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Hybrid FEC/ARQ Performance and Audio Quality over Wireless Links

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Abstract

Real time flows transmission over wireless networks often suffers from packet loss, delay and jitter. The main problem to implement a wireless network is the high and variable bit error rate in the radio link. In this paper, a hybrid ARQ/FEC scheme with limiting the slots number between the loss of the last retransmission of the original block n and the reception of block $n + \phi$ is analyzed in an environment of real-time applications. If a block is lost, the link-level error mechanism turns to ARQ for the transmission the block. The retransmission will be done a maximum number of times denoted by δ. If after δ trials the block n does not get through the wireless link, ARQ assume that the block cannot be locally recovered. In that case, the copy of block n can be recovered if the block $n+\phi$ is well received with the slots number S_n such that it less or equal to the slots number threshold S_{th}. The effect of link layer parameters like the offset ϕ, number of LL transmission attempts, slots number threshold S_{th} and LL packet size on the performance is evaluated. We show that the combination of FEC and ARQ with limiting the S_{th} , keeps a reasonable delay and increases the packet reject rate at the wireless link.

Keywords: Hybrid FEC/ ARQ, Markov Model, audio quality

1 Introduction

One of the most attractive and already functional real time services that is being provided is the transmission of voice over the Internet, which is called Internet telephony (VoIP: Voice over

IP). transmitting real time voice over the Internet has several advantages. One major advantage is price. Placing a voice call over a packet switched network like the Internet is much cheaper than making it through the traditional PSTN (Public Switched Telephone Network). Additional advantages are the possibility of sharing files and white-boards, having secured calls through encryption, caller identification, etc.

Real-time audio transmission is now widely used over the Internet and has become a very important application. Audio quality is still however an open problem due to the loss of audio packets and the variation of end-to-end delay (jitter). These two factors are a natural result of the simple best effort service provided by the current Internet. Indeed, the Internet provides a simple packet delivery service without any guarantee on bandwidth, delay or drop probability. The audio quality deteriorates (noise, poor interactivity) when packets cross a loaded part of the Internet. In the wait for some QoS facilities from the network side like resource reservation, call admission control , etc., the problem of audio quality must be studied and solved on an end-to-end basis. Some mechanisms must be introduced at the sender and/ or at receiver to compensate for packet losses and jitter. The jitter is often solved by some adaptive play-out algorithms at the receiver. Adaptive play-out mechanisms are treated in detail in [15] , and more recently in [9]. In this paper we focus on the problem of recovery from audio packet losses.

Various strategies have been proposed to combat this problem which can be classified along the following strategies: split-connection [1], proxy-based [7]. In current and future wireless technologies hybrid FEC/ARQ strategies are frequently used. Introduction of FEC consumes wireless resources but at the same time reduces the link loss rate. On the other hand, link losses can also be alleviated by using retransmission mechanisms as ARQ schemes but they increasing end-to end delay thus reducing end-to-end TCP throughput.

Mechanisms for recovering from packet losses can be classified as open loop mechanisms, or closed loop mechanisms[2]. Closed loop end to end mechanisms like automatic repeat request (ARQ) are not adequate for real time interactive applications since they increase considerably the end to end delay due to packet retransmission. Open loop mechanisms like FEC (Forward Error Correction) are better adapted to real time applications given that packet losses are recovered without the need of a retransmission. Some redundant information is transmitted with the basic audio flow. Once a packet is lost, the receiver uses (if possible) the redundant information to reconstruct the lost information. FEC schemes are recommended whenever the end to end delay is large so that a retransmission deteriorates the end to end quality.

FEC has been often used for loss recovery in audio communication tools. It is a sender-based repair mechanism. An efficient FEC scheme is one that is able to repair most of packet losses. Now, when FEC fails to recover from a loss, applications can resort to other receiver-based repair mechanisms. The FEC schemes proposed in the literature are often simple, so that the coding and the decoding of the redundancy can be quickly done without impacting the interactivity. In particular, the redundancy is computed over small blocks of audio packets. Well known audio tools as Rat[3], and Freephone [4], generally work by adding some redundant information of packet n to the next packet $n+1$, so that if packet n is dropped in the network, it could be recovered and played out in case packet $n+1$ is correctly received. The redundant information carried by a packet is generally obtained by coding the previous packet with a code of lower rate than that of the code used for coding the basic audio flow. For example, a basic audio packet can be coded with PCM and its copy with GSM or LPC. Thus, if the reconstruction succeeds, the lost packet is played out with a copy coded at a lower rate. This has shown to give better quality than playing nothing at the receiver.

In this paper, we address the problem of audio quality under hybrid FEC/ARQ scheme. We propose an analytical expression for the audio quality at the destination in function of the parameters of FEC scheme (the amount redundancy, offset ϕ), ARQ scheme (the limit number of retransmission), and the number of slots between the loss of the last retransmission of the original block n and the reception the redundant information carried in the block $n+\phi$. Then we evaluate analytically the performance of UDP as function of the parameters of the frame rate from the UDP source and the Packet loss probability. For the numerical analysis, we use four different utility functions in the expression of the audio quality.

The remainder of the paper is organized as follows. In section 2 we depict the network reference scenario. The analytical model and its solution are presented in section 3. In section 4 we present our numerical evaluation. We conclude this work in section 5.

2 Reference Network Scenario

The reference network scenario considered in this work is depicted the following figure.

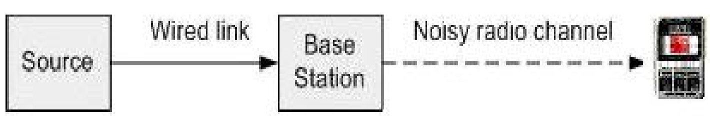

Figure 1: A model for a hybrid wired/wireless network. A connection extends over a wireless link through a base station.

In our model, we assume that the packets traverse both wired and wireless links. We consider the base station has large enough buffer and there is no buffer overflow of packets. A data packet that arrives at

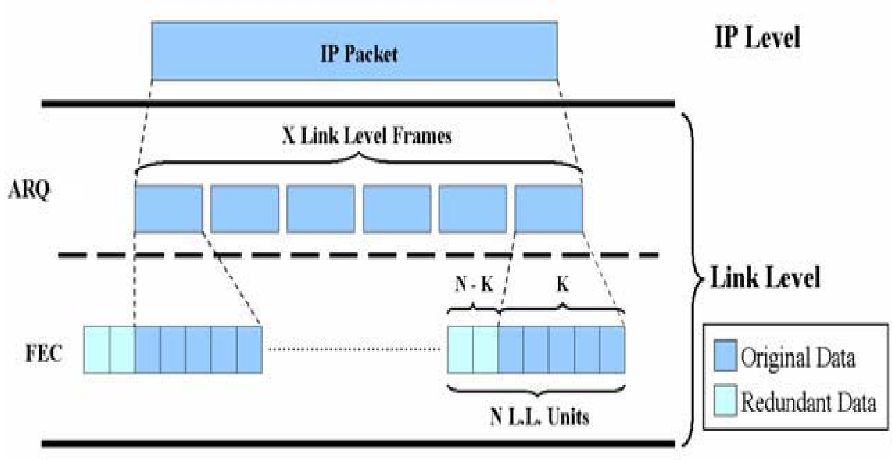

Figure 2: The hybrid model FEC/ARQ

the input of the wireless link is divided into X blocks. Note for the almost cases, the audio data packet are small. Thus, we assume that X as equal to one.

