A Model-free Periodic Adaptive Control for Freeway Traffic Density via Ramp Metering

CHI Rong-Hu[1] HOU Zhong-Sheng[2]

Abstract In this paper, a novel model-free periodic adaptive ramp metering scheme is presented for a freeway traffic system, which can be formulated as a general MIMO nonlinear system. The proposed method is model-free in nature, and the control inputs and the pseudo Jacobi parameters are updated periodically in a pointwise manner over the entire period, by directly using the I/O data obtained in the preceding periods. The geometrical tracking performance is shown with rigorous analysis. The simulation results illustrate the validity of the proposed method.

Key words Model-free adaptive control (MFAC), periodic control, MIMO nonlinear system, traffic density control, I/O data

DOI 10.3724/SP.J.1004.2010.01029

Ramp metering has been recognized as one of the most effective ways for combating freeway congestion[1], which is a typical regulating problem, and a number of control methods have been exploited[1−3]. However, these methods are model-based and require the exact model of the control system; thus they are difficult to design and construct for general nonlinear processes. In fact, the freeway traffic flow system is nonlinear, coupled, and uncertain, and its accurate model is hardly available in practice. Hence, we need a simple and robust control method that is insensitive to modeling uncertainties and suitable for nonlinear dynamics. Recently, Hou et al.[4] developed a new dynamical linearization method by introducing a concept of pseudo partial derivative (PPD) and proposed a model-free adaptive control (MFAC) scheme for general nonlinear systems. As a data-driven method in nature, it has received increasing attention from the control community[5−6].

It is worth pointing out that macroscopic traffic flow patterns are in general periodic everyday[7]. Ruling out the occasional occurrence of accidents, the routine traffic flow on freeway in the macroscopic level will show the inherent periodicity every day. In fact, the traffic periodicity is implicitly assumed in all fixed-time traffic control methods. However, the above-mentioned control approaches[1−6] are lack of the capability of learning from recurrent traffic processes to improve the tracking performance.

Considering the periodic or repetitive reference signals, recently some periodic adaptive control methods[8−10] have been proposed by means of the pointwise integral mechanism with the goal of ensuring the tracking/disturbance rejection of periodic references/disturbances. However, most of them focus on the cases in which the nonlinear plant dynamics can be linearly parameterized and the periodicity of parametric uncertainties must be known as a priori, which hinders further practical applications of these approaches[8−10].

Manuscript received December 15, 2008; accepted March 6, 2009
Supported by State Key Program (60834001) and General Program (60774022, 60974040) of National Natural Science Foundation of China
1. School of Automation and Electrical Engineering, Qingdao University of Science and Technology, Qingdao 266042, P. R. China
2. Advanced Control Systems Laboratory, School of Electronics and Information Engineering, Beijing Jiaotong University, Beijing 100044, P. R. China

In this paper, we introduce the basic ideas of periodic adaptive control and MFAC into a general MIMO nonlinear freeway traffic control system, where the only prior knowledge required is that the desired trajectory is periodic with a known periodicity. As a result, a novel model-free periodic adaptive control (MFPAC) approach is presented for on-ramp metering. The main distinct features of this method are summarized as follows:

1) It is model-free and only the I/O data of the corresponding point in previous periods, instead of previous time instances, is used to update the estimate of pseudo Jacobi matrix and the control input signal. Analogously, the convergence is exponential with respect to the number of periods, instead of the time instances.

2) The only prior knowledge required for the proposed method is the periodicity of the desired trajectory (e.g., the desired density and the mean speed of the freeway traffic flow). Obviously, it is much easier to classify the given known desired trajectories into periodic vs. nonperiodic ones.

1 Problem formulation and dynamical linearization

1.1 Traffic flow

The spatial and time-discretized traffic flow model[2] for a single freeway with one on-ramp and one off-ramp on each section is shown in Fig. 1.

