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A New Method for Finding an Optimal Solution of Fully Interval Integer Transportation Problems

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Abstract

A new method namely, separation method based on zero point method [9] is proposed for finding an optimal solution for integer transportation problems where transportation cost, supply and demand are intervals. The proposed method is a non-fuzzy method and also, has been developed without using the midpoint and width of the interval in the objective function. The solution procedure is illustrated with a numerical example. The separation method can be served as an important tool for the decision makers when they are handling various types of logistic problems having interval parameters. Further, the proposed method is extended to fuzzy transportation problems.

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Keywords: Interval integer transportation problem, Optimal solution, Zero point method, Fuzzy transportation problem.

1 Introduction

Various efficient methods were developed for solving transportation problems with the assumption of precise source, destination parameter, and the penalty factors. In real life problems, these conditions may not be satisfied always. To deal with inexact coefficients in transportation problems, many researchers [1-3, 5-8, 10, 11] have proposed fuzzy and interval programming techniques for solving them.

Das et al. [3] proposed a method, called fuzzy technique to solve interval transportation problem by considering the right bound and the midpoint of the interval. Sengupta and Pal [10] proposed a new fuzzy orientated method to solve interval transportation problems by considering the midpoint and width of the interval in the objective function.

In this paper, we propose a new method namely, separation method to find an optimal solution for integer transportation problems where transportation cost, supply and demand are intervals. We develop the separation method without using the midpoint and width of the interval in the objective function of the fully interval transportation problem which is a non-fuzzy method. The proposed method is based on zero point method [9]. The solution procedure is illustrated with a numerical example. The new method can be served as an important tool for the decision makers when they are handling various types of logistic problems having interval parameters. Further, the proposed method is extended to fuzzy transportation problems.

2 Preliminaries

Let D denote the set of all closed bounded intervals on the real line R. That is, $D = \{[a,b], a \le b$ and a and b are in R $\}$.

We need the following definitions of the basic arithmetic operators and partial ordering on closed bounded intervals which can be found in [6, 4].

Definition 1: Let $A = [a,b]$ and $B = [c,d]$ be in D. Then,

$A \oplus B = [a+c, b+d]$;

$A \Theta B = [a-d, b-c]$;

$kA = [ka, kb]$ if k is a positive real number;

$kA = [kb, ka]$ if k is a negative real number and

$A \otimes B = [p,q]$ where $p = \min\{ac, ad, bc, bd\}$ and $q = \max.\{ac, ad, bc, bd\}$

Definition 2: Let $A = [a,b]$ and $B = [c,d]$ be in D. Then,

$A \le B$ if $a \le c$ and $b \le d$

$A \ge B$ if $B \le A$, that is, $a \ge c$ and $b \ge d$ and

$A = B$ if $A \le B$ and $B \le A$, that is, $a = c$ and $b = d$.

3 Fully Interval Integer Transportation Problems

Consider the following fully interval integer transportation problem (FIITP):

$$\text{Minimize } [z_1, z_2] = \sum_{i=1}^{m} \sum_{j=1}^{n} [c_{ij}, d_{ij}] \otimes [x_{ij}, y_{ij}]$$

subject to

$$\sum_{j=1}^{n} [x_{ij}, y_{ij}] = [a_i, p_i],\ i = 1,2,...,m \quad (1)$$

$$\sum_{i=1}^{m} [x_{ij}, y_{ij}] = [b_j, q_j],\ j = 1,2,...,n \quad (2)$$

$$x_{ij} \ge 0,\ y_{ij} \ge 0,\ i = 1,2,...,m \text{ and } j = 1,2,...,n \text{ and are integers} \quad (3)$$

where c_{ij} and d_{ij} are positive real numbers for all i and j , a_i and p_i are positive real numbers for all i and b_j and q_j are positive real numbers for all j.

Definition 3: The set { $[x_{ij}, y_{ij}]$, for all $i = 1,2,...,m$ and $j = 1,2,...,n$ } is said to be a feasible solution of (FIITP) if they satisfy the equations (1), (2) and (3).