We shall consider the following scenario concerning the influence of redundancy on the packet size: Constant packet size model: we assume that the size of a packet is not affected by adding redundancy and thus redundancy is a overhead. The more the redundancy the less is the useful information carried by the packet.

In the wireless link, the data are transmitted in link level (LL) frames. We denote by B_{wl} the bandwidth of the wireless link, and D its round trip delay. In our model, we assume that the data are transmitted as packets ($X = 1$) of length K bits each. Each packet n includes, in addition to its encoded samples, information about packet $n - \phi$. However, the redundant information about packet $n - \phi$ is obtained with a low bit rate encoding of packet n. If the packet n is lost, the destination waits for packet $n + \phi$ decodes the redundant information with the slots number S_n such that S_n less or equal than S_{th}.

Note:

S_n : a random variable that indicates the number of slots between the loss of the last retransmission of the original packet n and the reception of copy (redundant information) which are located in the packet $n + \phi$.

S_{th} : indicates the number of slots threshold. The FEC field contains a more compressed data from audio flow. So even if the original data packet is lost, we can partially restore the quality of the original multimedia sample, by analyzing the lower rate data containing in this copy.

The packet size with redundant information is defined by N with $N > K$. The redundant ratio α is defined as the ratio of the amount of redundancy due to FEC, i.e. $\alpha = \frac{N-K}{K}$.

If a packet is lost, the link-level error mechanism turns to ARQ for the transmission the packet. The retransmission will be done a maximum number of times denoted by δ. If after δ trials the packet n does not get through the wireless link, ARQ assume that the packet cannot be locally recovered. In that case, the copy of packet n can be recovered if packet $n + \phi$ is well received with number of slots S_n less or equal to S_{th} . Finally, by using a shifted FEC fields, we allow to close a δ value much smaller than usual.

3 The Analytical frame Work

3.1 Markov model

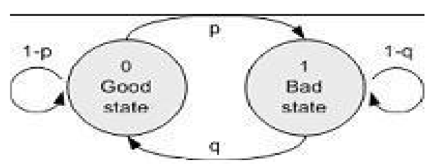

Figure 3: Two-state Markov model

The error on the link level PDU in case of Rayleigh fading is modeled with two-state discrete Markov chain (Gilber-Elliot channel [11]). That is , the probability matrix of the channel is given by

$$\begin{pmatrix} 1 - p & p \\ q & 1 - q \end{pmatrix}$$

Where $1 - p$ (respectively, q) is the probability that j^{th} packet transmission is successful, given that the $(j-1)^{th}$ packet transmission was successful (respectively, unsuccessful). Note that q^{-1} represents the average length of a burst of packet errors. In the absence of redundant information, the loss rate is

$$\pi_1 = \frac{p}{p+q}$$

In [12], the packet success/failure process on a flat Rayleigh fading channel was compute as the outcome of a comparison of the instantaneous signal-to-noise ratio to a threshold value γ. the channel is in the good state when the signal to noise ratio is higher than γ and in the bad state otherwise. As defined in [8], these probabilities can be compute with the relation

$$p = \frac{N(\Gamma_1)\Gamma_p}{\pi_0} \quad \text{and} \quad q = \frac{N(\Gamma_O)\Gamma_p}{\pi_0}$$

Where Γ_k is the SNR level relative to the Markov model state k, Γ_p is time slot and $N(\Gamma_k)$ is the level crossing rate of level Γ_k for the SNR process. This last one is defined as follows:

$$N(\Gamma_k) = \sqrt{\frac{N(2\pi\Gamma_k}{\gamma_0}} f_m \exp(-\frac{\Gamma_k}{\gamma_0})$$

f_m is the maximum Doppler frequency caused by motion at a certain speed. and γ_0 is the average SNR. Moreover, this SNR approach allows to define the steady state probability, π_0 as:

$$\pi_0 = \exp(-\frac{\Gamma_0}{\gamma_0}) - \exp(-\frac{\Gamma_1}{\gamma_0})$$

We can also use the simulation approach [12] to compute the average packet error and the conditional probability of a packet success give in that the previous packet was in error. These two quantities are then used to fully characterize a two-Markov approximation for the packet error process.

3.2 Packet loss model

In this section, we focus on the computation of the packet loss probability as function of our model parameters: p, q, ϕ, δ, S_n and S_{th}. However the packet loss process depends on the two-state Markov model.

Note that Y_n is a random variable that indicates a packet is correctly received or not, i.e. $Y_n = 1$, if a packet n is lost, and $Y_n = 0$ is a packet n is correctly received. Since the packet $n + \phi$ includes the redundant information about packet n, then a packet n is lost only if the packet n is lost and the packet $n + \phi$ is lost as well, or the packet n is lost and the packet $n + \phi$ is arrived at destination with the slots number S_n such that S_n bigger than S_{th}.

The IP packet loss probability, P_{loss}, correspond to the following relation

$$P_{loss} = P(Y_{n+\phi} = 1, Y_n = 1) + P(Y_{n+\phi} = 0, Y_n = 1)P(S_n > S_{th}) \qquad (1)$$

We denote

$$P_{\phi 1} = P(Y_{n+\phi} = 1, Y_n = 1) \quad \text{and} \quad P_{\phi 2} = P(Y_{n+\phi} = 0, Y_n = 1)$$

By substitution in (1)

$$P_{loss} = P_{\phi 1} + P_{\phi 2}[1 - P(S_n \leq S_{th})]$$

The probability $P_{\phi 1}$ can be calculated by computing all the scenario possible in the ϕ space. For example, for $\phi = 1$, the packet n is definitely lost only if the packet n is lost and the next packet $n+1$ is lost as well. Hence, in this case, the Markov chain remains in the bad state $(2\delta+2)$ slots which occurs with probability $\pi_1(1-q)^{2\delta+1}$. Moreover the probability that the first drop corresponds to the first transmission of packet n is $\frac{1}{1+\delta}$. Hence the probability P_ϕ is given by

$$P_{\phi 1} = \frac{1}{1+\delta}\pi_1(1-q)^{2\delta+1}$$

We can carry out a similar analysis and examine cases with different values of ϕ and δ. The table 2 in appendix provides some examples of $P_{\phi 1}$ for different values of ϕ and δ.

The results described in the table 3 in appendix, provides some examples of $P_{\phi 2}$ for different values of ϕ and δ. For example, for $\phi = 1$, and $\delta = 3$, the probability that the redundant information (in the packet $n + 1$) is correctly received take into account that the packet n is lost, is

$$P_{\phi 2} = \pi_1 q(1-q)^3 + \pi_1 q(1-q)^4 + \pi_1 q(1-q)^5 + \pi_1 q(1-q)^6$$

We can examine $P(S_n \leq S_{th})$ for different values of ϕ and δ such that

$$P(S_n \leq S_{th}) = P(S_n = \phi) + P(S_n = \phi + 1) + P(S_n = \phi + 2) + ... + P(S_n = S_{th})$$