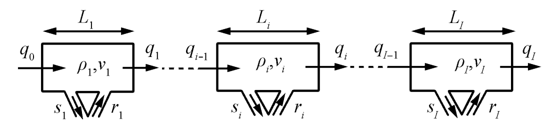

Fig. 1 Segments on a freeway with on/off ramp

Its formulation is given as follows:

$$\rho_i(t+1) = \rho_i(t) + \frac{N}{L_i}[q_{i-1}(t) - q_i(t) + r_i(t) - s_i(t)] \quad (1)$$

$$q_i(t) = \rho_i(t) v_i(t) \quad (2)$$

$$\begin{aligned} v_i(t+1) = {} & v_i(t) + \frac{N}{\tau}[V(\rho_i(t)) - v_i(t)] + \\ & \frac{N}{L_i} v_i(t)[v_{i-1}(t) - v_i(t)] - \frac{\nu T}{\tau L_i} \frac{\rho_{i+1}(t) - \rho_i(t)}{\rho_i(t) + \kappa} \end{aligned} \quad (3)$$

$$V(\rho_i(t)) = V_{\rm free}\left(1 - \frac{\rho_i(t)^l}{\rho_{\rm jam}}\right)^m \quad (4)$$

where N is the sample time interval; $t \in \{0, 1, \cdots, \infty\}$ is the t-th time interval; $i \in \{1, \cdots, I\}$ is the i-th section of a freeway, and I is the total number of sections; $\rho_i(t)$ (veh/lane/km) is the traffic density; $v_i(t)$ (km/h) is the mean traffic speed; $q_i(t)$ (veh/h) is the traffic flow leaving section i and entering section $i+1$; $r_i(t)$ (veh/h) is on-ramp traffic volume; $s_i(t)$ (veh/h) is off-ramp traffic volume, which is regarded as an unknown disturbance; L_i (km) is the length of freeway section; $v_{\rm free}$ and $\rho_{\rm jam}$ are the free speed and the maximum possible density per lane, respectively; τ, γ, κ, l, m, and ω are constant parameters that reflect particular characteristics of a given traffic system and depend on the freeway geometry, vehicle characteristics, drivers′ behaviors, etc.

Remark 1. The macroscopic traffic flow model (1) $\sim$ (4) in this paper only serves to formulate the traffic control problem more clearly and to provide a simulation evaluation. The design and analysis of the MFPAC does not require any information of the traffic flow model as shown in the following sections.

The boundary conditions of freeway traffic flow are summarized as: $\rho_0(t) = q_0(t)/v_1(t)$, $v_0(t) = v_1(t)$, $\rho_{I+1}(t) = \rho_I(t)$, and $v_{I+1}(t) = v_I(t)$.

1.2 General nonlinear representation and some assumptions

With the state space, we can also express the traffic dynamics (1) $\sim$ (4) in a general nonlinear form[1]

$$\boldsymbol{x}(t+1) = \boldsymbol{f}[\boldsymbol{x}(t), \boldsymbol{r}(t), \boldsymbol{d}(t)] \tag{5}$$

where the state vector $\boldsymbol{x}(t) \in \mathbf{R}^n$ comprises all traffic densities and mean speeds, as well as all ramp queues; the control vector $\boldsymbol{r}(t) \in \mathbf{R}^m$ comprises all controllable ramp volumes; the disturbance vector $\boldsymbol{d}(t) \in \mathbf{R}^p$ comprises all on-ramp demands and turning rates; $\boldsymbol{f}[\cdot,\cdot,\cdot] \in \mathbf{R}^n$ is an unknown vector valued function.

Assumption 1. The target trajectory $\boldsymbol{x}_d(t)$ of the desired traffic density and velocity, is periodic with a known common periodicity $T > 1$, i.e. $\boldsymbol{x}_d(t) = \boldsymbol{x}_d(t-T)$.

Assumption 2. The partial derivative of $\boldsymbol{f}[\cdot,\cdot,\cdot]$ with respect to control input $\boldsymbol{r}(t)$ is continuous.

