	> 0	T_3^0 = 0.063

6. Conclusion

When N is increasing, the optimal cycle time for the retailer is increasing. It implies that the retailer will order more quantity to get more interest earned offered by the supplier to compensate the loss of interest earned from longer trade credit period offered to his/her customer. When s is increasing the optimal cycle time for the retailer is not increasing.

We conclude that the retailer will order more quantity to get more interest earned offered by the supplier to compensate the loss of interest earned from longer trade credit period offered to his / her customer and the retailer will not order more quantity to take the benefits of the delay payments more frequently when the larger the differences between the unit selling price and the unit purchasing price.

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