And $\phi \leq S_{th} \leq \phi(1+\delta)$ The table 4 in appendix provides some examples of $P(S_n = i)$ for $\phi \leq i \leq \phi(1+\delta)$. We can carry a similar analysis in general case for $\phi = 1$, we obtain

$$P(S_n \leq S_{th}) = \pi_1[1 - (1 - q)^{\delta+1}]$$

3.3 Audio quality

In this section, we use previous work from [13],[5] and [14] to study the capability of our model and we define the audio quality received at the destination. The audio applications have strong delay constraint so that the quality deteriorates when the delay between the original and its copy increases. We assume that if the original information is lost , it can be reconstructed if the redundant information is correctly received with the number of slots S_n less or equal than S_{th}. We define the audio quality function $Q(\alpha, S_{th})$ as follows

$$Q(\alpha, S_{th}) = U(1 - \alpha)P(Y_n = 0) + U(\alpha)P(Y_n = 1)P(Y_{n+\phi} = 0/Y_n = 1)P(S_n \leq S_{th}) \qquad (2)$$

Where $U(\alpha)$ indicates how much the quality increases in function of the amount α. By using the results described in the tables (2,3,4) we can easily determine the following relations

$$P(Y_n = 1) = \pi_1(1 - q)^{\delta}, \qquad P(Y_n = 0) = 1 - \pi_1(1 - q)^{\delta}$$

$$P(Y_n = 1)P(Y_{n+\phi} = 0/Y_n = 1)P(S_n \leq S_{th}) = P(Y_n = 1) - P_{loss} \qquad (1)$$

Then it follows from (2)

$$Q(\alpha, S_{th}) = U(1 - \alpha)[1 - \pi_1(1 - q)^{\delta}] + U(\alpha)[\pi_1(1 - q)^{\delta} - P_{loss}]$$

4 Numerical analysis

4.1 Numerical framework

We apply the analytical framework developed in the previous sections. So we distinguish the relation linking the environment variables and the networks variables. In our experiments we have used a set of values for parameters which we tabulate in Table 1.

Variables	Values
Packet size (bits)	$mtu = 1536 * 8$
Frame number per packet	$X = 1$
Wired link round trip delay	$d = 0.02$
Wireless link round trip delay	$D = 0.02$
Wireless Bandwidth (bits/s.)	$B_{wl} = 6.10^6$
Ethernet Bandwidth (bits/s.)	$B_{eth} = 100.10^6$

Table 1: Parameters used in numerical examples

4.2 Packet loss probability P_{loss}

In figures (4,5,6) below, we present the variation of packet loss probability, P_{loss}, for $\phi = 1$, $\phi = 2$ and $\phi = 3$. The figures observation leads to following remarks:

- The packet loss probability P_{loss} decreases with δ, because the probability of well receive or recovered the original increase with the persistency δ.

- The P_{loss} decreases With the S_{th} because the probability $[P(Y_{n+\phi} = 0/Y_n = 1)P(S_n \leq S_{th}]$ (probability to recover the packet n in case when the packet $n + \phi$ is correctly received with S_n less or equal than S_{th}) increase with S_{th}.

- The P_{loss} increases with ϕ, because the offset raises the packets reject rate.

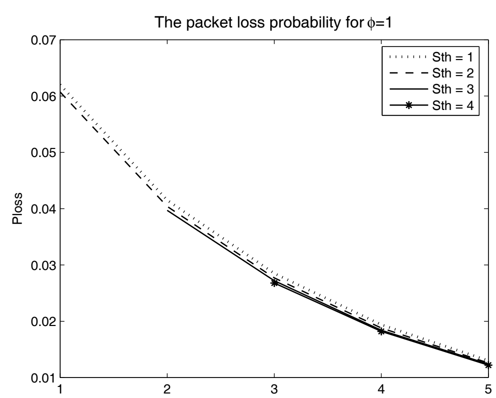

Figure 4: Packet loss probability for different values of S_{th}, $p = 0.05$, $q = 0.35$

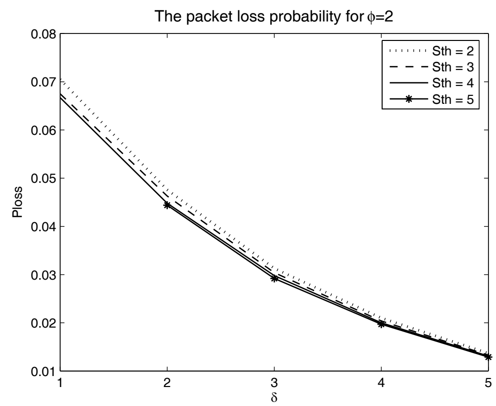

Figure 5: Packet loss probability for different values of S_{th}, $p = 0.05$, $q = 0.35$

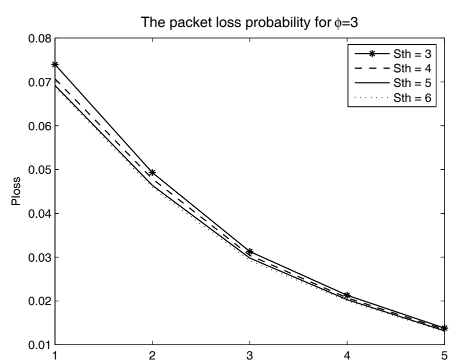

Figure 6: Packet loss probability for different values of S_{th}, $p = 0.05$, $q = 0.35$

4.3 Audio quality

Now, we plot the audio quality function based on the $U_0(\alpha)$ when the packet sizes remains unchanged after adding redundancy.

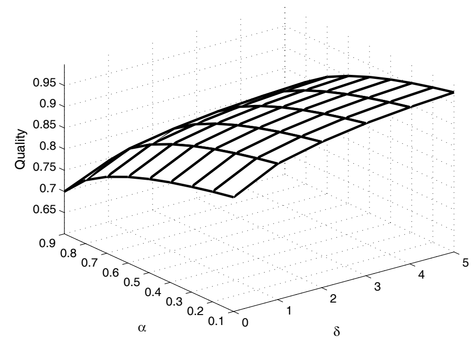

Figure 7: Flow quality given based on $U_2(\alpha) = \sqrt[10]{\alpha}$ FEC forms versus the FEC shift, the ARQ retransmission limit, δ , $\phi = 2$, $p = 0.05$, $q = 0.35$ and $S_{th} = 7$

In the figure(7), we observe that the retransmission process increases the audio quality. Indeed the probability of well receive the original increases with δ. The observation also show that the audio quality is a decreasing function on the amount of redundancy, because the more the redundancy, the less is the useful information carried by the packet ($U_0(1-\alpha)$ is an decreasing function of the α).

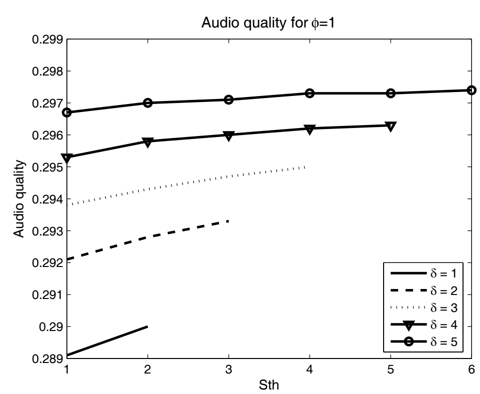

Figure 8: Flow quality based on U_0 versus the S_{th}, for different values of the ARQ retransmission limit, δ , $\alpha = 0.7$, $p = 0.05$ and $q = 0.35$.