Assumption 3. System (1) is generalized Lipschitz, i.e., $\forall t$ and $\Delta \boldsymbol{r}(t) \neq 0$,

$$\|\Delta \boldsymbol{x}(t+1)\| \leq M \|\Delta \boldsymbol{r}(t)\| \tag{6}$$

where M is a constant, $\Delta \boldsymbol{x}(t+1) = \boldsymbol{x}(t+1) - \boldsymbol{x}(t-T+1)$, and $\Delta \boldsymbol{r}(t) = \boldsymbol{r}(t) - \boldsymbol{r}(t-T)$.

Remark 2. Assumption 3 can be seen as some limitation to the rate of the change of the system output. It implies that the finite variation of the on-ramp traffic volume does not cause an infinite variation of the traffic density. Clearly, it holds in practice. Furthermore, we just need the existence of such a constant M without requiring the exact value.

1.3 Dynamical linearization

Theorem 1. For system (5) satisfying Assumptions 1 $\sim$ 3, there must exist a parameter matrix $\Phi(t)$, called pseudo Jacobi matrix, such that when $\|\Delta \boldsymbol{r}(t)\| \neq 0$,

$$\Delta \boldsymbol{x}(t+1) = \Phi(t) \Delta \boldsymbol{r}(t) \tag{7}$$

and $\|\Phi(t)\| \leq M$ with M is defined in Assumption 3.

Proof. From system (5), we have

$$\begin{aligned} &\Delta \boldsymbol{x}(t+1) = \\ &\quad \boldsymbol{f}[\boldsymbol{x}(t), \boldsymbol{r}(t), \boldsymbol{d}(t)] - \boldsymbol{f}[\boldsymbol{x}(t-T), \boldsymbol{r}(t-T), \boldsymbol{d}(t-T)] = \\ &\quad \boldsymbol{f}[\boldsymbol{x}(t), \boldsymbol{r}(t), \boldsymbol{d}(t)] - \boldsymbol{f}[\boldsymbol{x}(t), \boldsymbol{r}(t-T), \boldsymbol{d}(t)] + \\ &\quad \boldsymbol{f}[\boldsymbol{x}(t), \boldsymbol{r}(t-T), \boldsymbol{d}(t)] - \boldsymbol{f}[\boldsymbol{x}(t-T), \boldsymbol{r}(t-T), \boldsymbol{d}(t-T)] = \\ &\quad \frac{\partial \boldsymbol{f}^*}{\partial \boldsymbol{r}(t)} \Delta \boldsymbol{r}(t) + \boldsymbol{f}[\boldsymbol{x}(t), \boldsymbol{r}(t-T), \boldsymbol{d}(t)] - \\ &\quad \boldsymbol{f}[\boldsymbol{x}(t-T), \boldsymbol{r}(t-T), \boldsymbol{d}(t-T)] \end{aligned} \tag{8}$$

where $\frac{\partial \boldsymbol{f}^*}{\partial \boldsymbol{r}(t)} = \begin{bmatrix} \frac{\partial f_1^*}{\partial r_1(t)} & \frac{\partial f_1^*}{\partial r_2(t)} & \cdots & \frac{\partial f_1^*}{\partial r_m(t)} \\ \frac{\partial f_2^*}{\partial r_1(t)} & \frac{\partial f_2^*}{\partial r_2(t)} & \cdots & \frac{\partial f_2^*}{\partial r_m(t)} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n^*}{\partial r_1(t)} & \frac{\partial f_n^*}{\partial r_2(t)} & \cdots & \frac{\partial f_n^*}{\partial r_m(t)} \end{bmatrix} \in \mathbf{R}^{n \times m}$ is a matrix of the proper partial derivative values of $\boldsymbol{f}[\cdot,\cdot,\cdot]$ with respect to $\boldsymbol{r}(t)$ at some point in the time interval between t and $(t-T)$.

