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Convergence of Numerical Solutions to Stochastic Age-Dependent Population Equations Driven by Non-Gaussian Noise[1]

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Abstract

It presents and analyzes the Euler methods for stochastic age-dependent population equations driven by Poisson random jump measure; Under the Local Lipschitz condition, we prove that the Euler approximation solution converges to the exact solution in the mean-square sense. An example is given to illustrates our results.

Mathematics Subject Classifications: 65C30

Keywords: stochastic age-dependent population equations, compensated Poisson random measure, Euler method, numerical solution, strong convergence

1 Introduction

Population system are often subject to enviroment noise[1,2]. In the present investigation, the random behavior of the death and influence of external environment process are carefully incorporated into the age-dependent population equations to obtain a system of SDEs that model

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age-dependent population dynamics. Zhang Qi-min et al.[3,4] first showed the existence, uniqueness and exponential stability for stochastic age-dependent population equations perturbed by white noise; Mao [5] study the environmental brownian noise suppresses explosions in population dynamics. As SDEs, stochastic age-dependent population equations cannot be solved explicitly, we need the approximate numerical solutions to simulate such systems and obtain the corresponding numerical solutions to study their behavior characteristics. Zhang Qi-min et al.[7], Li Ronghua et al.[8]discussed the convergence of numerical solutions to stochastic age-dependent population equations. However, in the stochastic age-dependent population system, due to brusque variations from some rare events, the size of the population systems increases or decreases drastically, so the diffusion processes cannot better describe the dynamics of population density, we need to incorporate the jumps into stochastic age-dependent population equations to simulate such changes. Li Ronghua et al.[8] studied stochastic age-dependent population equations with Poisson jump process and given some results about the numerical analysis. In this paper, we mainly consider stochastic age-dependent population equations driven by Poisson random jump measures

$$dP_t = [-\frac{\partial P_t}{\partial a} - u(t,a)P_t + f(t,P_t)]dt + \int_E h(t,u,P_t)\tilde{N}(dt,du)$$

We first construct the Euler approximation solution for this system; and we relax the global lipschitz conditions on the coefficients which imposed in [8], under the Local lipschitz conditions, we present and prove that the Euler approximation solution converges to the exact solution in the mean-square sense.

2 Preliminaries and the Euler approximation

Let $V = H^1([0,A]) \equiv \{\varphi | \varphi \in L^p([0,A]), \frac{\partial \varphi}{\partial x_i} \in L^p([0,A])$, where $\frac{\partial \varphi}{\partial x_i}$ is generalized partial derivatives. $\}$ V is a Sobolev space. $H = L^p([0,A]), (P \geq 2)$ such that $V \hookrightarrow H \equiv H' \hookrightarrow V'$. V' is the dual space of V. We denote by $||\cdot||, |\cdot|$, and $||\cdot||_*$ the norm in V, H and V', respectively; by $\langle \cdot, \cdot \rangle$ the duality product between V, V', and by $(\cdot,\cdot)$ the scalar product in H.

Let $(\Omega, \mathcal{F}, P)$ be a complete probability space with a filtration $(\mathcal{F}_t)_{t\geq 0}$ satisfying the usual conditions (i.e., it is increasing and right continuous while $\mathcal{F}_0$ contains all P-null sets).

Let $C = C([0,T]; H)$ be the space of all continuous function from $[0,T]$ into H with sup-norm $||\psi||_C = \sup_{0\leq t\leq T} |\psi(s)|$, $L_V^p = L^p([0,T]; V)$ and $L_H^p = L^p([0,T]; H)$.