Definition 4: A feasible solution $\{[x_{ij}, y_{ij}]$, for $i = 1,2,...,m$ and $j = 1,2,...,n\}$ of the problem (FIITP) is said to be an optimal solution of (FIITP) if

$$\sum_{i=1}^{m}\sum_{j=1}^{n} [c_{ij}, d_{ij}] \otimes [x_{ij}, y_{ij}] \le \sum_{i=1}^{m}\sum_{j=1}^{n} [c_{ij}, d_{ij}] \otimes [u_{ij}, v_{ij}],$$

for $i = 1,2,...,m$ and $j = 1,2,...,n$ and for all feasible $\{[u_{ij}, v_{ij}]$ for $i = 1,2,...,m$ and $j = 1,2,...,n\}$.

Now, we prove the following theorem which finds a relation between optimal solutions of a fully interval integer transportation problem and a pair of induced transportation problems and also, is used in the proposed method.

Theorem 1: If the set $\{y^{\circ}_{ij}$ for all i and j $\}$ is an optimal solution of the upper bound transportation problem (UBITP) of (FIITP) where

(UBITP) Minimize $z_2 = \sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij} y_{ij}$

subject to

$$\sum_{j=1}^{n} y_{ij} = p_i,\ i = 1,2,...,m \quad (4)$$

$$\sum_{i=1}^{m} y_{ij} = q_j,\ j = 1,2,...,n \quad (5)$$

$$y_{ij} \ge 0,\ i = 1,2,...,m \text{ and } j = 1,2,...,n \text{ and are integers} \quad (6)$$

and the set $\{x^{\circ}_{ij}$ for all i and j $\}$ is an optimal solution of the lower bound transportation problem (LBITP) of (FIITP) where

(LBITP) Minimize $z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij}$

subject to

$$\sum_{j=1}^{n} x_{ij} = a_i\,,\ i = 1,2,...,m \qquad (7)$$

$$\sum_{i=1}^{m} x_{ij} = b_j\,,\ j = 1,2,...,n \qquad (8)$$

$$x_{ij} \geq 0\,,\ i = 1,2,...,m \text{ and } j = 1,2,...,n \text{ and are integers,} \qquad (9)$$

then the set of intervals $\{[x^{\circ}_{ij}, y^{\circ}_{ij}]$ for all i and j $\}$ is an optimal solution of the problem (FIITP) provided $x^{\circ}_{ij} \leq y^{\circ}_{ij}$, for all i and j.

Proof: Let $\{[x_{ij}, y_{ij}]$, for all i and j $\}$ be a feasible solution of the problem (FIITP).

Therefore, $\{ x^{\circ}_{ij}$, for all i and j $\}$ and $\{ y^{\circ}_{ij}$, for all i and j $\}$ are feasible solutions of the problems (UBITP) and (LBITP).

Now, since $\{ x^{\circ}_{ij}$, for all i and j $\}$ and $\{ y^{\circ}_{ij}$, for all i and j $\}$ are optimal solutions of (UBITP) and (LBITP), we have

$$\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}y^{\circ}_{ij} \leq \sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}y_{ij}\ ;\quad \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x^{\circ}_{ij} \leq \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

and $x^{\circ}_{ij} \leq y^{\circ}_{ij}$, $i = 1,2,...,m$ and $j = 1,2,...,n$.

This implies that, $\left[\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x^{\circ}_{ij},\ \sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}y^{\circ}_{ij}\right] \leq \left[\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij},\ \sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}y_{ij}\right]$

That is, $\sum_{i=1}^{m}\sum_{j=1}^{n} [c_{ij}, d_{ij}] \otimes [x^{\circ}_{ij}, y^{\circ}_{ij}] \leq \sum_{i=1}^{m}\sum_{j=1}^{n} [c_{ij}, d_{ij}] \otimes [x_{ij}, y_{ij}]$,

Now, since $\{ x^{\circ}_{ij}$, for all i and j $\}$ and $\{ y^{\circ}_{ij}$, for all i and j $\}$ satisfy (4) to (9) and $x^{\circ}_{ij} \leq y^{\circ}_{ij}$, for all i and j, we can conclude that the set $\{ [x^{\circ}_{ij}, y^{\circ}_{ij}]$ for all i and j $\}$ is a feasible solution of (FIITP).