The figures (8,9,10) show the behavior of flow quality based on U_0 versus the S_{th}, for different values of the ARQ retransmission limit δ, $\alpha = 0.7$, $p = 0.05$ and $q = 0.35$. However, we prefer focus the study over the delay effect on the quality. What is the audio quality enhancement comparing to the evolution

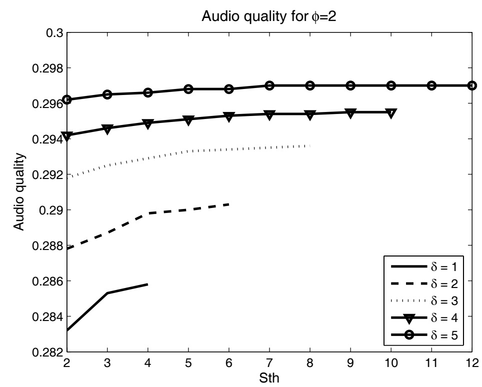

Figure 9: Flow quality based on U_0 versus the S_{th}, for different values of the ARQ retransmission limit, δ , $\alpha = 0.7$, $p = 0.05$ and $q = 0.35$.

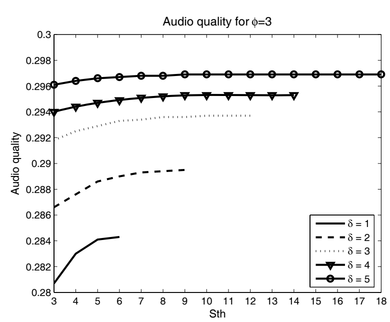

Figure 10: Flow quality based on U_0 versus the S_{th}, for different values of the ARQ retransmission limit, δ , $\alpha = 0.7$, $p = 0.05$ and $q = 0.35$.

of S_{th}? Then here, the figures show that for different values of δ, the audio quality is sensitive to the S_{th} parameter. Indeed, the audio quality increases with S_{th} which increase the probability of receive the redundancy $n + \phi$ with $S_n \leq S_{th}$, take account that the original packet n is lost. The delay lengthens when δ grows, in particular with large values of α. So, the audio quality will be deteriorated by an important delay.

We can observe, for each value of δ a critical setting $\{\delta,S_{th}\}=\{\delta,\phi(1+\delta)\}$ for which the audio quality reaches its maximum, and for each value of S_{th} the quality decreases with the offset because the ϕ raises the redundant reject rate.

We will next evaluate the quality achieved using different utility functions. The utility functions that we consider are used in [17],[12],[6] and are of the form proposed in [10],[16]:

$$U_0(\alpha) = \alpha, \quad U_1(\alpha) = \sqrt{\alpha}, \quad U_2(\alpha) = \sqrt[10]{\alpha}, \quad U_3(\alpha) = \mu(\alpha - \alpha_0) \sqrt[10]{\frac{1 - \cos(\Pi.\alpha)}{2}},$$

where

$$\mu(\alpha - \alpha_0) = \begin{cases} 1 & \alpha \geq \alpha_0 \\ 0 & \text{otherwise} \end{cases}$$

We denote α_0 as the FEC threshold for which the quality becomes equal to one.
All utilities are concave starting at some minimum and reach $U(\alpha) = 1$ for $\alpha = 1$. The least concave is the linear utility function U_0, which is thus proportional to the amount of information which is well received. The utility U_3 is zero for α less or equal than α_0. This is typical for real time applications with a minimum hard constraint. We shall plot the quality function with these four utility functions for the case when the packet sizes remains unchanged after adding redundancy.
We observe in the figures (11,12,13,14) that:

- As expected, U_2 give an upper bound for the quality. More generally, for two utility functions, U_j and U_i, if $U_i \geq U_j$ for all α, then the corresponding quality is larger. This is confirmed in the figures, taking into account that $U_2 \geq U_1 \geq U_0$.
- $U_3 \geq U_2$ for $\alpha \leq 0.5$, then the corresponding quality of U_3 is larger than that of U_2 , and conversely for $\alpha > 0.5$.

4.4 UDP throughput performance

The UDP throughput is defined as follows:

$$th_{UDP} = FR.(1 - P_{loss})$$

Where FR is the Frame Rate from the UDP source.

We analyze the UDP throughput. The figures (15,16,17,18) provide an interesting set of simulation. The figure 15 shows that the high throughput are reachable for small values of ϕ and large values of δ.

The maximal throughput can be reached for high δ values because the packet loss probability decreases. Moreover, when the FEC offset increases, the throughput decreases, particulary with a lower number of retransmission.

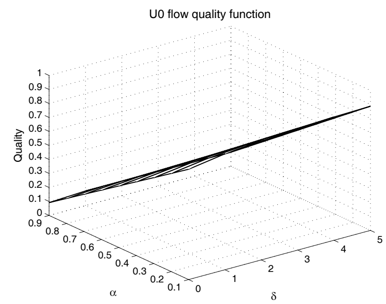

Figure 11: Flow quality based on U_0 versus the ARQ retransmission limit, δ ,$\phi = 2$, $p = 0.05$, $q = 0.35$ and $S_{th} = 12$.

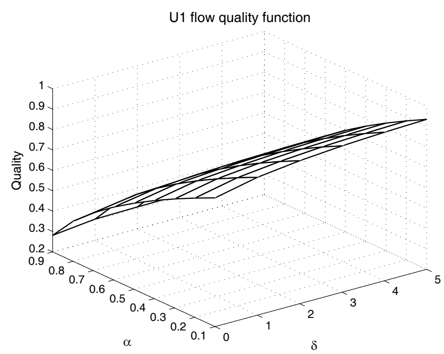

Figure 12: Flow quality based on U_1 versus the ARQ retransmission limit, δ ,$\phi = 2$, $p = 0.05$, $q = 0.35$ and $S_{th} = 12$.

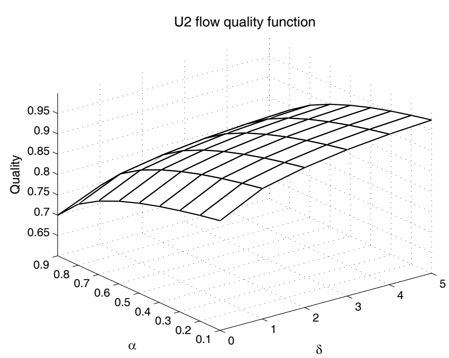

Figure 13: Flow quality based on U_2 versus the ARQ retransmission limit, δ , $\phi = 2$, $p = 0.05$, $q = 0.35$ and $S_{th} = 12$.

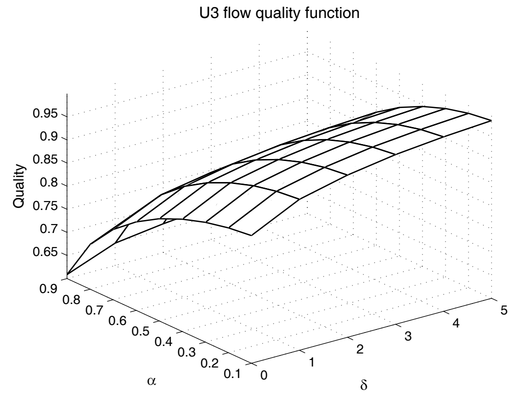

Figure 14: Flow quality based on U_3 versus the ARQ retransmission limit, δ , $\phi = 2$, $p = 0.05$, $q = 0.35$ and $S_{th} = 12$.

The figures (16,17,18), show the great throughput improvement is obtained by using the hybrid ARQ/FEC scheme. We can observe for each value of δ, a critical $\{\delta, S_{th}\} = \{\delta, \phi(1+\delta)\}$ setting for which the throughput reaches its maximum.