Consider n-dimensional stochastic pantograph equations with Poisson jump random measures:

$$\begin{cases} dP_t = [-\frac{\partial P_t}{\partial a} - u(t,a)P_t + f(t,P_t)]dt + \int_E h(t,u,P_t)\tilde{N}(dt,du), & in\ Q = (0,T)\times(0,A), \\ P(0,a) = P_0(a), & in \quad [0,A], \\ P(t,a) = \int_0^A \beta(t,a)P(t,a)da, & in \quad [0,T], \end{cases} \tag{2.1}$$

where $T > 0, A > 0, Q = (0,T) \times (0,A)$, $f : R^+ \times L^p_H \to H$ and $h : R^+ \times E \times L^p_H \to H$. where $\tilde{N}(dt,du) = N(dt,du) - \Pi(du)dt$ is a compensated Poisson random measures in $R^+ \times E$. We refer to [9] for the background on Probability theory and to [10] for stochastic differential equations.

For systerm (2.1), the discrete implicit Euler approximation on $t \in \{0, h, 2h, \cdots\}$ is given by the iterative scheme

$$Q_t^{n+1} = Q_t^n - [\frac{\partial Q_t^{n+1}}{\partial a} + u(t,a)Q_t^n - f(t,Q_t^n)]h + \int_E h(t,u,Q_t^n)\tilde{N}(h,du), \tag{2.2}$$

with initial value $Q_t^0 = P(0,a), Q^n(t,0) = \int_0^A \beta(t,a)Q_t^n da$,$n \geq 1$. Here,$Q_t^n$ is the approximation to $P(t_n, a)$, the time increment is $h = T/N$, for some sufficiently large integer N such that $h << 1$.

For convenience, We extend our discrete numerical solution to continuous time. First we define the step function: $Z_t = Z(t,a) = \sum\limits_{n=0}^{N} Q_t^n I_{[nh,(n+1)h)}(t)$, where I_G is the indicator function for the set G.

Then we define the continuous-time approximation

$$Q_t = Q_0 - \int_0^t [\frac{\partial Q_s}{\partial a} + u(s,a)Z_s - f(s,Z_s)]ds + \int_0^t \int_E h(s,u,Z_s)\tilde{N}(ds,du). \tag{2.3}$$

In this paper, we impose the following conditions:
i)$u(t,a), \beta(t,a)$ are continuous in $\bar{Q}$ such that

$$0 \leq u_0 \leq u(t,a) \leq \bar{\alpha} < \infty, \qquad 0 \leq \beta(t,a) \leq \bar{\beta} < \infty;$$

ii) There exists constant $\gamma > 0$ such that $x \in V$

$$|\frac{\partial x}{\partial a}|^2 \leq \gamma |x|^2;$$

iii)(**the Local Lipschitz condition**) For every $d \geq 1$, there exists a positive constant $K_d \geq 0$, such that for all $x, y \in C, u \in E$ and $||x||_C \vee ||y||_C \leq d$,

$$|f(t,x) - f(t,y)|^2 \vee \int_E |h(t,u,x) - h(t,u,y)|^2 \Pi(du) \leq K_d(||x-y||_C^2); \tag{2.4}$$

iv)(**the Linear growth condition**) there exists K such that for all $x, y \in C, u \in E$,

$$|f(t,x)|^2 \vee \int_E |h(t,u,x)|^2 \Pi(du) \leq K(1 + ||x||_C^2). \tag{2.5}$$

3 Boundness of the exact solutions and numerical solutions

In this section, we show that the exact solution and the implicit approximate solutions has bounded Pth moments.

Theorem 3.1 Under condition ii), we get

$$E(\sup_{0\le t\le T}|P_t|^P)\le H_1,\qquad E(\sup_{0\le t\le T}|Q_t|^P)\le H_2, \tag{3.1}$$

where H_1, H_2 are positive constants independent of h.

