Applied Mathematical Sciences, Vol. 4, 2010, no. 43, 2105 - 2117

Slip Effect on Peristaltic Transport of Micropolar Fluid

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Abstract

In this paper, we investigate peristaltic transport of a micropolar fluid in a channel when no slip boundary condition is inadequate. The long wavelength and low Reynolds number approximations are used to get solution. The effects of coupling number, micropolar parameter and slip parameter on pumping region, friction force and trapping are analyzed and presented by graphs. It is found that the pumping region increases with increasing coupling number while it decreases with increasing micropolar parameter and slip parameter. The size of trapped bolus increases by increasing coupling number and micropolar parameter while it decreases by increasing slip parameter.

Mathematics Subject Classification: 76Z05

Keywords: Peristaltic transport; micropolar fluid; slip condition

1. Introduction

The mechanism involved in transportation of bio fluids from one place to another due to muscular contraction or expansion of tube/channel containing the fluid, is called peristalsis. Peristalsis is used in urine flow from kidney to blader, swallowing food through the esophagus, movement of chyme in gastro-intestinal tract, intra-uterine fluid motion, flow of spermatozoa in the ductus efferentes of the male reproductive tract, movement of ovum in the female fallopian tube, transport of lymph in the lymphatic vessels and the vasomotion of small blood vessels such arterioles, venules and capillaries. Some worms also use peristaltic motion as a means of locomotion. Roller and finger pumps also operate on this principle.

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From first investigation made by Latham [15], peristalsis has attracted several researchers due to its great application in engineering and physiology. Shapiro et al. [2] studied the peristaltic flow of Newtonian fluid through channel and tube under long wavelength and low Reynolds number assumptions in wave frame of reference. Fung and Yih [17], Yin and Fung [4] studied the peristaltic flow of Newtonian fluid through channel and tube respectively using perturbation technique in laboratory frame of reference. The behavior of most of the physiological fluids is known to be non-Newtonian. The model of micropolar fluid introduced by Eringen [1] represents fluids consisting of rigid, randomly oriented (or spherical) particles suspended in a viscous medium where the deformation of the particles is ignored. Micropolar fluids exhibit some microscopic effects arising from the local structure and micromotion of the fluid elements. Furthermore, they can sustain couple stresses. The micropolar fluid is considered to model the blood flow in small arteries and the calculation of theoretical velocity profiles is observed in good agreement with the experimental data. Several researchers [5, 6, 8-11, 13, 14] contributed towards the study of peristaltic transport of micropolar fluid under different situations assuming no slip boundary condition at walls of the vessels.

But, in real system there is always a certain amount of slip and no slip boundary condition is no longer valid. Many studies [3, 7, 12, 16] are made to investigate slip effect on peristaltic transport of fluids. However, to the our best knowledge, no attentions are given to analyze slip effect on peristaltic transport of micropolar fluid. So in this study we analyze the effect of slip boundary condition on peristaltic transport of micropolar fluid in a channel.

2. Mathematical Formulation

We consider peristaltic transport of a micropolar fluid in a channel of width $2a$. Let the motion of the walls of the channel be governed by sinusoidal wave, then

$$H = a + b\sin\frac{2\pi}{\lambda}\left(X - ct\right) \tag{1}$$

where b, λ, c are amplitude, wavelength and velocity of the wave and t is time.

The flow is unsteady in the laboratory frame (X, Y) and steady in the wave frame (x, y) moving with velocity c. The relation between laboratory frame and wave frame is given by

$$x = X - ct, \quad y = Y, \quad u = U - c, \; v = V. \tag{2}$$

In the absence of body force and body couple, the governing equations of steady flow of an incompressible micropolar fluid are

$$\nabla.\vec{v} = 0, \tag{3}$$

$$\rho\left(\vec{v}.\nabla\vec{v}\right) = -\nabla p + \kappa\nabla\times\vec{w} + (\mu+\kappa)\nabla^2\vec{v}, \tag{4}$$