Thus, the set of intervals $\{[x^{\circ}_{ij}, y^{\circ}_{ij}]$, for all i and j $\}$ is an optimal solution of the problem (FIITP).

Hence the theorem.

3 Separation method

We, now introduce a new algorithm namely, separation method for finding an optimal solution for a fully interval integer transportation problem.

The separation method proceeds as follows.

Step 1. Construct the UBITP of the given FIITP.

Step 2. Solve the UBITP using the zero point method. Let { y°_{ij}, for all i and j } be an optimal solution of the UBITP.

Step 3. Construct the LBITP of the given FIITP.

Step 4. Solve the LBITP with the upper bound constraints $x_{ij} \le y^{\circ}_{ij}$, for all i and j using the zero point method. Let { x°_{ij}, for all i and j } be the optimal solution of LBITP with $x^{\circ}_{ij} \le y^{\circ}_{ij}$, for all i and j.

Step 5. The optimal solution of the given FIITP is $\{[x^{\circ}_{ij}, y^{\circ}_{ij}]$, for all i and j } (by the Theorem 1).

The proposed algorithm is illustrated by the following example.

Example 1: Consider the following FIITP

					Supply
	[1,2]	[1,3]	[5,9]	[4,8]	[7,9]
	[1,2]	[7,10]	[2,6]	[3,5]	[17,21]
	[7,9]	[7,11]	[3,5]	[5,7]	[16,18]
Demand	[10,12]	[2,4]	[13,15]	[15,17]	[40,48]

Now, the UBITP of the given problem is given below:

					Supply
	2	3	9	8	9
	2	10	6	5	21
	9	11	5	7	18
Demand	12	4	15	17	48

Now, using the zero point method, the optimal solution to the UBITP is $y^{\circ}_{11} = 5, y^{\circ}_{12} = 4, y^{\circ}_{21} = 7, y^{\circ}_{24} = 14, y^{\circ}_{33} = 15$ and $y^{\circ}_{34} = 3$.

Now, the LBITP of the given problem with the upper bounded constraints is given below:

					Supply
	1	1	5	4	7
	1	7	2	3	17
	7	7	3	5	16
Demand	10	2	13	15	40

and also, $x_{ij} \le y^{\circ}_{ij}$, $i = 1,2,...,m$ and $j = 1,2,...,n$ and are integers.

Now, using the zero point method, using the zero point method, the optimal solution to the LBITP with the upper bounded constraints is $x^{\circ}_{11} = 5$, $x^{\circ}_{12} = 2$, $x^{\circ}_{21} = 5$, $x^{\circ}_{24} = 12$, $x^{\circ}_{33} = 13$ and $x^{\circ}_{34} = 3$.

Thus, an optimal solution to the given FIITP is $[x^{\circ}_{11}, y^{\circ}_{11}] = [5,5]$, $[x^{\circ}_{21}, y^{\circ}_{21}] = [2,4]$, $[x^{\circ}_{21}, y^{\circ}_{21}] = [5,7]$, $[x^{\circ}_{24}, y^{\circ}_{24}] = [12,14]$, $[x^{\circ}_{33}, y^{\circ}_{33}] = [13,15]$ and $[x^{\circ}_{34}, y^{\circ}_{34}] = [3,3]$ and also, the minimum transportation cost is [102,202].

4 Interval Transportation Problems

Consider the following interval integer transportation problem (IITP):

(IITP) Minimize $[z_1, z_2] = \sum_{i=1}^{m} \sum_{j=1}^{n} [c_{ij}, d_{ij}] w_{ij}$

subject to

$$\sum_{j=1}^{n} w_{ij} \in [a_i, p_i],\ i = 1,2,...,m$$

$$\sum_{i=1}^{m} w_{ij} \in [b_j, q_j],\ j = 1,2,...,n$$

$$w_{ij} \ge 0,\ i = 1,2,...,m \text{ and } j = 1,2,...,n \text{ and integers}$$

where c_{ij} and d_{ij} are positive real numbers for all i and j , a_i and p_i are positive real numbers for all i and b_j and q_j are positive real numbers for all j.