Proof:By the Holder inequality, it is easy to see from (2.1) that

$$\begin{aligned}|P_t|^P &\le 5^{P-1}[|P_0|^P+|-\int_0^t\frac{\partial P_s}{\partial a}ds|^P+|-\int_0^t u(s,a)P_s ds|^P\\ &\quad+|\int_0^t f(s,P_s)ds|^P+|\int_0^t\int_E h(s,u,P_s)\tilde N(ds,du)|^P]\\ &\le 5^{P-1}[|P_0|^P+t^{P-1}\int_0^t|\frac{\partial P_s}{\partial a}|^P ds+{u_0}^{P-1}\int_0^t|P_s|^P ds\\ &\quad+t^{P-1}\int_0^t|f(s,P_s)|^P ds+|\int_0^t\int_E h(s,u,P_s)\tilde N(ds,du)|^P].\end{aligned}$$

Hence, for any $t_1\in[0,T]$,

$$\begin{aligned}E(\sup_{0\le t\le t_1}|P_t|^P) &\le 5^{P-1}[E|P_0|^P+T^{P-1}E\int_0^{t_1}|\frac{\partial P_s}{\partial a}|^P ds+{u_0}^{P-1}E\int_0^{t_1}|P_s|^P ds\\ &\quad+T^{P-1}E\int_0^{t_1}|f(s,P_s)|^P ds+E\sup_{0\le t\le t_1}|\int_0^t\int_E h(s,u,P_s)\tilde N(ds,du)|^P]\end{aligned} \tag{3.2}$$

By condition iv), we compute that

$$\begin{aligned}E\int_0^{t_1}|f(s,P_s)|^P &= E\int_0^{t_1}[|f(s,P_s)|^2]^{\frac{P}{2}}ds\\ &\le E\int_0^{t_1}[K(1+||P_s||_C^2]^{\frac{P}{2}}ds\\ &\le K^{\frac{P}{2}}E\int_0^{t_1}(1+||P_s||_C^2)^{\frac{P}{2}}ds\\ &\le K^{\frac{P}{2}}2^{\frac{P}{2}-1}E\int_0^{t_1}(1+||P_s||_C^P)ds\\ &\le K^{\frac{P}{2}}2^{\frac{P}{2}-1}[T+\int_0^{t_1}E\sup_{0\le t\le s}|P_t|^P ds].\end{aligned} \tag{3.3}$$

We also compute, using the Burkholder-Davis-Gundy inequality,

$$E\sup_{0\le t\le t_1}|\int_0^t\int_E h(s,u,P_s)\tilde N(ds,du)|^P \le C_P E(\int_0^{t_1}\int_E|h(s,u,P_s)|^2\Pi(du)ds)^{\frac{P}{2}}$$

$$\begin{aligned} &\leq C_P T^{\frac{P}{2}-1} E\int_0^{t_1}[\int_E |h(h(s,u,P_s))|^2\Pi(du)]^{\frac{P}{2}}ds \\ &\leq C_P T^{\frac{P}{2}-1} E\int_0^{t_1}[K(1+||P_s||_C^2]^{\frac{P}{2}}ds \\ &\leq C_P(2T)^{\frac{P}{2}-1}K^{\frac{P}{2}}\int_0^{t_1}(1+||P_s||_C^P)ds \\ &\leq C_P(2T)^{\frac{P}{2}-1}K^{\frac{P}{2}}[T+\int_0^{t_1}E(\sup_{0\leq t\leq s}|P_t|^P)ds]. \quad (3.4) \end{aligned}$$

Substituting (3.3), (3.4)into (3.2), we then derive the following inequalities:

$$\begin{aligned} E(\sup_{0\leq t\leq t_1}|P_t|^P) &\leq 5^{P-1}E|P_0|^P + T^{P-1}E\int_0^{t_1}|\frac{\partial P_s}{\partial a}|^P ds + (K^{\frac{P}{2}}2^{\frac{P}{2}-1}T^P + C_P(2T)^{\frac{P}{2}-1}K^{\frac{P}{2}}T) \\ &\quad +[{u_0}^{P-1} + K^{\frac{P}{2}}2^{\frac{P}{2}-1} + C_P(2T)^{\frac{P}{2}-1}K^{\frac{P}{2}}]\int_0^{t_1}E(\sup_{0\leq t\leq s}|P_t|^P)ds \\ &\leq 5^{P-1}E|P_0|^P + T^{P-1}E\int_0^{t_1}|\frac{\partial P_s}{\partial a}|^P ds + C_1 + C_2\int_0^{t_1}E(\sup_{0\leq t\leq s}|P_t|^P)ds, \end{aligned}$$

