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Chaos Control in the Problem of a Satellite

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Abstract

Analytically control term has been determined to control chaos in the problem of a satellite. Computational studies reveal the suppression of chaos which is in good agreement with the analytical investigations.

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1 Introduction

During the last decade or so, since chaos can be harmful in several contexts therefore much attention has been paid to the studies of chaos control. The meaning of control here is to reduce or suppress chaos by means of a perturbation so that the original structure of the system under investigation is kept unaltered [4, 6]. To suppress chaos is a long-standing and extremely interesting problem in several branches of physics and other disciplines.

Elliptically orbiting planer oscillations of satellites in the solar system make an interesting study, and significant contributions to this end can be found in the work of, e.g. Maciejewski [6], Singh and Demin [9], Singh [8], and Khan [5], all of whom have studied the influence of certain perturbative forces such as solar radiation pressure, tidal force and air resistance. In the present work, we consider the spin-orbit coupling problem for a satellite, and in the equation of motion we address the effect of solar radiation pressure and tidal torque. This type of model is generally used for investigation of the rotational motion of natural satellites. We use control theory of Hamiltonian system based on

[2, 3] to achieve control of chaos in the system. In dynamical systems, the Hamiltonian ones are difficult to control due to their special geometry and the absence of phase space attractors. In this article, the problem we address is how to suppress or control the chaos arising in Hamiltonian system of the satellite. We have employed the method of control of chaos based on [1]. For the perturbed Hamiltonian $H = H_0 + eV$, we compute analytically the control term f of order $o(e^2)$ such that $H = H_0 + eV + f$ has more regular behaviour than the original system.

2 Control Theory of Hamiltonian Systems

Let $\mathcal{A}$ be the Lie algebra of real functions defined on phase space. For $H \in \mathcal{A}$, let $\{H\}$ be the linear operator action on $\mathcal{A}$ such that

$$\{H\}H' = \{H, H'\},$$

for any $H' \in \mathcal{A}$ where $\{.,.\}$ is the Poisson bracket. The time-evolution of a function $V \in \mathcal{A}$ following the flow of H is given by

$$\frac{dV}{dt} = \{H\}V,$$

which is formally solved as

$$V(t) = e^{t\{H\}}V(0),$$

if H is time independent, and where

$$e^{t\{H\}} = \sum_{n=0}^{\infty} \frac{t^n}{n!}\{H\}^n.$$

Any element $V \in \mathcal{A}$ such that $\{H\}V = 0$, is constant under the flow of H, i.e.

$$\forall \ \ t \in \mathbb{R}, \qquad e^{t\{H\}}V = V.$$

Let us now fix a Hamiltonian $H_0 \in \mathcal{A}$. The vector space $\mathrm{Ker}\{H_0\}$ is the set of constants of motion and it is a sub-algebra of $\mathcal{A}$. The operator $\{H_0\}$ is not invertible since a derivation has always a non-trivial kernel. For instance $\{H_0\}(H_0^\alpha) = 0$ for any α such that $H_0^\alpha \in \mathcal{A}$. Hence we consider a pseudo-inverse of $\{H_0\}$. We define a linear operator Γ on $\mathcal{A}$ such that

$$\{H_0\}^2\Gamma = \{H_0\}, \tag{2.1}$$

i.e.

$$\forall \ V \in \mathcal{A}, \quad \{H_0, \{H_0, \Gamma V\}\} = \{H_0, V\}.$$

If the operator Γ exists, it is not unique in general. Any other choice Γ' satisfies $Rg(\Gamma' - \Gamma) \in Ker(\{H_0\}^2)$.