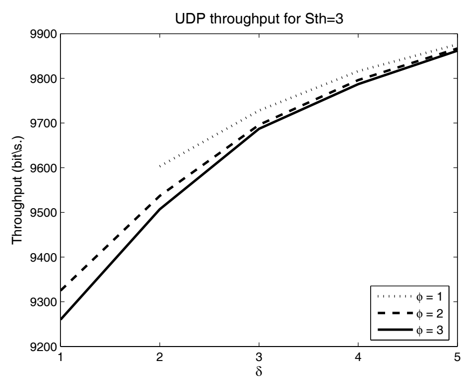

Figure 15: UDP throughput versus the ARQ retransmission limit, δ, for different values of the FEC shift,ϕ. $p = 0.05$, $q = 0.35$

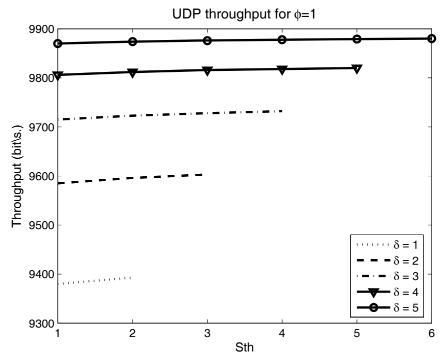

Figure 16: UDP throughput versus the S_{th}, for different values of the ARQ retransmission limit, δ. $p = 0.05$, $q = 0.35$

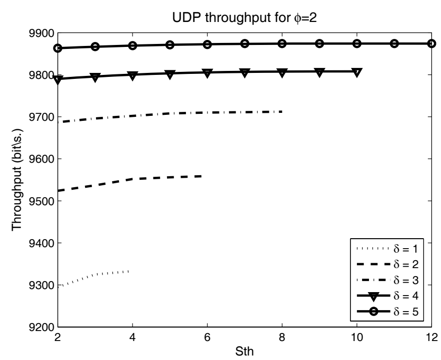

Figure 17: UDP throughput versus the S_{th}, for different values of the ARQ retransmission limit, δ. $p = 0.05$, $q = 0.35$

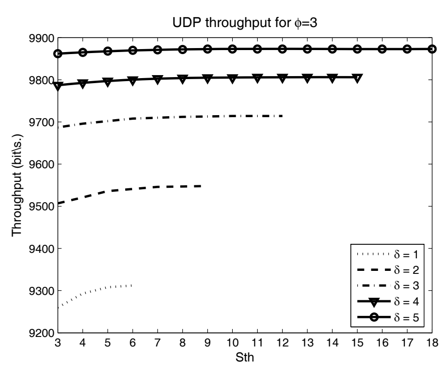

Figure 18: UDP throughput versus the S_{th}, for different values of the ARQ retransmission limit, δ. $p = 0.05$, $q = 0.35$

5 Conclusion

In this paper, we studied the effect of hybrid FEC/ARQ scheme keeping a reasonable number of slots between the loss of the last retransmission the original packet n and the reception of copy which are located in the packet $n + \phi$ (redundant information); when the packet n is lost, we can partially recover the packet $n + \phi$ if it is well received with the number of slots S_n less or equal to S_{th}.

We obtained the analytical expression of audio quality in the case of constant packet size model. The simulated results show that the small values of S_{th} increase the number of rejected packet, and return the delay reasonable. Moreover,the quality reaches it's maximum for $S_{th} = \phi(1 + \delta)$, it is very sensitive to the packets loss rate and the delay in an environment of real-time application.

In real-time flows, the hybrid FEC/ARQ scheme with limiting the slots number S_{th} enable to tune the communication parameters in function the user performances needs.

For any type of communication (real-time) different $\{\delta, \phi\}$ profiles can be used to support high throughput and low latency.

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Appendix

ϕ	δ	$P_{\phi 1}$
$\phi = 1$	$\delta = 0$	$\pi_1(1-q)$
$\phi = 1$	$\delta = 1$	$\frac{1}{2}\pi_1(1-q)^3$
$\phi = 1$	$\delta = 2$	$\frac{1}{3}\pi_1(1-q)^5$
$\phi = 1$	$\delta = 3$	$\frac{1}{4}\pi_1(1-q)^7$
$\phi = 1$	$\delta = 4$	$\frac{1}{5}\pi_1(1-q)^9$
$\phi = 1$	$\delta = 5$	$\frac{1}{6}\pi_1(1-q)^{11}$
$\phi = 2$	$\delta = 0$	$\pi_1[pq + (1-q)^2]$
$\phi = 2$	$\delta = 1$	$\frac{1}{2}\pi_1[pq(1-q)^2 + pq(1-q)^3 + (1-q)^5]$
$\phi = 2$	$\delta = 2$	$\frac{1}{3}\pi_1[pq(1-q)^4 + pq(1-q)^5 + pq(1-q)^6 + (1-q)^8]$
$\phi = 2$	$\delta = 3$	$\frac{1}{4}\pi_1[pq(1-q)^6 + pq(1-q)^7 + pq(1-q)^8 + pq(1-q)^9 + (1-q)^{11}]$
$\phi = 2$	$\delta = 4$	$\frac{1}{5}\pi_1[pq(1-q)^8 + pq(1-q)^9 + pq(1-q)^{10} + pq(1-q)^{11} + pq(1-q)^{12} + (1-q)^{14}]$
$\phi = 2$	$\delta = 5$	$\frac{1}{6}\pi_1[pq(1-q)^{10} + pq(1-q)^{11} + pq(1-q)^{12} + pq(1-q)^{13} + pq(1-q)^{14} + pq(1-q)^{15}$ $+(1-q)^{17}]$
$\phi = 3$	$\delta = 0$	$\pi_1[pq(1-p) + 2pq(1-q) + (1-q)^3]$
$\phi = 3$	$\delta = 1$	$\frac{1}{2}\pi_1[pq(1-q)^2(1-p) + p^2q^2(1-q)^2 + 2pq(1-q)^4 + p^2q^2(1-q)^3 + 2pq(1-q)^5$ $+pq(1-q)^3(1-p) + (1-q)^7]$
$\phi = 3$	$\delta = 2$	$\frac{1}{3}\pi_1[pq(1-q)^4(1-p) + p^2q^2(1-q)^4 + p^2q^2(1-q)^5 + 2pq(1-q)^7 + pq(1-q)^5(1-p)$ $+p^2q^2(1-q)^5 + p^2q^2(1-q)^6 + 2pq(1-q)^8 + pq(1-q)^6(1-p) + p^2q^2(1-q)^6$ $+p^2q^2(1-q)^7 + 2pq(1-q)^9 + (1-q)^{11}]$
$\phi = 3$	$\delta = 3$	$\frac{1}{4}\pi_1[pq(1-q)^6(1-p) + p^2q^2(1-q)^6 + p^2q^2(1-q)^7 + 2p^2q^2(1-q)^8 + 2pq(1-q)^{10}$ $+pq(1-q)^7(1-p) + p^2q^2(1-q)^7 + 3p^2q^2(1-q)^9 + 2pq(1-q)^{11} + pq(1-q)^8(1-p)$ $+p^2q^2(1-q)^8 + 2p^2q^2(1-q)^{10} + 2pq(1-q)^{12} + pq(1-q)^9(1-p) + p^2q^2(1-q)^{11}$ $+2pq(1-q)^{13} + (1-q)^{15}]$
$\phi = 3$	$\delta = 4$	$\frac{1}{5}\pi_1[pq(1-q)^8(1-p) + p^2q^2(1-q)^8 + 2p^2q^2(1-q)^9 + 3p^2q^2(1-q)^{10}$ $+4p^2q^2(1-q)^{11} + 2pq(1-q)^{13} + pq(1-q)^9(1-p) + 4p^2q^2(1-q)^{12} + 2pq(1-q)^{14}$ $+pq(1-q)^{10}(1-p) + 3p^2q^2(1-q)^{13} + pq(1-q)^{11}(1-p) + 2p^2q^2(1-q)^{14}$ $+2pq(1-q)^{16} + pq(1-q)^{12}(1-p) + p^2q^2(1-q)^{15} + 2pq(1-q)^{17} + 2pq(1-q)^{15}$ $+(1-q)^{19}]$
$\phi = 3$	$\delta = 5$	$\frac{1}{6}\pi_1[pq(1-q)^{10}(1-p) + p^2q^2(1-q)^{10} + 2p^2q^2(1-q)^{11} + 3p^2q^2(1-q)^{12}$ $+4p^2q^2(1-q)^{13} + 5p^2q^2(1-q)^{14} + 2pq(1-q)^{16} + pq(1-q)^{11}(1-p)$ $+5p^2q^2(1-q)^{15} + 2pq(1-q)^{17} + pq(1-q)^{12}(1-p) + 4p^2q^2(1-q)^{16}$ $+2pq(1-q)^{18} + pq(1-q)^{13}(1-p) + 3p^2q^2(1-q)^{17} + 2pq(1-q)^{19} + pq(1-q)^{14}(1-p)$ $+2p^2q^2(1-q)^{18} + 2pq(1-q)^{20} + pq(1-q)^{15}(1-p)$ $+p^2q^2(1-q)^{19} + 2pq(1-q)^{21} + (1-q)^{23}]$