where C_1, C_2 dependent only on K, P and T, but independent of h. By the well-known Gronwall inequality, we find that

$$E(\sup_{0\leq t\leq t_1}|P_t|^P) \leq [5^{P-1}E|P_0|^P + T^{P-1}E\int_0^{t_1}|\frac{\partial P_s}{\partial a}|^P ds + C_1]e^{C_2 T}.$$

Similarly, we have $E(\sup_{0\leq t\leq T}|Q_t|^P) \leq [5^{P-1}E|P_0|^P + T^{P-1}E\int_0^T|\frac{\partial P_s}{\partial a}|^P ds + C_1]e^{C_2 T}$. hence the required assertion must hold. □

4 convergence of the Euler methods

In this section, we derive our strong convergence result, Theorem 4.3 and prove it.

The following lemma shows that the continuous-time approximation remains close to the step functions in a strong sense.

Lemma4.1 Under conditions ii) and iv), for each $t \in [0, T]$,

$$E[|Q_t - Z_t|^2] \leq C_1 h, \quad (4.1)$$

whereC_1dependent only on K, P and T, but independent of h.

Proof: For any $t \in [0, T]$, choose a n such that $t \in [nh, (n+1)h)$. Then

$$\begin{aligned} Q_t - Z_t &= Q_t - Q_t^n \\ &= -\int_{nh}^t \frac{\partial Q_s}{\partial a}ds - \int_{nh}^t u(s,a)Z_s ds \\ &\quad + \int_{nh}^t f(s,Z_s)ds + \int_{nh}^t\int_E h(s,u,Z_s)\tilde{N}(ds,du). \end{aligned}$$

Applying basic inequality $|a+b+c+d|^2 \leq 4|a|^2+4|b|^2+4|c|^2+4|d|^2$, Holder inequality and martingale isometries, we have

$$\begin{aligned}
E|Q_t - Z_t|^2 &\leq 4E|\int_{nh}^{t} \frac{\partial Q_s}{\partial a} ds|^2 + 4E|\int_{nh}^{t} -u(s,a)Z_s ds|^2 \\
&\quad +4E|\int_{nh}^{t} f(s,Z_s)ds|^2 + 4E|\int_{nh}^{t}\int_{E} h(s,u,Z_s)\tilde{N}(ds,du)|^2 \\
&\leq 4hE\int_{nh}^{t} |\frac{\partial Q_s}{\partial a}|^2 ds + 4hu_0^2 E\int_{nh}^{t} |Z_s|^2 ds \\
&\quad +4hE\int_{nh}^{t} |f(s,Z_s)|^2 ds + 4E\int_{nh}^{t}\int_{E} |h(s,u,Z_s)|^2 \Pi(du)ds \\
&\leq 4h\gamma E\int_{nh}^{t} |Q_s|^2 ds + 4hu_0^2 E\int_{nh}^{t} |Z_s|^2 ds + (4h+4)E\int_{nh}^{t} K(1+||Z_s||_C^2)ds \\
&\leq (4hKT+4Kh) + (4h\gamma + 4hu_0^2 + 4hK + 4K)\int_{nh}^{t} E\sup_{0\leq u\leq s}|Q_u|^2 ds \\
&\leq (4hKT+4Kh) + (4h\gamma + 4hu_0^2 + 4hK + 4K)h[E\sup_{0\leq u\leq s}|Q_u|^P]^{\frac{2}{P}} \\
&\leq [(4Kh+4K) + (4h\gamma + 4hu_0^2 + 4hK + 4K)(H_2)^{\frac{2}{P}}]h
\end{aligned}$$

The proof is completed. □

For each $d>0$, define the stopping times $\tau_d = inf\{t\in[0,T] : |P_t| \geq d\}$ and $\rho_d = inf\{t \in [0,T] : |Q_t| \geq d\}$, let $\theta_d = \tau_d \wedge \rho_d$. The following corollary follows directly from Lemma 4.1.

