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Shape Optimization of Cantilever Beams Using Neural Network

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Abstract

In this study, shape optimisation of cantilever beams has been carried out using neural network. Considering different geometrical parameters, finite element analyses of cantilever beams are carried out. Using these results, a back propagation neural network is trained. Successfully trained networks are further used for shape optimisation of newer problems. Thus optimised beams are further validated with finite element analyses results and found to be in closer match.

Keywords: Profile; Stress; Displacement; Finite element; Cantilever; Neural network

INTRODUCTION

The objective of shape optimization is to find the shape which is optimal in the sense that it minimizes a certain cost functional while satisfying given constraints. Analytical methods for solving shape optimization problems have been used for a long time. The first known attempt at developing a mathematical formulation for shape optimization dates back to Galileo in 1638, who found that minimum weight cantilever is a parabolic beam. Use of numerical methods for shape optimization became main interest of scientists in this field after the invention of computers, In last 40 years a lot of progresses have been made in this field. Most of the shape optimization problems solved so far can

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be classified into gradientbased and gradientless methods. Both of these methods have their pros and cons and selection of a particular method depends upon the type and size of the problem, number of design variables and convergence required.
Durelli (1981) et al. adopted a step-by-step procedure for modifying hole boundaries in a two dimensional photoelastic model until the tensile and compressive boundary stresses were approximately constant. Mattheck (1990) based on the interesting observation that “living structures” appear to be able to add material in region of high stresses and to reduce material in region of low stresses to bring about an optimal shape that produces a constant von Mises stress distribution on the free surface. Hasengawa (1992) proposed two gradientless methods namely, boundary changing methods, in which co-ordinates are changed, and thickness changing methods, in which thickness is changed. Pathak(2000) used design elements, fuzzy set theory and artificial neural networks in a gradientless method of shape optimisation. Hsu (1993) developed a new method for optimization called as **‘curvature function method’**. They have solved various problems such as cantilever beam, fillet and torque arm. Ghoddosian(1998) have extended curvature function method to find optimum shape of shell structures. He has successfully solved one circular and one spherical shell problems. In recent years application of artificial intelligence based techniques have formed important place in structural engineering and shape optimization is not untouched to that. Most important among them is evolutionary method, application of genetic algorithms (GA), neural networks etc. Nicholas Ali (2003) reported shape optimization of very large planer and space problems using GA. The proposed clubbing of FEA and GA finds lighter and reasonable structural design. In this remeshing is avoided and particularly the computation burden and errors caused by sensitivity analysis are eliminated completely. From the literature survey it is observed that shape optimization is one of the most complex problem which requires multidisciplinary knowledge like Numerical Mathematics; Finite Element Method (FEM); Computer Aided Design(CAD) etc. Since most of the design engineers are not well versed in these areas, shape optimization is still beyond their reach. To overcome this difficulty, to some extent, application of neural network is proposed in this study. The stress and displacement obtained from finite element analyses are used for training of neural network. Successfully trained network is used for prediction of shapes. Several thus designed beams are compared with finite element analysis results and both are found to be in good match.

ARTIFICIAL NEURAL NETWORK

Artificial neural network attempts to imitate the learning activities of the brain. The human brain is composed of approximately 10^{11} neurons (nerve cells) of different types. In a typical neuron, we can find the nucleus, where the connections with other neurons are made through a network of fibers called dendrites. Extending out from the nucleus is the axon, which transmits , by means of a complex chemical process, electric potentials to the neurons with which the axon is connected to

(Fig.1). When the signals received by the neuron equal or surpass their threshold, it "triggers", sending the axon an electric signal of constant level and duration. In this way the message is transferred from one neuron to the other.

In an artificial neural network (ANN), the artificial neuron or the processing unit may have several input paths corresponding to the dendrites. The units combine usually, by a simple summation, the weighted values of these paths (Fig.2). The weighted value is passed to the neuron, where it is modified by threshold function such as sigmoid function (Fig.3). The modified value is directly presented to the next neuron. In Fig.4 a 3-4-2 feed forward back propagation artificial neural network is shown. The connections between various neurons are strengthened or weakened according to the experiences obtained during the training. The algorithm for training the back propagation neural network can be explained in the following steps-

Step 1 – Select the number of hidden layers, number of iterations, tolerance of the mean square error and initialize the weights and bias functions.

Step 2 – Present the normalized input –output pattern sets to the network. At each node of the network except the nodes on input layer, calculate the weighted sum of the inputs, add bias and apply sigmoid function

Step 3- Calculate total mean error . If error is less than permissible limit, the training process is stopped. Otherwise,

Step 4 –Change the weights and bias values based on generalized delta rule and repeat step 2.

The mathematical formulations of training the network can be found in Ref.4.