Let $\boldsymbol{\xi}(t) = \boldsymbol{f}[\boldsymbol{x}(t), \boldsymbol{r}(t-T), \boldsymbol{d}(t)] - \boldsymbol{f}[\boldsymbol{x}(t-T), \boldsymbol{r}(t-T), \boldsymbol{d}(t-T)]$. Consider an equation with matrix $\Xi(t) \in \mathbf{R}^{n \times m}$ as follows

$$\xi(t) = \Xi(t) \Delta \boldsymbol{r}(t) \tag{9}$$

When $\|\Delta \boldsymbol{r}(t)\| \neq 0$, we must have the solution $\Xi(t) = \boldsymbol{\xi}(t) \Delta \boldsymbol{r}^{\mathrm{T}}(t) / (\Delta \boldsymbol{r}^{\mathrm{T}}(t) \Delta \boldsymbol{r}(t))$ for (9).

Let $\Phi(t) = \frac{\partial \boldsymbol{f}^*}{\partial \boldsymbol{r}(t)} + \Xi(t)$, (7) can be obtained directly from (8) and (9). In terms of Assumption 3, we have $\|\Phi(t)\| \leq M$. $\square$

2 Model-free periodic adaptive controller for on-ramp metering

2.1 Controller design

Given a target trajectory $\boldsymbol{x}_d(t)$ of the desired traffic density and velocity with the known period T, the control target is to find a sequence of appropriate ramp volumes $\boldsymbol{r}(t)$ such that the tracking error $\boldsymbol{e}(t) = \boldsymbol{x}_d(t) - \boldsymbol{x}(t)$ converges to zero over the entire period as the period number approaches to infinity.

Define an index function of control input as

$$J(\boldsymbol{r}(t)) = \|\boldsymbol{e}(t+1)\|^2 + \lambda \|\boldsymbol{r}(t) - \boldsymbol{r}(t-T)\|^2 \tag{10}$$

where λ is a positive weighting factor.

Using the optimal condition $\frac{1}{2} \frac{\partial J}{\partial \boldsymbol{r}(t)} = 0$, and according to (7), we have

$$\boldsymbol{r}(t) = \boldsymbol{r}(t-T) + \frac{\eta \Phi^{\mathrm{T}}(t)}{\lambda + \|\Phi(t)\|^2} \boldsymbol{e}(t-T+1) \tag{11}$$

where $\eta > 0$ is a step-size constant series, which is added to have the generality of the algorithm (11) and will be used in stability analysis later.

Because $\Phi(t)$ is unknown and not available, we present the periodic learning control law as

$$\boldsymbol{r}(t) = \begin{cases} \boldsymbol{r}(t-T) + \dfrac{\eta \hat{\Phi}^{\mathrm{T}}(t)}{\lambda + \|\hat{\Phi}(t)\|^2} \boldsymbol{e}(t-T+1), & \\ & t \in \{T, T+1, \cdots\} \\ \boldsymbol{r}_0, & t \in \{0, \cdots, T-1\} \end{cases} \tag{12}$$

where $\boldsymbol{r}_0$ denotes the initial input values in the first period, and $\hat{\Phi}(t)$ is to learn parameter $\Phi(t)$ and updated in terms of the optimal solution of the following criterion index function

$$\begin{aligned} J(\hat{\Phi}(t)) = \|\Delta \boldsymbol{x}(t-T+1) - \hat{\Phi}(t) \Delta \boldsymbol{r}(t-T)\|^2 + \\ \mu \| b\hat{\Phi}(t) - \hat{\Phi}(t-T)\|^2 \end{aligned} \tag{13}$$

Using the optimal condition $\frac{\partial J}{\partial \hat{\Phi}(t)} = 0$, we can obtain the parameter updating law as follows

$$\hat{\Phi}(t) = \begin{cases} \hat{\Phi}(t-T) + \\ \dfrac{\beta [\Delta \boldsymbol{x}(t-T+1) - \hat{\Phi}(t-T) \times \Delta \boldsymbol{r}(t-T)] \Delta \boldsymbol{r}^{\mathrm{T}}(t-T)}{\mu + \|\Delta \boldsymbol{r}(t-T)\|^2}, \\ & t \in \{T, T+1, \cdots\} \\ \hat{\Phi}_0, & t \in \{0, \cdots, T-1\} \end{cases} \tag{14}$$

where $\mu > 0$ is the positive weighting factor, $\beta \in (0, 2)$ is a step-size constant series added to make the generality of the algorithm (14), and $\hat{\Phi}_0$ can be chosen arbitrarily.