Corollary 4.2 Under conditions ii),iv), for each $t \in [0,T]$,

$$E[|Q_{t\wedge\theta_d} - Z_{t\wedge\theta_d}|^2] \leq C_1(d)h, \tag{4.2}$$

where$C_1(d)$dependent only on K, P and T, but independent of h.

Theorem 4.3 Under conditions ii), iii),and iv), then the numerical solution (2.3) will converge to the exact solution of Eq.(2.1), i.e.,

$$\lim_{h\to 0} E[\sup_{0\leq t\leq T}|Q_t - P_t|^2] = 0, \qquad \forall T>0.$$

Proof: Let $e_t = Q_t - P_t$, obviously,

$$E[\sup_{0\leq t\leq T}|e_t|^2] = E[\sup_{0\leq t\leq T}|e_t|^2 I_{\{\tau_d > T, \rho_d > T\}}] + E[\sup_{0\leq t\leq T}|e_t|^2 I_{\{\tau_d \leq T or \rho_d \leq T\}}].$$

By the young inequality

$$AB \leq \delta\frac{A^m}{m} + \frac{1}{\delta^{\frac{n}{m}}}\frac{B^n}{n} \quad \forall A,B,\delta>0 \;\; when \frac{1}{m}+\frac{1}{n} = 1 \; (m,n>0)$$

We obtain

$$\begin{aligned}
E[\sup_{0\leq t\leq T}|e_t|^2 I_{\{\tau_d\leq T or \rho_d \leq T\}}] &\leq E[(\delta\sup_{0\leq t\leq T}|e_t|^P)^{\frac{2}{P}}(\frac{1}{\delta^{\frac{2}{P-2}}} I_{\{\tau_d\leq T or \rho_d\leq T\}})^{\frac{P-2}{P}}] \\
&\leq \frac{2\delta}{P} E[\sup_{0\leq t\leq T}|e_t|^P] + \frac{P-2}{P\delta^{\frac{2}{P-2}}} P(\tau_d \leq T or \rho_d \leq T).
\end{aligned}$$

Hence

$$\begin{aligned} E[\sup_{0\le t\le T}|e_t|^2] &\le E[\sup_{0\le t\le T}|e_t|^2 I_{\{\theta_d>T\}}] + \frac{2\delta}{P}E[\sup_{0\le t\le T}|e_t|^P] \\ &+\frac{P-2}{P\delta^{\frac{2}{P-2}}}P(\tau_d\le T or \rho_d\le T). \end{aligned} \tag{4.3}$$

Now, by **Theorem 3.1**, we get

$$\begin{aligned} P(\tau_d\le T)=E[I_{\{\tau_d>T\}}] &\le E[\frac{|P_{\tau_d}|^P}{d^P}I_{\{\tau_d>T\}}] \\ &\le \frac{1}{d^P}E[\sup_{0\le t\le T}|P_t|^P]\le\frac{H_1}{d^P}. \end{aligned}$$

Similary, we have

$$P(\rho_d\le T)\le\frac{H_2}{d^P}.$$

Thus

$$P(\tau_d\le T or \rho_d\le T)\le P(\tau_d\le T)+P(\rho_d\le T)\le\frac{H_1+H_2}{d^P}. \tag{4.4}$$