METHODOLOGY

In Fig. 6, a cantilever beam of span L is shown. Let X, Y and Z be the width at fixed support, middle and free end respectively. P is the tip load on the cantilever beam. Due to design constraint, minimum free end width is restricted to Z_{min}. The other dimensions can be normalized with respect to this as:

$$k_1 = X/Z_{min},\ k_2 = Y/Z_{min},\ k_3 = Z/Z_{min}$$

Various combinations of these geometrical parameters are framed and analysed using FEM. Results for these cases are recorded in terms of peak bending stress and displacement. These data are used for training of the neural network. Successfully trained network is employed for shape optimisation of newer problems. The flowchart of the methodology used for shape optimization using neural network is shown in Fig. 5

APPLICATION OF NEURAL NETWORK

In this study a cantilever beam of span 10000 mm is considered. Three value of X namely 200, 250 and 300mm are accounted. Z_{min} is considered as 50mm. Two values of Z viz. 75

mm and 50mm are accounted. In this way, two values of k_3 i.e. 1 and 1.5 are adopted. Considering these variations in geometrical parameters, 17 cases are framed (Table 1). Constant thickness of 150 mm is considered for all the cases. Beams are divided in 20, nine nodded elements making up 205 nodes (Fig 7.). An automatic mesh generator has been developed to generate finite element mesh for these cases. Young's Modulus of $2x10^5$ MPa and Poisson's ratio 0.3 is accounted. Considering these data, linear elastic finite element analyses of the 17 cases are carried out and maximum value of bending stresses and displacements are noted for each case (Table 1). The finite element analysis results are given in Table 1. Stresses and displacements, thus obtained, are used for training the neural network. For this a 3-5-2 size, back propagation neural network is adopted. The input parameters are k_3, stress and displacement and output parameters are k_1 and k_2. It took 543268 epochs to converge to an error tolerance limit of 0.01. The trained network is used for predicting geometrical parameters for new testing patterns given in Table 2. To validate the results, obtained from the neural network, design variables 'X' and 'Y' are calculated from the projected k_1 and k_2 and finite element analyses considering these data are carried out. The stress and displacement results obtained from FEA and corresponding percentage error are given in Table 3. Maximum error in stress and displacement predictions are 8.09% and 9.09% respectively. The comparative results obtained from neural network (NNT) and Finite Element Analyses (FEM) for stress and displacement are shown with the help of bar diagrams in Fig. 8 and Fig 9. This may be acceptable at first hand design. Based on the requirement, it may be further improved using more rigorous approaches.

CONCLUSION

In this study, an application of neural network is demonstrated on shape optimization problems. It overcomes some of the drawbacks of conventional approaches of shape optimisation, like high computational time and large memory requirement. It is observed that proposed approach works efficiently for optimizing shape accounting displacement and stress criteria. It offers a handy tool for design engineers who are not familiar with the theoretical and computational aspects of shape optimization.

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Table 1: Training Patterns

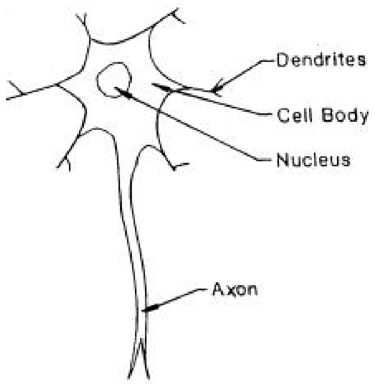
S.No	Input parameters			Output parameters	
	k_3	Stress (σ) MPa	Displacement (δ) mm	k_1	k_2
1.	1	67.05	26.5	6	5
2.	1	66.54	43.0	6	4
3.	1	66.03	85.9	6	3
4.	1	65.53	266	6	2
5.	1	96.15	48.9	5	4
6.	1	95.83	91.8	5	3
7.	1	95.55	252	5	2
8.	1	149	107	4	3
9.	1	150.62	263	4	2
10.	1.5	66.99	25.6	6	5
11.	1.5	66.48	40.7	6	4
12.	1.5	65.97	48.1	6	3
13.	1.5	96.10	48.9	5	4
14.	1.5	95.79	86.2	5	3
15.	1.5	95.52	222	5	2
16.	1.5	150.05	103	4	3
17.	1.5	150.70	244	4	2

Table 2: Testing Patterns

S.No.	k_3	Stress (MPa)	Displacement (mm)
1.	1	135	110
2.	1	130	90
3.	1	125	80
4.	1	120	70
5.	1	115	80
6.	1	105	70
7.	1.5	140	90
8.	1.5	135	100
9.	1.5	115	70