Let $w_{ij} = \alpha x_{ij} + (1-\alpha) y_{ij}$, $0 \le \alpha \le 1$ and x_{ij} and y_{ij} are integers with $x_{ij} \le y_{ij}$ for all i and j.

Consider the following FIITP

Minimize $[z_1, z_2] = \sum_{i=1}^{m} \sum_{j=1}^{n} [c_{ij}, d_{ij}] \otimes [x_{ij}, y_{ij}]$

subject to

$$\sum_{j=1}^{n} [x_{ij}, y_{ij}] = [a_i, p_i],\ i = 1,2,...,m$$

$$\sum_{i=1}^{m} [x_{ij}, y_{ij}] = [b_j, q_j],\ j = 1,2,...,n$$

$$[x_{ij}, y_{ij}] \ge 0,\ i = 1,2,...,m \text{ and } j = 1,2,...,n \text{ and are integers}$$

The above problem can be solved by using separation method. Let { $[x^{\circ}_{ij}, y^{\circ}_{ij}]$, for all i and j } be an optimal solution of the above problem. Since w_{ij} , for all i and j are integers, choose any α° such that $w^{\circ}_{ij} = \alpha^{\circ} x^{\circ}_{ij} + (1-\alpha^{\circ}) y^{\circ}_{ij}$, for all i and j are integers. Then, optimal solutions to IITP are given below.

$$w^{\circ}_{ij} = \alpha^{\circ} x^{\circ}_{ij} + (1-\alpha^{\circ}) y^{\circ}_{ij} \;\; 0 \le \alpha^{\circ} \le 1 .$$

The solving procedure of obtaining optimal solutions to IITP using the separation method is illustrated by the following example.

Example 2: Consider the following IITP with integer real decision variables

					Supply
	[3,5]	[2,6]	[2,4]	[1,5]	[7,9]
	[4,6]	[7,9]	[7,10]	[9,11]	[17,21]
	[4,8]	[1,3]	[3,6]	[1,2]	[16,18]
Demand	[10,12]	[2,4]	[13,15]	[15,17]	[40,48]

Let $w_{ij} = \alpha x_{ij} + (1-\alpha) y_{ij},\ 0 \le \alpha \le 1$ and x_{ij} and y_{ij} are integers with $x_{ij} \le y_{ij}$ for all i and j.

Now, we consider the following FIITP with variables $[x_{ij}, y_{ij}]$ for all i and j corresponding to the given IITP.
Now, the UBITP of the FIITP is given below.

					Supply
	5	6	4	5	9
	6	9	10	11	21
	8	3	6	2	18
Demand	12	4	15	17	48

Now, using the zero point method, the optimal solution to the UBITP is $y^{\circ}_{13} = 9, y^{\circ}_{21} = 12, y^{\circ}_{22} = 3,\ y^{\circ}_{23} = 6,\ y^{\circ}_{32} = 1$ and $y^{\circ}_{34} = 17$.

Now, the LBITP of the FIITP with the bounded constraints is given below.

					Supply
	3	2	2	1	7
	4	7	7	9	17
	4	1	3	1	16
Demand	10	2	13	15	40

and also, $x_{ij} \le y^{\circ}_{ij}$, $i = 1,2,...,m$ and $j = 1,2,...,n$ and are integers.

Now, using the zero point method, using the zero point method, the optimal solution to the LBITP with bounded constraints is $x^{\circ}_{13} = 7$, $x^{\circ}_{21} = 10$, $x^{\circ}_{22} = 1$, $x^{\circ}_{23} = 6$, $x^{\circ}_{32} = 1$ and $x^{\circ}_{34} = 15$.