Note

$$\begin{aligned} E[\sup_{0\le t\le T}|e_t|^P] &\le 2^{P-1}E[\sup_{0\le t\le T}|P_t|^P+\sup_{0\le t\le T}|Q_t|^P] \\ &\le 2^{P-1}[H_1+H_2]. \end{aligned} \tag{4.5}$$

Now,

$$E[\sup_{0\le t\le T}|e_t|^2 I_{\{\theta_d>T\}}]=E[\sup_{0\le t\le T}|e_{t\wedge\theta_d}|^2]=E[\sup_{0\le t\le T}|P_{t\wedge\theta_d}-Q_{t\wedge\theta_d}|^2].$$

where

$$\begin{aligned} |P(t\wedge\theta_d)-Q(t\wedge\theta_d)|^2 &\le 4|-\int_0^{t\wedge\theta_d}(\frac{\partial P_s}{\partial a}-\frac{\partial Q_s}{\partial a})ds|^2+4|\int_0^{t\wedge\theta_d}u(s,a)(P_s-Z_s)ds|^2 \\ &+4|\int_0^{t\wedge\theta_d}[f(s,P_s)-f(s,Z_s)]ds|^2 \\ &+4|\int_0^{t\wedge\theta_d}\int_E[h(s,u,P_s)-h(s,u,Z_s)]\tilde{N}(ds,du)|^2. \end{aligned}$$

So, for any $0\le t_1\le T$, by the Doob martingale inequality, we have

$$\begin{aligned} E[\sup_{0\le t\le t_1}|P_{t\wedge\theta_d}-Q_{t\wedge\theta_d}|^2] &\le 4TE\int_0^{t_1\wedge\theta_d}|\frac{\partial P_s}{\partial a}-\frac{\partial Q_s}{\partial a}|^2ds+4Tu_0^2E\int_0^{t_1\wedge\theta_d}|P_s-Z_s|^2ds \\ &+ 4TE\int_0^{t\wedge\theta_d}|f(s,P_s)-f(s,Z_s)|^2ds \end{aligned}$$

$$
\begin{aligned}
&+ \quad 4E[\sup_{0\le t\le t_1} |\int_0^{t\wedge\theta_d}\int_E [h(s,u,P_s)-h(s,u,Z_s]\tilde{N}(ds,du)|^2] \\
&\le \quad 4TE\int_0^{t_1\wedge\theta_d} |\frac{\partial P_s}{\partial a}-\frac{\partial Q_s}{\partial a}|^2 ds + 4Tu_0^2 E\int_0^{t_1\wedge\theta_d}|P_s-Z_s|^2 ds \\
&+ \quad 4TE\int_0^{t\wedge\theta_d}|f(s,P_s)-f(s,Z_s)|^2 ds \\
&+ \quad 16E[\int_0^{t\wedge\theta_d}\int_E |h(s,u,P_s)-h(s,u,Z_s)|^2\Pi(du)ds].
\end{aligned}
$$

Using condition i) and **corollary 4.2**, we derive that

$$
\begin{aligned}
E[\sup_{0\le t\le t_1}|P_{t\wedge\theta_d}-Q_{t\wedge\theta_d}|^2] \quad &\le \quad 4T\gamma E\int_0^{t_1\wedge\theta_d}(|P_s-Q_s|^2)ds + 4Tu_0^2E\int_0^{t_1\wedge\theta_d}|P_s-Z_s|^2 ds \\
&+ \quad 4TK_dE\int_0^{t_1\wedge\theta_d}||P_s-Z_s||_C^2 ds + 16K_dE\int_0^{t_1\wedge\theta_d}||P_s-Z_s||_C^2 ds \\
&\le \quad 4T\gamma E\int_0^{t_1\wedge\theta_d}(|P_{s\wedge\theta_d}-Q_{s\wedge\theta_d}|^2)ds \\
&\quad +[4Tu_0^2+4TK_d+16K_d]E\int_0^{t_1\wedge\theta_d}(|P_s-Q_s|^2+|Q_s-Z_s|^2)ds \\
&\le \quad 4T\gamma E\int_0^{t_1\wedge\theta_d}(|P_{s\wedge\theta_d}-Q_{s\wedge\theta_d}|^2)ds \\
&\quad +[4Tu_0^2+4TK_d+16K_d]E\int_0^{t_1}(|P_{s\wedge\theta_d}-Q_{s\wedge\theta_d}|^2+|Q_{s\wedge\theta_d}-Z_{s\wedge\theta_d|^2})ds \\
&\le \quad (4Tu_0^2+4TK_d+16K_d)C_dTh \\
&\quad +(4T\gamma+4Tu_0^2+4TK_d+16K_d)\int_0^{t_1}E\sup_{0\le u\le s}|P_{u\wedge\theta_d}-Q_{u\wedge\theta_d}|^2 ds \\
&= \quad M_1h+M_2\int_0^{t_1}E\sup_{0\le u\le s}|P_{u\wedge\theta_d}-Q_{u\wedge\theta_d}|^2 ds.
\end{aligned}
$$