$$\rho j\left(\vec{v}.\nabla\vec{w}\right) = -2\kappa\vec{w} + \kappa\nabla\times\vec{v} - \gamma\left(\nabla\times\nabla\times\vec{w}\right) + (\alpha+\beta+\gamma)\nabla\left(\nabla.\vec{w}\right). \tag{5}$$

where $\vec{v}, \vec{w}, p, \rho$ and j are the velocity vector, the microrotation vector, the fluid pressure, the fluid density and microgyration parameter. The material constants $\mu, \kappa, \alpha, \beta$ and γ satisfy [5]:

$$2\mu + \kappa \geq 0, \quad \kappa \geq 0, \quad 3\alpha + \beta + \gamma \geq 0, \ \gamma \geq |\beta|. \tag{6}$$

We take the velocity vector $\vec{v} = (u, v, 0)$ and microrotation vector $\vec{w} = (0,0,w)$. We nondimensionalize the variables as follows:

$$x' = \frac{x}{\lambda}, \quad y' = \frac{y}{a}, \quad u' = \frac{u}{c}, \ v' = \frac{v}{\delta c}, \quad w' = \frac{a\,w}{c}, \ t' = \frac{c\,t}{\lambda}, \ \phi = \frac{b}{a},$$

$$j' = \frac{j}{a^2}, \quad p' = \frac{a^2 p}{c\,\lambda\mu}, \ h = \frac{H}{a}, \quad \delta = \frac{a}{\lambda}, \quad R_e = \frac{\rho\,c\,a}{\mu}. \tag{7}$$

where R_e is the Reynolds number and δ is the wave number.

Using these nondimensional variables in Eqs. (1), (3),(4) and (5) and primes and applying low Reynolds number and long wavelength approximation, we get

$$h(x) = 1 + \phi \sin 2\pi\, x, \tag{8}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{9}$$

$$N\frac{\partial w}{\partial y} + \frac{\partial^2 u}{\partial y^2} = (1-N)\frac{\partial p}{\partial x}, \tag{10}$$

$$\frac{\partial p}{\partial y} = 0, \tag{11}$$

$$-2\,w - \frac{\partial u}{\partial y} + \left(\frac{2-N}{M^2}\right)\frac{\partial^2 w}{\partial y^2} = 0. \tag{12}$$

where $N = \kappa/(\mu+\kappa)$ is the coupling number $(0 \leq N \leq 1)$ $M^2 = a^2\kappa(2\mu+\kappa)/\gamma(\mu+\kappa)$ is the micropolar parameter.

In the wave frame the boundary conditions are:

$$\frac{\partial u}{\partial y} = 0, \qquad w = 0. \qquad \text{at } y = 0. \tag{13}$$

$$u = -1 \mp \beta\frac{\partial u}{\partial y}, \quad w = 0. \qquad \text{at } y = \pm h. \tag{14}$$

where $\beta (= L/a)$ is the dimensionless slip parameter and L is the dimensional slip parameter.

3. Solution

From Eq. (11) it is clear that p does not depend on y, so Eq. (10) may be written as

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left[(1-N)y\frac{dp}{dx} - N\,w\right] \tag{15}$$

Integrating Eq. (15) w.r.t. to y, we get

$$\frac{\partial u}{\partial y} = (1-N)y\frac{dp}{dx} - N\,w + C_1(x) \tag{16}$$

where $C_1(x)$ is constant of integration.

Using boundary condition (13) in Eq. (16) we get $C_1(x) = 0$, so Eq. (16) becomes

$$\frac{\partial u}{\partial y} = (1-N)y\frac{dp}{dx} - N\,w \tag{17}$$

Putting the value of $(\partial u/\partial y)$ from Eq. (17) in Eq. (12), we get

$$\frac{\partial^2 w}{\partial y^2} - M^2 w = M^2\left(\frac{1-N}{2-N}\right)y\frac{dp}{dx} \tag{18}$$

The general solution of above equation may be written as

$$w = -\left(\frac{1-N}{2-N}\right)y\frac{dp}{dx} + C_2\cosh M\,y + C_3\sinh M\,y \tag{19}$$