In order to ensure the condition $\|\Delta \boldsymbol{r}(t)\| \neq 0$ holds for any t, and periodic estimate algorithm (14) has stronger ability in tracking performance, we present a reset algorithm as follows:

$$\hat{\Phi}(t) = \hat{\Phi}_0, \quad \text{if} \quad \|\hat{\Phi}(t)\| \leq \epsilon \quad \text{or} \quad \|\Delta \boldsymbol{r}(t)\| \leq \epsilon \tag{15}$$

where ϵ is a small positive constant.

Remark 3. It should be noted that in theory, $\|\Delta \boldsymbol{r}(t)\| \to 0$ as $t \to \infty$ by the proposed method. However, in control practice, a perfect tracking is never achieved due to the influence of disturbances and some other factors. Thus, the contradiction of (15) with the theory results can be neglected in practice.

Remark 4. For the proposed MFPAC (12), (14), and (15), what we need is to tune the parameters η and β in a small range with properly fixed values of λ and μ, without requiring any priori knowledge of the dynamic system.

2.2 Convergence analysis

Assumption 4. The pseudo Jacobi matrix $\Phi(t)$ is positive (or negative) for all t and $\Phi(t) = 0$ holds only for some finite time points.

Remark 5. This assumption is similar to the limitation of control input direction. For example, the traffic flow density will increase (or not decrease at least) when the on-ramp metering traffic volume increases in practice.

Theorem 2. For freeway traffic control system (5) satisfying Assumptions $1 \sim 4$, the presented model-free periodic adaptive control (12), (14), and (15) can guarantee that:

1) The parameter estimation value $\hat{\Phi}(t)$ is bounded;

2) The tracking error $\boldsymbol{e}(t)$ converges to zero exponentially and point-wisely as t approaches to infinity.

Proof. The proof consists of two parts. Part 1 derives the boundedness of $\hat{\Phi}(t)$. Part 2 proves the asymptotic convergence of the tracking error.

Part 1. The Boundedness of $\hat{\Phi}(t)$.

Case 1. For $t \in \{0, \cdots, T-1\}$, clearly $\hat{\Phi}(t)$ is bounded.

Case 2. For $t \in \{T, T+1, \cdots\}$, first when $\|\Delta \boldsymbol{r}(t)\| \leq \epsilon$, clearly $\hat{\Phi}(t)$ is bounded according to (15). When $\|\Delta \boldsymbol{r}(t)\| > \epsilon$, subtracting $\Phi(t)$ from both sides of (14), we have

$$\tilde{\Phi}(t) = \left(1 - \frac{\beta \Delta \boldsymbol{r}(t-T) \Delta \boldsymbol{r}^{\mathrm{T}}(t-T)}{\mu + \|\Delta \boldsymbol{r}(t-T)\|^2}\right) \tilde{\Phi}(t-T) - (\Phi(t) - \Phi(t-T)) \tag{16}$$

where $\tilde{\Phi}(t) = \hat{\Phi}(t) - \Phi(t)$.

Let $\Delta\Phi(t) = \Phi(t) - \Phi(t-T)$. Substituting (7) into (16), we have

$$\|\tilde{\Phi}(t)\| \leq \left\|1 - \frac{\beta \Delta \boldsymbol{r}(t-T) \Delta \boldsymbol{r}^{\mathrm{T}}(t-T)}{\mu + \|\Delta \boldsymbol{r}(t-T)\|^2}\right\| \|\tilde{\Phi}(t-T)\| + 2M \tag{17}$$