Table 2: Values for $P_{\phi 1}$

ϕ	δ	$P_{\phi 2}$
$\phi=1$	$\delta=0$	$\pi_1 q$
$\phi=1$	$\delta=1$	$\pi_1 q(1-q)+\pi_1 q(1-q)^2$
$\phi=1$	$\delta=2$	$\pi_1 q(1-q)^2+\pi_1 q(1-q)^3+\pi_1 q(1-q)^4$
$\phi=1$	$\delta=3$	$\pi_1 q(1-q)^3+\pi_1 q(1-q)^4+\pi_1 q(1-q)^5+\pi_1 q(1-q)^6$
$\phi=1$	$\delta=4$	$\pi_1 q(1-q)^4+\pi_1 q(1-q)^5+\pi_1 q(1-q)^6+\pi_1 q(1-q)^7+\pi_1 q(1-q)^8$
$\phi=1$	$\delta=5$	$\pi_1 q(1-q)^5+\pi_1 q(1-q)^6+\pi_1 q(1-q)^7+\pi_1 q(1-q)^8+\pi_1 q(1-q)^9+\pi_1 q(1-q)^{10}$
$\phi=2$	$\delta=0$	$\pi_1 q(1-p)+\pi_1 q(1-q)$
$\phi=2$	$\delta=1$	$\pi_1 q(1-p)(1-q)+\pi_1 q(1-p)(1-q)^2+\pi_1 q(1-q)^3+\pi_1 pq^2(1-q)+\pi_1 pq^2(1-q)^2$ $+\pi_1 q(1-q)^4$
$\phi=2$	$\delta=2$	$\pi_1 q(1-p)(1-q)^2+\pi_1 q(1-p)(1-q)^3+\pi_1 q(1-p)(1-q)^4+\pi_1 q(1-q)^5+\pi_1 q^2 p(1-q)^2$ $+2\pi_1 q^2 p(1-q)^3+2\pi_1 q^2 p(1-q)^4+\pi_1 q(1-q)^6+\pi_1 pq^2(1-q)^5+\pi_1 q(1-q)^7$
$\phi=2$	$\delta=3$	$\pi_1 q(1-p)(1-q)^3+\pi_1 q(1-p)(1-q)^4+\pi_1 q(1-p)(1-q)^5+\pi_1 q(1-p)(1-q)^6$ $+\pi_1 q(1-q)^7+\pi_1 q^2 p(1-q)^3+2\pi_1 q^2 p(1-q)^4+3\pi_1 q^2 p(1-q)^5+3\pi_1 q^2 p(1-q)^6$ $+\pi_1 q(1-q)^8+2\pi_1 pq^2(1-q)^7+\pi_1 q(1-q)^9+\pi_1 pq^2(1-q)^8+\pi_1 q(1-q)^{10}$
$\phi=2$	$\delta=4$	$\pi_1 q(1-p)(1-q)^4+\pi_1 q(1-p)(1-q)^5+\pi_1 q(1-p)(1-q)^6+\pi_1 q(1-p)(1-q)^7$ $+\pi_1 q(1-p)(1-q)^8+\pi_1 q(1-q)^9+\pi_1 q^2 p(1-q)^4+2\pi_1 q^2 p(1-q)^5+3\pi_1 q^2 p(1-q)^6$ $+4\pi_1 q^2 p(1-q)^7+4\pi_1 q^2 p(1-q)^8+\pi_1 q(1-q)^{10}+3\pi_1 pq^2(1-q)^9+\pi_1 q(1-q)^{11}$ $+2\pi_1 pq^2(1-q)^{10}+\pi_1 q(1-q)^{12}+\pi_1 pq^2(1-q)^{11}+\pi_1 q(1-q)^{13}$
$\phi=2$	$\delta=5$	$\pi_1 q(1-p)(1-q)^5+\pi_1 q(1-p)(1-q)^6+\pi_1 q(1-p)(1-q)^7+\pi_1 q(1-p)(1-q)^8$ $+\pi_1 q(1-p)(1-q)^9+\pi_1 q(1-p)(1-q)^{10}+\pi_1 q(1-q)^{11}+\pi_1 q^2 p(1-q)^5$ $+2\pi_1 q^2 p(1-q)^6+3\pi_1 q^2 p(1-q)^7+4\pi_1 q^2 p(1-q)^8+5\pi_1 q^2 p(1-q)^9$ $+5\pi_1 q^2 p(1-q)^{10}+\pi_1 q(1-q)^{12}+4\pi_1 pq^2(1-q)^{11}+\pi_1 q(1-q)^{13}$ $+3\pi_1 pq^2(1-q)^{12}+\pi_1 q(1-q)^{14}+2\pi_1 pq^2(1-q)^{13}+\pi_1 q(1-q)^{15}$ $+\pi_1 pq^2(1-q)^{14}+\pi_1 q(1-q)^{16}$
$\phi=3$	$\delta=0$	$\pi_1 q(1-q)^2+\pi_1 q(1-p)(1-q)+\pi_1 pq^2+\pi_1 q(1-p)^2$
$\phi=3$	$\delta=1$	$\pi_1 q(1-p)(1-q)^3+\pi_1 q(1-p)(1-q)^4+\pi_1 q(1-q)^5+\pi_1 q(1-p)^2(1-q)^2$ $+2\pi_1 pq^2(1-p)(1-q)^2+3\pi_1 pq^2(1-q)^3+\pi_1 q(1-p)^2(1-q)+2\pi_1 pq^2(1-p)(1-q)$ $+\pi_1 pq^2(1-q)^2+2\pi_1 pq^2(1-q)^4+\pi_1 q(1-q)^6+\pi_1 p^2 q^3(1-q)+\pi_1 p^2 q^3(1-q)^2$