We define the non-resonant operator $\mathcal{N}$ and the resonant operator $\mathcal{R}$ as

$$\begin{aligned} \mathcal{N} &= \{H_0\}\Gamma \\ \mathcal{R} &= 1 - \mathcal{N}, \end{aligned}$$

where the operator 1 is the identity in the algebra of linear operators acting on $\mathcal{A}$. We notice that Equation (2.1) becomes

$$\{H_0\}\mathcal{R} = 0$$

which means that the range Rg $\mathcal{R}$ of the operator $\mathcal{R}$ is included in Ker$\{H_0\}$. A consequence is that any element $\mathcal{R}V$ is constant under the flow of H_0, i.e. $e^{t\{H_0\}}\mathcal{R}V = \mathcal{R}V$. We notice that when $\{H_0\}$ and Γ commute, $\mathcal{R}$ and $\mathcal{N}$ are projectors i.e. $\mathcal{R}^2 = \mathcal{R}$ and $\mathcal{N}^2 = \mathcal{N}$. Moreover, in this we have $Rg\mathcal{R} = Ker\{H_0\}$, i.e. the constant of motion are the elements $\mathcal{R}V$ where $V \in \mathcal{A}$.

Let us now assume that H_0 is integrable with action-angle variables $(A, \varphi) \in B \times T^n$ where B is an open set of $\mathcal{R}^n$ and T^n is the n-dimensional torus, so that $H_0 = H_0(A)$ and the Poisson bracket $\{H, H'\}$ between two Hamiltonians is

$$\{H, H'\} = \frac{\partial H}{\partial A} \cdot \frac{\partial H'}{\partial \varphi} - \frac{\partial H}{\partial \varphi} \cdot \frac{\partial H'}{\partial A}$$

The operator $\{H_0\}$ acts on V given by

$$V = \sum_{k \in Z^n} V_k(A) c^{ik\cdot\varphi}$$

as

$$\{H_0\}V(A,\varphi) = \sum_k i\omega(A).kV_k(A)e^{ik\cdot\varphi}$$

where the frequency vector is given by

$$\omega(A) = \frac{\partial H_0}{\partial A}.$$

A possible choice of Γ is

$$\Gamma V(A,\varphi) = \sum_{\substack{k \in Z^n \\ \omega(A)\cdot k \neq 0}} \frac{V_k(A)}{i\omega(A)\cdot k} e^{ik\cdot\varphi}$$

We notice that this choice of Γ commutes with $\{H_0\}$.

For a given $V \in \mathcal{A}$, $\mathcal{R}V$ is the resonant part of V and $\mathcal{N}V$ is the non-resonant part:

$$\mathcal{R}V = \sum_k V_k(A)\chi(\omega(A)\cdot k = 0)e^{ik\cdot\varphi} \tag{2.2}$$

$$\mathcal{N}V = \sum_k V_k(A)\chi(\omega(A)\cdot k \neq 0)e^{ik\cdot\varphi} \tag{2.3}$$

where $\chi(\alpha)$ vanishes when proposition α is wrong and it is equal to 1 when α is true.

From these operators defined for the integrable part H_0, we construct a control term for the perturbed Hamiltonian $H_0 + V$ where $V \in \mathcal{A}$, i.e. we construct f such that $H_0 + V + f$ is canonically conjugate to $H_0 + \mathcal{R}V$.

If H_0 is resonant and $\mathcal{R}V = 0$, the controlled Hamiltonian $H = H_0 + V + f$ is conjugate to H_0.

In the case $\mathcal{R}V = 0$, the series (6) which gives the expansion of the control term f, can be written as

$$f(V) = \sum_{s=2}^{\infty} f_s, \tag{2.4}$$

where f_s is of order ε^s and given by the recursion formula

$$f_s = -\frac{1}{s}\{\Gamma V, f_{s-1}\} \tag{2.5}$$

where $f_1 = V$.

3 Application to the Problem of A Satellite

The spin-orbit coupling problem for a satellite studied by J. Maciejewski [6] together with effects of solar radiation pressure which is of the order of eccentricity and tidal torque has been investigated in our present manuscript.