Noting that for $\|\Delta \boldsymbol{r}(t)\| > \epsilon$, $\mu > 0$, and $\beta \in (0, 2)$, we have

$$0 < \left\|1 - \frac{\beta \Delta \boldsymbol{r}(t-T) \Delta \boldsymbol{r}^{\mathrm{T}}(t-T)}{\mu + \|\Delta \boldsymbol{r}(t-T)\|^2}\right\| \leq d_1 < 1 \tag{18}$$

For any $t \in \{pT, pT+1, \cdots, (p+1)T-1\}$ and noticing $t_0 = t - pT$, we have

$$\|\tilde{\Phi}(t)\| \leq d_1 \|\tilde{\Phi}(t-T)\| + 2M \leq \cdots \leq d_1^p \|\tilde{\Phi}(t_0)\| + \frac{2M}{1-d_1} \tag{19}$$

Since $t_0 \in \{0, \cdots, T-1\}$ and when $t \to \infty$, $p = (t-t_0)/T \to \infty$, according to (19),

$$\lim_{p\to\infty} \|\tilde{\Phi}(t)\| \leq \lim_{p\to\infty} d_1^p \max_{t_0 \in \{0,\cdots,T-1\}} \{\|\tilde{\Phi}(t_0)\|\} + \frac{2M}{1-d_1} \leq \max_{t_0 \in \{0,\cdots,T-1\}} \{\|\tilde{\Phi}(t_0)\|\} + \frac{2M}{1-d_1} \tag{20}$$

Hence, $\tilde{\Phi}(t)$ is bounded. Because $\|\Phi(t)\| \leq M$, then $\hat{\Phi}(t)$ is bounded for all t.

Part 2. The exponential convergence.

For any $t \in \{T, T+1, \cdots\}$, the dynamics of the tracking error can be expressed as follows in terms of (7)

$$\begin{aligned}&\|\boldsymbol{e}(t+1)\| = \|\boldsymbol{x}_d(t+1) - \boldsymbol{x}(t+1)\| = \\ &\|\boldsymbol{x}_d(t-T+1) - \boldsymbol{x}(t-T+1) - \Phi(t)\Delta r(t)\| = \\ &\|\boldsymbol{e}(t-T+1) - \Phi(t)\Delta \boldsymbol{r}(t)\| \leq \\ &\left\|1 - \frac{\eta\Phi(t)\hat{\Phi}^{\mathrm{T}}(t)}{\lambda + \|\hat{\Phi}(t)\|^2}\right\| \|\boldsymbol{e}(t-T+1)\| = \\ &d_2(t)\|\boldsymbol{e}(t-T+1)\|\end{aligned} \tag{21}$$

where $d_2(t) = \|1 - \frac{\eta\Phi(t)\hat{\Phi}^{\mathrm{T}}(t)}{\lambda + \|\hat{\Phi}(t)\|^2}\|$.

For any $t \in \{pT, pT+1, \cdots, (p+1)T-1\}$ and $t_0 = t - pT$, we have

$$\|\boldsymbol{e}(t+1)\| \leq \prod_{i=1}^{p} d_2(t_0 + kT) \|\boldsymbol{e}(t_0+1)\| \leq \prod_{i=1}^{p} d_2(t_0 + kT) \max_{t_0 \in \{0,\cdots,T-1\}} \{\|\boldsymbol{e}(t_0+1)\|\} \tag{22}$$

From Assumption 4 and resetting algorithm (15), we know that $\Phi(t)\hat{\Phi}^{\mathrm{T}}(t)$ is nonnegative, and $\Phi(t)\hat{\Phi}^{\mathrm{T}}(t) = 0$ only holds at some finite time instants. Thus, except for these finite instants where $d_2(t_0 + kT) = 1$, we can choose η and λ appropriately such that $\forall k = 1, 2, \cdots$ and $\forall t_0 \in \{0, \cdots, T-1\}$, $0 < d_2(t_0 + kT) < 1$ strictly holds.