Table 3: Values for $P_{\phi 2}$

ϕ	δ	$P(S_n = i)$
$\phi = 1$	$\delta = 0$	$P(S_n = 1) = \pi_1 q$
$\phi = 1$	$\delta = 1$	$P(S_n = 1) = \pi_1 q; \quad P(S_n = 2) = \pi_1 (1-q) q$
$\phi = 1$	$\delta = 2$	$P(S_n = 1) = \pi_1 q\ ; \quad P(S_n = 2) = \pi_1 (1-q) q; \quad P(S_n = 3) = \pi_1 q (1-q)^2$
$\phi = 1$	$\delta = 3$	$P(S_n = 1) = \pi_1 q; \quad P(S_n = 2) = \pi_1 q (1-q); \quad P(S_n = 3) = \pi_1 q (1-q)^2$ $P(S_n = 4) = \pi_1 q (1-q)^3$
$\phi = 1$	$\delta = 4$	$P(S_n = 1) = \pi_1 q; \quad P(S_n = 2) = \pi_1 q (1-q); \quad P(S_n = 3) = \pi_1 q (1-q)^2;$ $P(S_n = 4) = \pi_1 q (1-q)^3; \quad P(S_n = 5) = \pi_1 q (1-q)^4$
$\phi = 1$	$\delta = 5$	$P(S_n = 1) = \pi_1 q$ $P(S_n = 2) = \pi_1 q (1-q)$ $P(S_n = 3) = \pi_1 q (1-q)^2$ $P(S_n = 4) = \pi_1 q (1-q)^3$ $P(S_n = 5) = \pi_1 q (1-q)^4$ $P(S_n = 6) = \pi_1 q (1-q)^5$
$\phi = 2$	$\delta = 0$	$P(S_n = 2) = \pi_1 q (1-p) + \pi_1 q (1-q)$
$\phi = 2$	$\delta = 1$	$P(S_n = 2) = \pi_1 q (1-p)$ $P(S_n = 3) = \pi_1 [(1-q)^2 q + (1-q) q (1-p) + q^2 p]$ $P(S_n = 4) = \pi_1 [(1-q)^3 q + q^2 p (1-q)]$
$\phi = 2$	$\delta = 2$	$P(S_n = 2) = \pi_1 q (1-p)$ $P(S_n = 3) = \pi_1 [q (1-p)(1-q) + q^2 p]$ $P(S_n = 4) = \pi_1 [q (1-q)^3 + q (1-q)^2 (1-p) + 2 q^2 p (1-q)]$ $P(S_n = 5) = \pi_1 [q (1-q)^4 + 2 q^2 p (1-q)^2]$ $P(S_n = 6) = \pi_1 [q (1-q)^5 + q^2 p (1-q)^3]$
$\phi = 2$	$\delta = 3$	$P(S_n = 2) = \pi_1 q (1-p)$ $P(S_n = 3) = \pi_1 [q (1-p)(1-q) + q^2 p]$ $P(S_n = 4) = \pi_1 [q (1-q)^2 (1-p) + 2 q^2 p (1-q)]$ $P(S_n = 5) = \pi_1 [q (1-q)^4 + q (1-p)(1-q)^3 + 3 q^2 p (1-q)^2]$ $P(S_n = 6) = \pi_1 [q (1-q)^5 + 3 q^2 p (1-q)^3]$ $P(S_n = 7) = \pi_1 [q (1-q)^6 + 2 q^2 p (1-q)^4]$ $P(S_n = 8) = \pi_1 [q (1-q)^7 + q^2 p (1-q)^5]$
$\phi = 2$	$\delta = 4$	$P(S_n = 2) = \pi_1 q (1-p)$ $P(S_n = 3) = \pi_1 [q (1-p)(1-q) + q^2 p]$ $P(S_n = 4) = \pi_1 [q (1-p)(1-q)^2 + 2 q^2 p (1-q)]$ $P(S_n = 5) = \pi_1 [q (1-p)(1-q)^3 + 3 q^2 p (1-q)^2]$ $P(S_n = 6) = \pi_1 [q (1-p)(1-q)^4 + q (1-q)^5 + 4 q^2 p (1-q)^3]$ $P(S_n = 7) = \pi_1 [q (1-q)^6 + 4 q^2 p (1-q)^4]$ $P(S_n = 8) = \pi_1 [q (1-q)^7 + 3 q^2 p (1-q)^5]$ $P(S_n = 9) = \pi_1 [q (1-q)^8 + 2 q^2 p (1-q)^6]$ $P(S_n = 10) = \pi_1 [q (1-q)^9 + q^2 p (1-q)^7]$

$\phi = 2$	$\delta = 5$	$P(S_n = 2) = \pi_1 q(1-p)$ $P(S_n = 3) = \pi_1[q(1-p)(1-q) + q^2p]$ $P(S_n = 4) = \pi_1[q(1-p)(1-q)^2 + 2q^2p(1-q)]$ $P(S_n = 5) = \pi_1[q(1-p)(1-q)^3 + 3q^2p(1-q)^2]$ $P(S_n = 6) = \pi_1[q(1-p)(1-q)^4 + 4q^2p(1-q)^3]$ $P(S_n = 7) = \pi_1[q(1-q)^6 + q(1-p)(1-q)^5 + 5q^2p(1-q)^4]$ $P(S_n = 8) = \pi_1[q(1-q)^7 + 5q^2p(1-q)^5]$ $P(S_n = 9) = \pi_1[q(1-q)^8 + 4q^2p(1-q)^6]$ $P(S_n = 10) = \pi_1[q(1-q)^9 + 3q^2p(1-q)^7]$ $P(S_n = 11) = \pi_1[q(1-q)^{10} + 2q^2p(1-q)^8]$ $P(S_n = 12) = \pi_1[q(1-q)^{11} + q^2p(1-q)^9]$
$\phi = 3$	$\delta = 0$	$P(S_n = 3) = \pi_1[q(1-p)(1-q) + q(1-p)^2 + q(1-q)^2 + q^2p]$
$\phi = 3$	$\delta = 1$	$P(S_n = 3) = \pi_1 q(1-p)^2$ $P(S_n = 4) = \pi_1[q(1-p)^2(1-q) + 2q^2p(1-p) + q(1-p)(1-q)^2 + q^2p(1-q)]$ $P(S_n = 5) = \pi_1[2q^2p(1-p)(1-q) + 3q^2p(1-q)^2 + q(1-p)(1-q)^3 + q(1-q)^4 + q^3p^2]$ $P(S_n = 6) = \pi_1[q^3p^2(1-q) + 2q^2p(1-q)^3 + q(1-q)^5]$
$\phi = 3$	$\delta = 2$	$P(S_n = 3) = \pi_1 q(1-p)^2$ $P(S_n = 4) = \pi_1[q(1-p)^2(1-q) + 2q^2p(1-p)]$ $P(S_n = 5) = \pi_1[4q^2p(1-p)(1-q) + q(1-p)^2(1-q)^2 + q(1-p)(1-q)^3 + q^3p^2$ $+q^2p(1-q)^2]$ $P(S_n = 6) = \pi_1[4q^2p(1-p)(1-q)^2 + 3q^3p^2(1-q) + 3q^2p(1-q)^3 + q(1-p)(1-q)^4]$ $P(S_n = 7) = \pi_1[4q^3p^2(1-q)^2 + 5q^2p(1-q)^4 + 2q^2p(1-p)(1-q)^3 + q(1-p)(1-q)^5$ $+q(1-q)^6]$ $P(S_n = 8) = \pi_1[3q^3p^2(1-q)^3 + 4q^2p(1-q)^5 + q(1-q)^7]$ $P(S_n = 9) = \pi_1[q^3p^2(1-q)^4 + 2q^2p(1-q)^6 + q(1-q)^8]$
$\phi = 3$	$\delta = 3$	$P(S_n = 3) = \pi_1 q(1-p)^2$ $P(S_n = 4) = \pi_1[2q^2p(1-p) + q(1-p)^2(1-q)]$ $P(S_n = 5) = \pi_1[4q^2p(1-p)(1-q) + q^3p^2 + q(1-p)^2(1-q)^2]$ $P(S_n = 6) = \pi_1[q^2p(1-q)^3 + 6q^2p(1-p)(1-q)^2 + 3q^3p^2(1-q) + q(1-p)^2(1-q)^3$ $+q(1-p)(1-q)^4]$ $P(S_n = 7) = \pi_1[3q^2p(1-q)^4 + 6q^3p^2(1-q)^2 + 6q^2p(1-p)(1-q)^3 + q(1-p)(1-q)^5]$ $P(S_n = 8) = \pi_1[5q^2p(1-q)^5 + 8q^3p^2(1-q)^3 + 4q^2p(1-p)(1-q)^4 + q(1-p)(1-q)^6]$ $P(S_n = 9) = \pi_1[7q^2p(1-q)^6 + 8q^3p^2(1-q)^4 + 2q^2p(1-p)(1-q)^5 + q(1-q)^8$ $+q(1-p)(1-q)^7]$ $P(S_n = 10) = \pi_1[6q^2p(1-q)^7 + 6q^3p^2(1-q)^5 + q(1-q)^9]$ $P(S_n = 11) = \pi_1[3q^2p(1-q)^8 + 4q^3p^2(1-q)^6 + q(1-q)^{10}]$ $P(S_n = 12) = \pi_1[2q^2p(1-q)^9 + q^3p^2(1-q)^7 + q(1-q)^{11}]$