Thus, an optimal solution to the FIITP is $[x^{\circ}_{13}, y^{\circ}_{13}] = [7,9]$, $[x^{\circ}_{21}, y^{\circ}_{21}] = [10,12]$, $[x^{\circ}_{22}, y^{\circ}_{22}] = [1,3]$, $[x^{\circ}_{23}, y^{\circ}_{23}] = [6,6]$, $[x^{\circ}_{32}, y^{\circ}_{32}] = [1,1]$ and $[x^{\circ}_{34}, y^{\circ}_{34}] = [15,17]$ and also, the minimum transportation cost is [119,232].

Now, $w^{\circ}_{ij} = \alpha x^{\circ}_{ij} + (1-\alpha) y^{\circ}_{ij}$, $0 \le \alpha \le 1$ are integers for all i and j.

S.No.	α	Solution	z -value	Number of units transported
1	0	$w^{\circ}_{13} = 9$, $w^{\circ}_{21} = 12$, $w^{\circ}_{22} = 3$, $w^{\circ}_{23} = 6$, $w^{\circ}_{32} = 1$ and $w^{\circ}_{34} = 17$	[147,232]	48
2	0.5	$w^{\circ}_{13} = 8$, $w^{\circ}_{21} = 11$, $w^{\circ}_{22} = 2$, $w^{\circ}_{23} = 6$, $w^{\circ}_{32} = 1$ and $w^{\circ}_{34} = 16$	[133,211]	45
3	1	$w^{\circ}_{13} = 7$, $w^{\circ}_{21} = 10$, $w^{\circ}_{22} = 1$, $w^{\circ}_{23} = 6$, $w^{\circ}_{32} = 1$ and $w^{\circ}_{34} = 15$	[119,190]	40

Note: This type of solutions set is very much useful for decision makers to select a solution according to their needs, since the set of solutions to the ITP is a function of the number of units transported.

5 Fully fuzzy integer transportation problems

Consider the following fuzzy integer transportation problem (FFITP) where

(FFITP) Minimize $\tilde{z} \approx \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij}\, \tilde{x}_{ij}$

subject to

$$\sum_{j=1}^{n} \tilde{x}_{ij} = \tilde{a}_i, \; i = 1,2,...,m$$

$$\sum_{i=1}^{m} \tilde{x}_{ij} = \tilde{b}_j, \; j = 1,2,...,n$$

$$\tilde{x}_{ij} \geq \tilde{0},\ \ i = 1,2,...,m \text{ and } j = 1,2,...,n \text{ and are integers}$$

where
m = the number of supply points ;
n = the number of demand points ;
$\tilde{x}_{ij}$ is the uncertain number of units shipped from supply point i to demand point j ;
$\tilde{c}_{ij}$ is the uncertain cost of shipping one unit from supply point i to the demand point j ;
$\tilde{a}_i$ is the uncertain supply at supply point i and
$\tilde{b}_j$ is the uncertain demand at demand point j.

A trapezoidal fuzzy number (a,b,c,d) can be represented as an interval number form as follows.

$$(a,b,c,d) = [a+(b-a)\alpha, d-(d-c)\alpha]\ ;\ 0 \leq \alpha \leq 1. \tag{10}$$

Using the relation (10), we can convert the given fuzzy transportation problem into an interval transportation problem . Using the separation method, we obtain an optimal solution to the interval transportation. Then, again using the relation (10), we can obtain an optimal solution to the given fuzzy transportation problem.

The solution procedure of obtaining an optimal solution to a fuzzy transportation problem using the separation method is illustrated by the following example.

Example 3: Consider the following FFITP:

					Supply
	(1,2,3,4)	(1,3,4,6)	(9,11,12,14)	(5,7,8,11)	(1,6,7,12)
	(0,1,2,4)	(-1,0,1,2)	(5,6,7,8)	(0,1,2,3)	(0,1,2,3)
	(3,5,6,8)	(5,8,9,12)	(12,15,16,19)	(7,9,10,12)	(5,10,12,17)
Demand	(5,7,8,10)	(1,5,6,10)	(-1,3,4,8)	(1,2,3,4)	

The given fuzzy transportation problem is a balanced one.

Now, the FIITP corresponding to the above problem is given below.