Since $t_0 \in \{0, \cdots, T-1\}$ and when $t \to \infty$, $p = (t-t_0)/T \to \infty$, according to (22)

$$\lim_{t\to\infty} \|\boldsymbol{e}(t+1)\| \leq \lim_{t\to\infty} \prod_{i=1}^{p} d_2(t_0 + kT) \max_{t_0 \in \{0,\cdots,T-1\}} \{\|\boldsymbol{e}(t_0+1)\|\} \tag{23}$$

Since $\forall k = 1, 2, \cdots$ and $\forall t_0 \in \{0, \cdots, T-1\}$, $0 < d_2(t_0 + kT) < 1$ strictly holds, we have $\lim_{t\to\infty} \prod_{k=1}^{p} d_2(t_0+kT) = 0$. Also, because initial error $\boldsymbol{e}(t_0+1), t_0 \in \{0, \cdots, T-1\}$ is bounded, clearly (23) implies that $\boldsymbol{e}(t) \to 0$ as $t \to \infty$. □

3 Simulation study

In order to verify the effectiveness of the MFPAC approach, consider a freeway subdivided into 12 sections, and the length of each section is 0.5 km. Assume there is an on-ramp located in section 7 and one off-ramp located on section 4. For all the sections, the initial density and the initial mean speed are 30 veh/lan/km and 50 km/h, respectively. Other parameters used in this model are listed as: $V_{\text{free}} = 80$ km/h, $\rho_{\text{jam}} = 30$ veh/lane/km, $l = 0.5$, $m = 1.7$, $\kappa = 13$, $\tau = 0.01$, $N = 0.00417$ h, $\gamma = 35$, $r_i(t) = 0$ veh/h, and $\alpha = 0.95$.

The exiting flow $s(t)$ in the off-ramp is regarded as a large exogenous disturbance in the simulation. Also to show the sensitivity of the proposed control law to the random disturbances, a random disturbance uniformly distributed on the interval $(-15, 15)$ is added to the initial traffic flow on the mainstream for all the time instants, that is the initial traffic volume entering section 1 is $1\,500(1 + 0.01\, randn)$ veh/h, where the $randn$ function generates arrays of random numbers whose elements are normally distributed with mean value 0, variance 1, and standard deviation 1.

The control objective is to apply the proposed model-free periodic adaptive control mechanism to generate a proper value of $r_i(t)$ to drive the traffic density $\rho_i(t)$ track the desired traffic density $\rho_{i,d}(t) = 30 + 3\sin(t\pi/25)$ with the disturbances occurring at on-ramp and off-ramp. Clearly, the desired traffic density is periodic with a known periodicity $T = 50N$, where $N = 0.00417$ h is the sampling time.

Through substantial simulations, the parameters of the proposed model-free periodic adaptive control are chosen as: $\eta = 16$, $\beta = 0.0001$, $\mu = 0.01$, $\lambda = 0.001$, $\epsilon = 0.00005$. By using $|e_{k,i}|_{\sup}$ to record the maximum absolute tracking error of section i during the k-th period, i.e. $|e_{k,i}|_{\sup} = \sup\limits_{t\in\{(k-1)T,\cdots,kT-1\}} |\rho_i(t) - \rho_{d,i}(t)|, k = 0, 1, 2, \cdots$, the learning convergence of MFPAC is shown in Fig. 2.

For the purpose of comparison, we also apply the model-free adaptive control method to the freeway traffic density control as done in [6]. Fig. 3 shows the learning convergence of MFAC vs. period number.

From the simulation results, we can see that the proposed MFPAC has the ability of learning from the corresponding point in previous periods, as a result, the maximum absolute tracking error over the entire period converges to zero in a pointwise manner as the period number approaches to infinity (Fig. 2). While by virtue of no learning from preceding periods, the traditional MFAC performs poorly for the periodic tracking task (Fig. 3).