By the Gronwall inequality, for any $t_1\in[0,T]$, we find that

$$
E[\sup_{0\le t\le T}|P_{t\wedge\theta_d}-Q_{t\wedge\theta_d}|^2]\le M_1he^{M_2T}. \tag{4.6}
$$

Substituting (4.4)-(4.6)into (4.3),

$$
E[\sup_{0\le t\le T}|e_t|^2] \quad \le \quad M_1he^{M_2T}+\frac{\delta}{P}2^P[H_1+H_2]+\frac{P-2}{P\delta^{\frac{2}{P-2}}}\frac{H_1+H_2}{d^P}. \tag{4.7}
$$

Now, given any $\epsilon>0$, we can select δ sufficiently small for $\frac{\delta}{P}2^P[H_1+H_2]<\frac{\epsilon}{3}$ then choose d so large for $\frac{P-2}{P\delta^{\frac{2}{P-2}}}\frac{H_1+H_2}{d^P}<\frac{\epsilon}{3}$ and finally choose h sufficiently small such that $M_1he^{M_2T}<\frac{\epsilon}{3}$ As a result

$$
E[\sup_{0\le t\le T}|Q_t-P_t|^2]<\epsilon.
$$

and the proof is complete. □

5 An example

In this section, we present an example which illustrates the Theorem 4.3. Consider the following stochastic age-dependent population equations driven by Poisson random jump measures

$$\begin{cases} dP_t = [-\frac{\partial P_t}{\partial a} - \frac{1}{(1-a)^p} - tP_t]dt + \int_0^{+\infty} u\varphi(P_t)\tilde{N}(dt,du), & in\ Q = (0,T)\times(0,1), \\ P(0,a) = e^{-\frac{1}{(1-a)^p}}, & in \quad [0,1], \\ P(t,a) = \displaystyle\int_0^1 \frac{1}{(1-a)^p} P(t,a)da & in \quad [0,T]. \end{cases} \tag{5.1}$$

Here $\tilde{N}(dt,du) = N(dt,du) - \Pi(du)dt$ is a compensated Poisson random measures in $R^+ \times R^+$, $\varphi(\cdot) : R \to R$ are Lipschitz continuous function. We can set this problem in our formulation by taking $H = L^p([0,1]), V = W_0^1([0,1]),\ u(t,a) = \beta(t,a) = \frac{1}{(1-a)^p},\ f(t,P) = -tP$, and $h(t,u,P) = u\varphi(P),\ P(0,a) = e^{-\frac{1}{(1-a)^p}}$.

Clearly, $u(t,a)$ and $\beta(t,a)$ satisfy condition (i), the operators f and h satisfy conditions iii), iv). Consequently, the approximate solution will converge to the exact solution of (2.1) for any $(t,a) \in (0,T)\times(0,1)$ in the sense of theorem 4.3, provided h is sufficiently small. □

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