The equation of motion for satellite under consideration is

$$(1 + e\cos v)\frac{d^2q}{dv^2} - 2e\sin v\frac{dq}{dv} + \frac{\beta e(1+e\cos v)^5}{(1-e^2)^4}\frac{dq}{dv} - 4e\sin v + n^2\sin q + 2\varepsilon_1 e(1+e\cos v)^{-3}\sin\left(\frac{q}{2}+v\right) = 0\,. \tag{3.1}$$

The Hamiltonian for the above equation after ignoring higher order terms of e can be written as

$$H = \frac{p^2}{2} - 2p - n^2\cos q - e\left\{p^2\cos v + \beta pq + n^2\cos q\cos v + 4\varepsilon_1\cos\left(\frac{q}{2}+v\right)\right\} \tag{3.2}$$

In order to apply the control theory [10], we need to put the Hamiltonian in an autonomous form. We consider v as an additional angle whose conjugate action is E. Then in the autonomous form Hamiltonian can be perceived as

$$\begin{aligned} H(p,q,E,v) &= \frac{p^2}{2} - 2p - n^2\cos q + E \\ &\quad - e\left\{p^2\cos v + \beta pq\cos v + n^2\cos q + 4\varepsilon_1\cos\left(\frac{q}{2}+v\right)\right\}. \end{aligned} \quad (3.3)$$

where the actions are $A = (p, E)$ and the angles are $\phi = (q, v)$.

The unperturbed Hamiltonian to be used for constructing the operator Γ is

$$H_0 = \frac{p^2}{2} - 2p - n^2\cos q + E \quad (3.4)$$

The action of $\{H_0\}$ and Γ on
$V = -e\{p^2\cos v + \beta pq + n^2\cos q\cos v + 4\varepsilon_1\cos\left(\frac{q}{2}+v\right)$ is, $V \in \mathcal{A}$ is expressed as:

$$\begin{aligned} \{H_0\}V &= e\Big[n^2(p-2)\sin q\cos v + 2p\varepsilon_1\sin\left(\frac{q}{2}+v\right) \\ &\quad -\beta p^2 + 2\beta p + p^2\sin v + n^2\cos q\sin v\Big] \end{aligned} \quad (3.5)$$

$$\Gamma V = e\left[\frac{n^2\sin q\cos v}{(p-2)} + \frac{2\varepsilon_1\sin\left(\frac{q}{2}+v\right)}{p} - \frac{\beta}{p^2} + \frac{2\beta}{p} + \frac{\sin v}{p^2}\right] \quad (3.6)$$

for $p \neq 0, 2$.

The control term f is given by

$$\begin{aligned} f &= \frac{-1}{2}\{\Gamma V, V\} \\ &= \frac{-1}{2}\left\{\frac{\partial \Gamma V}{\partial p}\cdot\frac{\partial V}{\partial q} - \frac{\partial \Gamma V}{\partial q}\frac{\partial V}{\partial p}\right\} \end{aligned}$$

The explicit expression of $f$for $p = 1$ is given by

$$\begin{aligned} f &= \frac{1}{2}e^2\Big[n^4\sin^2 q\cos^2 v + 4n^2\varepsilon_1\sin\left(\frac{q}{2}+v\right)\cos v\sin q \\ &\quad + 2n^2\sin v\cos v\sin q + 4\varepsilon_1^2\sin^2\left(\frac{q}{2}+v\right) + 4\varepsilon_1\sin v\sin\left(\frac{q}{2}+v\right) \\ &\quad - \beta n^2\sin q\cos v - 2\beta\varepsilon_1\sin\left(\frac{q}{2}+v\right) - 2\beta\sin v + 2n^2\cos^2 v\cos q \\ &\quad - 2\varepsilon_1\cos v\cos\left(\frac{q}{2}+t\right) + \beta qn^2\cos v\cos q - \beta q\varepsilon_1\cos\left(\frac{q}{2}+v\right)\Big]. \end{aligned} \quad (3.7)$$