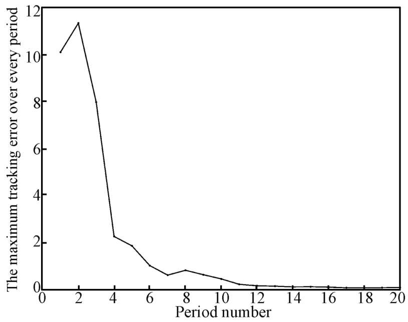

Fig. 2 The convergence of maximum tracking error in section 7 by using the proposed MFPAC

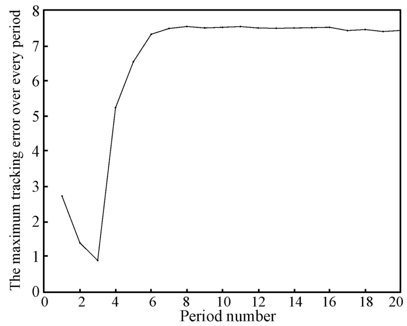

Fig. 3 The convergence of maximum tracking error in section 7 by using the traditional MFAC

Remark 6. The mean speed of traffic flow is strictly related to the traffic density. Thus, we can just control the traffic density to improve the transportation capability of the freeway system.

4 Conclusion

In this paper, a novel model-free periodic adaptive control approach is presented for freeway traffic control by taking the advantage of its distinct feature of periodicity. It is model-free without requiring any other knowledge of the system except for the I/O data. The proposed method is updated by the I/O data derived from the corresponding point in previous periods; thus it has the ability of learning from preceding periods and performs well for periodic tracking tasks. Both convergence analysis and simulation results illustrate the validity of the presented methods.

References

1 Papageorgiou M, Kotsialos A. Freeway ramp metering: an overview. *IEEE Transactions on Intelligent Transportation Systems*, 2002, **3**(4): 271−281

2 Parageorgiou M, Blosseville J M, Hadj-Salem H. Modeling and real time control on traffic flow on the southern part of Bpulevard Peripherique in Paris, part I: modeling, part II: coordinated on-ramp metering. *Transportation Research Part A: General*, 1990, **24**(5): 345−370

3 Akiyama T, Okushima M. Advanced fuzzy traffic controller for urban expressway. *International Journal of Innovative Computing, Information and Control*, 2006, **2**(2): 339−355

4 Hou Z S, Huang W H. The model-free learning adaptive control of a class of SISO nonlinear systems. In: Proceedings of the American Control Conference. Albuquerque, USA: IEEE, 1997. 343−344

5 Chi R H, Hou Z S. Optimal higher order learning adaptive control approach for a class of SISO nonlinear systems. *Journal of Control Theory and Applications*, 2005, **3**(3): 247−251

6 Chi R H, Hou Z S. A model-free adaptive control approach for freeway traffic density via ramp metering. *International Journal of Innovative Computing, Information and Control*, 2008, **4**(11): 2823−2832

7 Hou Z S, Xu J X, Zhong H W. Freeway traffic control using iterative learning control based ramp metering and speed signaling. *IEEE Transactions on Vehicular Technology*, 2007, **56**(2): 466−477

8 Xu J X. A new periodic adaptive control approach for time-varying parameters with known periodicity. *IEEE Transactions on Automatic Control*, 2004, **49**(4): 579−583

9 Sun M X, Ge S S. Adaptive repetitive control for a class of nonlinearly parameterized systems. *IEEE Transactions on Automatic Control*, 2006, **51**(10): 1684−1688

10 Sun Y P, Li J M, Zhang G. Adaptive repetitive learning control for a class of nonlinear time-varying systems. In: Proceedings of the 26th Chinese Control Conference. Zhangjiajie, China: IEEE, 2007. 766−770

CHI Rong-Hu Received his Ph. D. degree from Beijing Jiaotong University in 2007. He is currently an associate professor at the School of Automation and Electrical Engineering, Qingdao University of Science and Technology. His research interest covers adaptive control, model-free adaptive control, learning control, and intelligent transportation systems. Corresponding author of this paper. E-mail: rhchi@163.com

HOU Zhong-Sheng Professor at the Advanced Control Systems Laboratory, Beijing Jiaotong University. His research interest covers the fields of the model free adaptive control, learning control, and intelligent transportation systems.
E-mail: houzhongsheng@china.com