Table 3: Validation

Sr. No	k_1	k_2	X (mm)	Y (mm)	Stress (MPa)			Displacement (mm)		
					NNT	FEM	% Error	NNT	FEM	% Error
1.	4.09	4.09	205	140	135	142.87	5.51	110	121	9.09
2.	4.13	3.21	207	160	130	139.98	7.13	90	90.50	0.55
3.	4.20	3.43	210	172	125	136	8.09	80	77.80	2.83
4.	4.29	3.68	215	184	120	129.74	7.51	70	66.20	5.74
5.	4.43	3.34	221	167	115	122.88	6.41	80	78.80	1.52
6.	4.72	3.55	236	137	105	107.78	2.58	70	65.80	6.38
7.	4.01	3.19	200	159	140	149	6.04	90	91.90	2.07
8.	4.08	2.91	214	107	135	144	6.25	100	108	7.41
9.	4.41	3.51	221	176	115	122.85	6.39	70	68.80	1.74



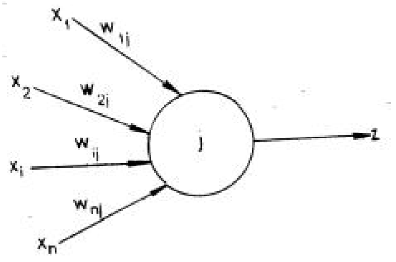