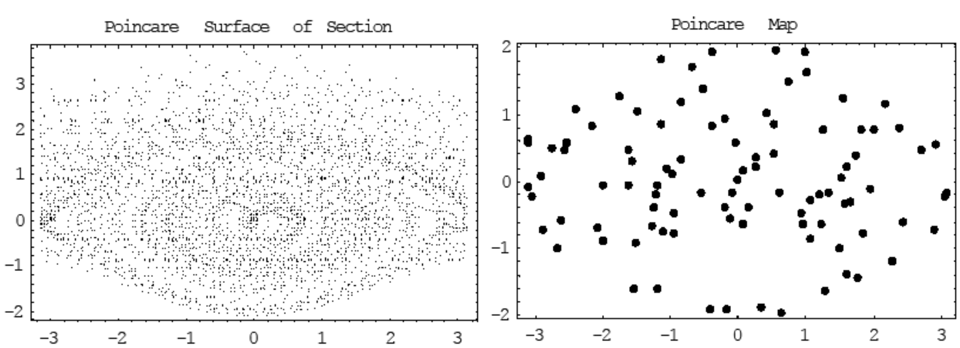
We can decrease the amplitude of the control term by considering a control parameter α in the expression of f i.e.

$$
\begin{aligned}
f \;=\; & \frac{1}{2}\alpha e^2 \Big[n^4 \sin^2 q \cos^2 v + 4n^2\varepsilon_1 \sin\left(\frac{q}{2}+v\right)\cos v \sin q \\
& + 2n^2 \sin v \cos v \sin q + 4\varepsilon_1^2 \sin^2\left(\frac{q}{2}+v\right) + 4\varepsilon_1 \sin v \sin\left(\frac{q}{2}+v\right) \\
& - \beta n^2 \sin q \cos v - 2\beta\varepsilon_1 \sin\left(\frac{q}{2}+v\right) - 2\beta \sin v + 2n^2 \cos^2 v \cos q \\
& - 2\varepsilon_1 \cos v \cos\left(\frac{q}{2}+t\right) + \beta q n^2 \cos v \cos q - \beta q \varepsilon_1 \cos\left(\frac{q}{2}+v\right) \Big]. \qquad (3.8)
\end{aligned}
$$

4 Results and Discussions

Figure 1 depicts Poincare surface of section and Poincare map of the Hamiltonian given by equation (3.3) without the inclusion of control term for $e = 0.1$, $n = 1.0$, $\varepsilon_1 = 0.008$, $\beta = 0.002$ which exhibits chaotic behavior. Figure 2 depicts the Poincare surface of section and Poincare Map of the same Hamiltonian with the inclusion of control term given by (3.8) for $e = 0.1$, $n = 1.0$, $\varepsilon_1 = 0.008$, $\beta = 0.002$ and $\alpha = 0.9$.

From these figures we observed that for $e = 0.1$ the system exhibits chaotic behavior without the control term. With the addition of the control term the chaos is suppressed for a particular value of the control parameter $\alpha = 0.9$.

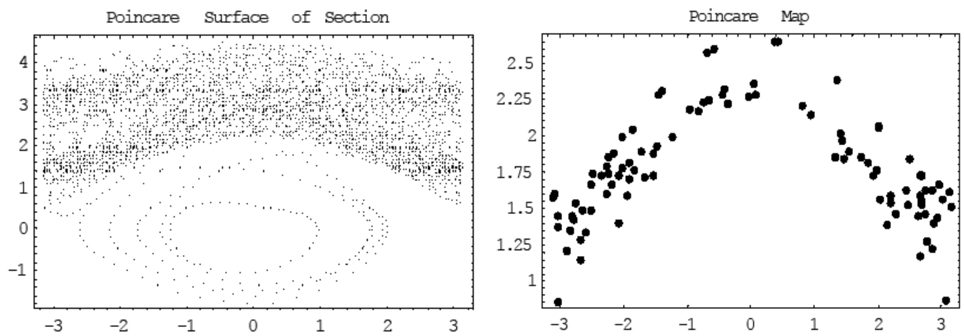


(a) (b)

Figure 1

(a) Poincare surface of section for $e = 0.1$ without the control term.

(b) Poincare Map for $e = 0.1$ without the control term.



(a) (b)

Figure 2
(a) Poincare surface of section for $e = 0.1$ with the control term.
(b) Poincare Map for $e = 0.1$ with the control term.

5 Conclusion

Remarkably we have found that the system is able to suppress chaos for the particular value of the controlling parameter α which asserts that the control term obtained analytically is very effective to control the system under consideration. Consequently, our analytical and computational studies are in good agreement.

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