					Supply
	[1+ α ,4- α]	[1+2 α ,6-2 α]	[9+2 α ,14-2 α]	[5+2 α ,11-3 α]	[1+5 α ,12-5 α]
	[0+ α , 4-2 α]	[-1+ α ,2- α]	[5+ α , 8- α]	[0+ α ,3- α]	[0+ α ,3- α]
	[3+2 α ,8-2 α]	[5+3 α ,12-3 α]	[12+3 α ,19-3 α)	[7+2 α ,12-2 α]	[5+5 α ,17-5 α]
Demand	[5+2 α ,10-2 α]	[1+4 α ,10-4 α]	[-1+4 α , 8-2 α]	[1+ α ,4- α]	[6+11 α ,32-9 α]

Now, the UBITP of the FIITP is given below.

					Supply
	$4-\alpha$	$6-2\alpha$	$14-2\alpha$	$11-3\alpha$	$12-5\alpha$
	$4-2\alpha$	$2-\alpha$	$8-\alpha$	$3-\alpha$	$3-\alpha$
	$8-2\alpha$	$12-3\alpha$	$19-3\alpha$	$12-2\alpha$	$17-5\alpha$
Demand	$10-2\alpha$	$10-4\alpha$	$8-2\alpha$	$4-\alpha$	$32-9\alpha$

Now, using the zero point method, an optimal solution to the UBITP is given below.

					Supply
		$10-4\alpha$	$2-\alpha$		
			$3-\alpha$		
	$10-2\alpha$		$3-2\alpha$	$4-\alpha$	
Demand					

Now, the LBITP of the FIITP with the constraints is given below.

					Supply
	$1+\alpha$	$1+2\alpha$	$9+2\alpha$	$5+2\alpha$	$1+5\alpha$
	$0+\alpha$	$-1+\alpha$	$5+\alpha$	$0+\alpha$	$0+\alpha$
	$3+2\alpha$	$5+3\alpha$	$12+3\alpha$	$7+2\alpha$	$5+5\alpha$
Demand	$5+2\alpha$	$1+4\alpha$	$-1+4\alpha$	$1+\alpha$	

and also, $x_{ij} \leq y^{\circ}_{ij}$, $i = 1,2,...,m$ and $j = 1,2,...,n$ and are integers.

Now, using the zero point method, an optimal solution to the LBITP is given below.

					Supply
		$1+4\alpha$	α		
			α		
	$5+2\alpha$		$-1+2\alpha$	$1+\alpha$	
Demand					

Therefore, an optimal solution to the FIITP is given below.

				Supply
	[1+4 α ,10-4 α]	[α ,2- α]		[1+5 α ,12-5 α]
		[α , 3- α]		[0+ α ,3- α]
[5+2 α ,10-2 α]		[-1+2 α , 3-2 α]	[1+ α , 4- α]	[5+5 α ,17-5 α]
Demand [5+2 α ,10-2 α]	[1+4 α ,10-4 α]	[1+2 α ,6-2 α]	[1+ α ,4- α]	

Thus, the fuzzy optimal solution for the given FFITP is $\tilde{x}_{12} \approx (1,5,6,10)$, $\tilde{x}_{13} \approx (0,1,1,2)$, $\tilde{x}_{23} \approx (0,1,2,3)$, $\tilde{x}_{31} \approx (5,7,8,10)$, $\tilde{x}_{23} \approx (-1,1,1,3)$ and $\tilde{x}_{34} \approx (1,2,3,4)$ with the fuzzy objective value $\tilde{z} = (4,100,144,297)$ and the crisp value of the optimum fuzzy transportation cost for the problem, z is 126.75.

Remark : The fuzzy transportation problem with crisp decision variables can also be solved by using the separation method similar to the method of solving interval integer transportation problems.

6 Conclusion

The separation method based on the zero point method provides an optimal value of the objective function for the fully interval transportation problem. This method is a systematic procedure, both easy to understand and to apply and also it is a non-fuzzy method. The proposed method provides more options and can be served an important tool for the decision makers when they are handling various types of logistic problems having interval parameters.

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