Fig.2. A single processing unit

Fig.1. A typical biological neuron

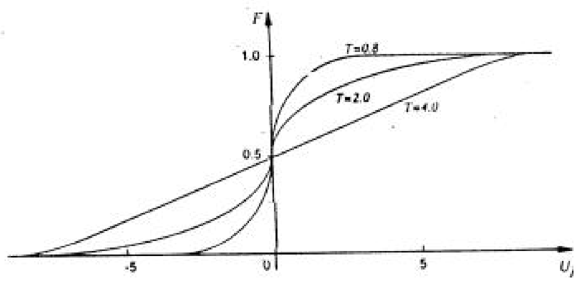


Fig.3. The sigmoid function

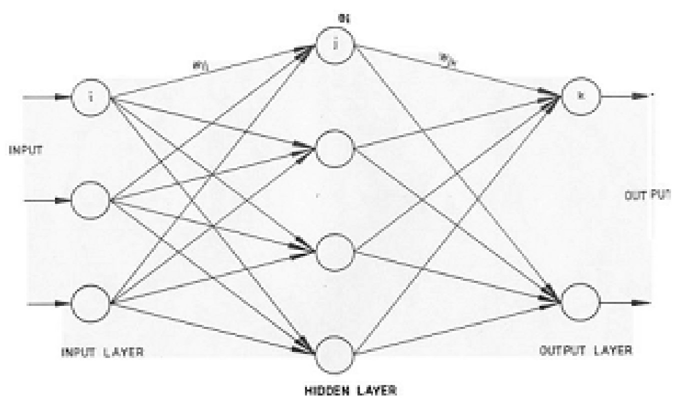


Fig.4. Neural network

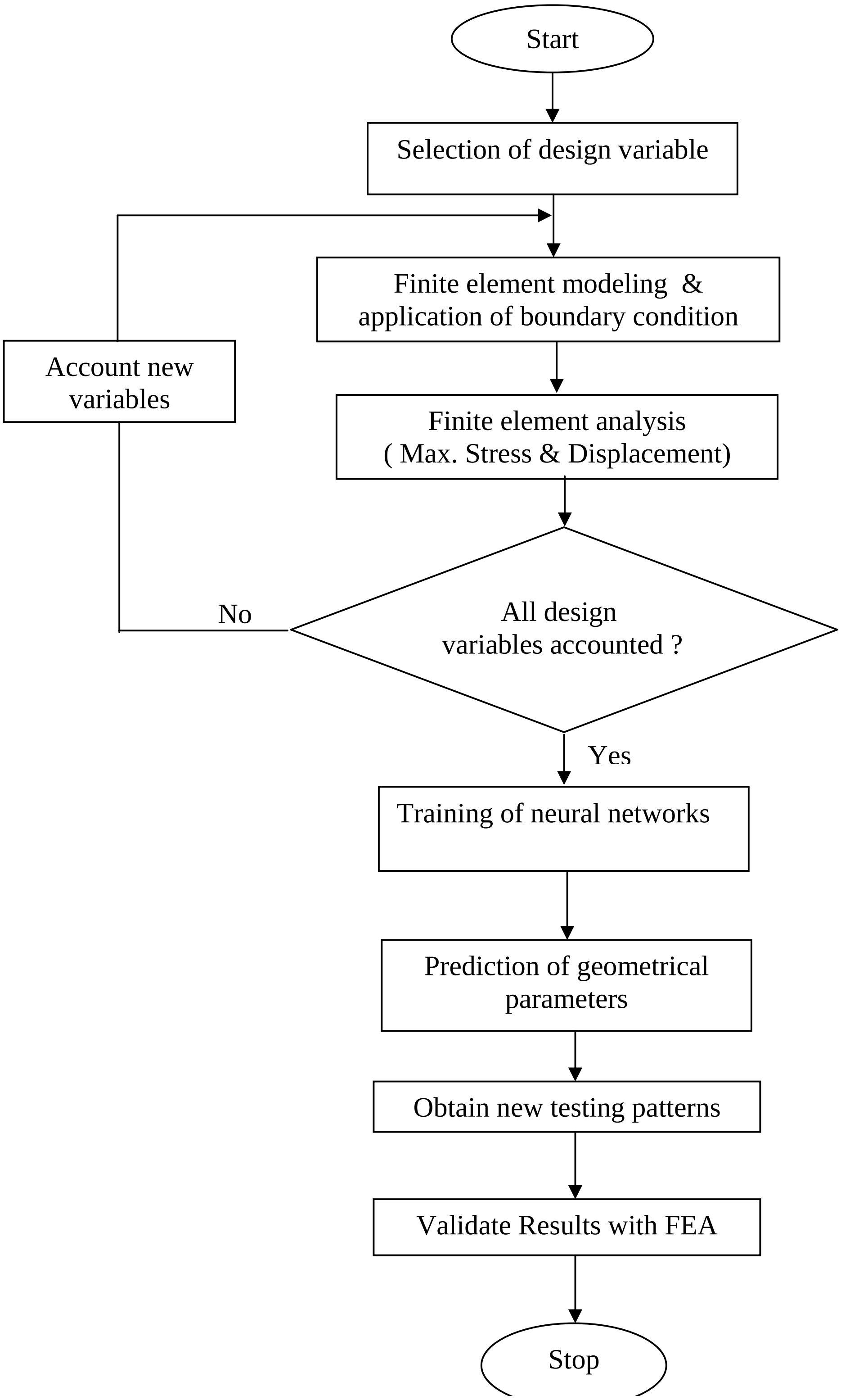


Fig. 5 Flowchart

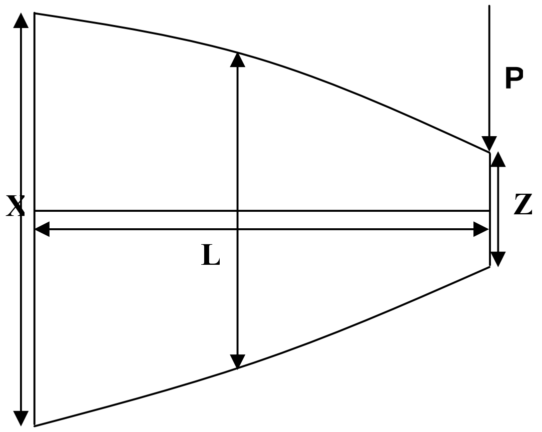


Fig. 6: Cantilever Beam

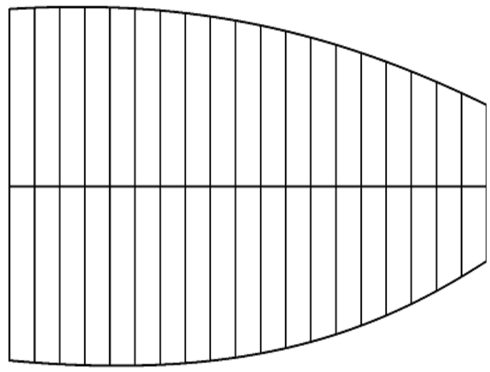


Fig.7 Finite Element Model

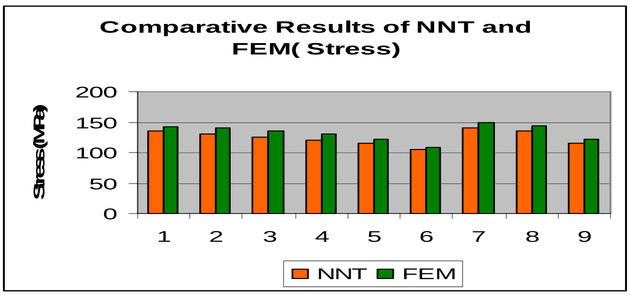


Fig. 8 Stress Comparison

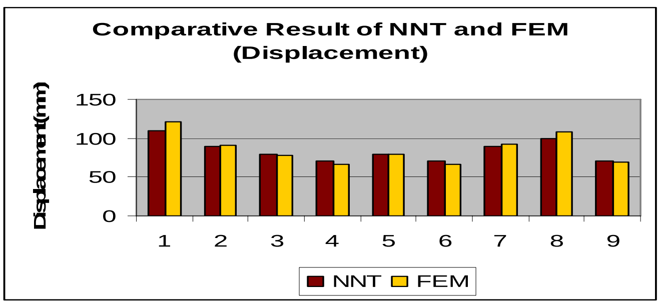


Fig. 9 Displacement Comparison

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