$\phi=3$	$\delta=4$	$P(S_n=3)=\pi_1 q(1-p)^2$ $P(S_n=4)=\pi_1[2q^2p(1-p)+q(1-p)^2(1-q)]$ $P(S_n=5)=\pi_1[4q^2p(1-p)(1-q)+q^3p^2+q(1-p)^2(1-q)^2]$ $P(S_n=6)=\pi_1[6q^2p(1-p)(1-q)^2+3q^3p^2(1-q)+q(1-p)^2(1-q)^3]$ $P(S_n=7)=\pi_1[q(1-p)(1-q)^5+q(1-p)^2(1-q)^4+2q^2p(1-q)^4+7q^2p(1-p)(1-q)^3$ $+6q^3p^2(1-q)^2]$ $P(S_n=8)=\pi_1[q(1-p)(1-q)^6+3q^2p(1-q)^5+8q^2p(1-p)(1-q)^4+10q^3p^2(1-q)^3]$ $P(S_n=9)=\pi_1[q(1-p)(1-q)^7+5q^2p(1-q)^6+6q^2p(1-p)(1-q)^5+13q^3p^2(1-q)^4]$ $P(S_n=10)=\pi_1[q(1-p)(1-q)^8+7q^2p(1-q)^7+4q^2p(1-p)(1-q)^6+14q^3p^2(1-q)^5]$ $P(S_n=11)=\pi_1[q(1-q)^{10}+q(1-p)(1-q)^9+9q^2p(1-q)^8+2q^2p(1-p)(1-q)^7$ $+13q^3p^2(1-q)^6]$ $P(S_n=12)=\pi_1[q(1-q)^{11}+8q^2p(1-q)^9+10q^3p^2(1-q)^7]$ $P(S_n=13)=\pi_1[q(1-q)^{12}+6q^2p(1-q)^{10}+6q^3p^2(1-q)^8]$ $P(S_n=14)=\pi_1[q(1-q)^{13}+4q^2p(1-q)^{11}+3q^3p^2(1-q)^9]$ $P(S_n=15)=\pi_1[q(1-q)^{14}+2q^2p(1-q)^{12}+q^3p^2(1-q)^{10}]$
$\phi=3$	$\delta=5$	$P(S_n=3)=\pi_1 q(1-p)^2$ $P(S_n=4)=\pi_1[2q^2p(1-p)+q(1-p)^2(1-q)]$ $P(S_n=5)=\pi_1[4q^2p(1-p)(1-q)+q^3p^2+q(1-p)^2(1-q)^2]$ $P(S_n=6)=\pi_1[6q^2p(1-p)(1-q)^2+3q^3p^2(1-q)+q(1-p)^2(1-q)^3]$ $P(S_n=7)=\pi_1[8q^2p(1-p)(1-q)^3+6q^3p^2(1-q)^2+q(1-p)^2(1-q)^4]$ $P(S_n=8)=\pi_1[10q^2p(1-p)(1-q)^4+10q^3p^2(1-q)^3+q^2p(1-q)^5+q(1-p)^2(1-q)^6$ $+q(1-p)(1-q)^6]$ $P(S_n=9)=\pi_1[q(1-p)(1-q)^7+3q^2p(1-q)^6+10q^2p(1-p)(1-q)^5+15q^3p^2(1-q)^4]$ $P(S_n=10)=\pi_1[q(1-p)(1-q)^8+5q^2p(1-q)^7+8q^2p(1-p)(1-q)^6+19q^3p^2(1-q)^5]$ $P(S_n=11)=\pi_1[q(1-p)(1-q)^9+7q^2p(1-q)^8+6q^2p(1-p)(1-q)^7+21q^3p^2(1-q)^6]$ $P(S_n=12)=\pi_1[q(1-p)(1-q)^{10}+9q^2p(1-q)^9+4q^2p(1-p)(1-q)^8+21q^3p^2(1-q)^7]$ $P(S_n=13)=\pi_1[q(1-q)^{12}+q(1-p)(1-q)^{11}+11q^2p(1-q)^{10}+2q^2p(1-p)(1-q)^9$ $+19q^3p^2(1-q)^8]$ $P(S_n=14)=\pi_1[q(1-q)^{13}+10q^2p(1-q)^{11}+15q^3p^2(1-q)^9]$ $P(S_n=15)=\pi_1[q(1-q)^{14}+8q^2p(1-q)^{12}+10q^3p^2(1-q)^{10}]$ $P(S_n=16)=\pi_1[q(1-q)^{15}+6q^2p(1-q)^{13}+6q^3p^2(1-q)^{11}]$ $P(S_n=17)=\pi_1[q(1-q)^{16}+4q^2p(1-q)^{14}+3q^3p^2(1-q)^{12}]$ $P(S_n=18)=\pi_1[q(1-q)^{17}+2q^2p(1-q)^{15}+q^3p^2(1-q)^{13}]$ $P(S_n=19)=0$ $P(S_n=S_{th})=0$ $P(S_n\leq S_{th})=P(S_n=3)+...+P(S_n=S_{th})$

Table 4: probability $P(S_n=i)$ for different values of ϕ and δ