Using Eq. (19) in Eq. (17), we get

$$u = \left(\frac{1-N}{2-N}\right)y^2\frac{dp}{dx} - C_2\frac{N}{M}\sinh My - C_3\frac{N}{M}\cosh My + C_4 \tag{20}$$

where C_2, C_3, C_4 are integrating constants, which are evaluated by using the boundary conditions (13) & (14). Thus the resulting solutions are given by

$$u = \left(\frac{1-N}{2-N}\right)y^2\frac{dp}{dx} - \frac{(1-N)h}{(2-N)M\sinh Mh}\frac{dp}{dx}\big[N(\cosh My - M\beta\sinh Mh - \cosh Mh) + M(2\beta+h)\sinh Mh\big] - 1 \tag{21}$$

$$w = \left(\frac{1-N}{2-N}\right)\frac{dp}{dx}\left[\frac{h\sinh My}{\sinh Mh} - y\right] \tag{22}$$

The corresponding stream function ψ is given by

$$u = \frac{\partial \psi}{\partial y} \qquad \text{and}\; v = -\frac{\partial \psi}{\partial x} \tag{23}$$

Using (23) and (21) we get

$$\psi = \left(\frac{1-N}{2-N}\right)\frac{y^3}{3}\frac{dp}{dx} - \frac{(1-N)h}{(2-N)M\sinh Mh}\frac{dp}{dx}\left[\frac{N}{M}\sinh My - (NM\beta\sinh Mh + N\cosh Mh)y + (Mh\sinh Mh + 2\beta M\sinh mh)y\right] - y \tag{24}$$

The nondimensional mean flow in wave frame is given by

$$q = \int_0^h u\,dy \quad = -h + \left(\frac{1-N}{2-N}\right)\frac{dp}{dx}\,f(h) \tag{25}$$

$$f(h) = \left[\frac{h^3}{3} - \frac{h}{M\sinh Mh}\left\{\frac{N}{M}\sinh Mh - N\,h(M\beta\sinh Mh + \cosh Mh) + M\,h(2\beta+h)\sinh Mh\right\}\right] \tag{26}$$

The relation between the dimensionless mean flow Θ in laboratory frame and q in wave frame is

$$\Theta = q + 1 \tag{27}$$

From Eq. (25) we get

$$\frac{dp}{dx}=\left(\frac{2-N}{1-N}\right)\frac{(q+h)}{f(h)} \tag{28}$$

The non-dimensional pressure rise per wavelength Δp and the friction force F at the wall are

$$\Delta p=\int_0^1\left(\frac{dp}{dx}\right)dx \tag{29}$$

$$F=\int_0^1 -h\left(\frac{dp}{dx}\right)dx \tag{30}$$

4. Numerical results and discussion

In this section, we have carried out numerical calculations and plotted graphs to study effects of the coupling number N, the micropolar parameter M and the slip parameter β on the pumping region, the friction force at the wall and the trapping phenomenon. In the case of free pumping that is when $\Delta p=0$, the corresponding time mean flow rate is denoted by Θ_0.The maximum pressure against which the peristalsis work as a pump, that is, Δp corresponding to $\Theta=0$ is denoted by p_0.In the case of copumping that is when $\Delta p<0$, the pressure assist the flow.

Figs. 1-3 show the variation of pressure rise Δp with flow rate Θ for various values of coupling number N, micropolar parameter M and slip parameter β respectively. It is observed that the pumping region $(0\le\Delta p\le p_0)$ increases as the coupling number N increases while it decreases as the micropolar parameter M and the slip parameter β increase. Moreover the pumping region is more for a micropolar fluid in comparison to that of a Newtonian fluid.

Figs. 4-6 present the variation of friction force F with flow rate Θ for various values of coupling number N, micropolar parameter M and slip parameter β respectively. From these figures it is noted that there exists a critical value of Θ below which F resists the flow and above which F assist the flow. Below this critical value of Θ, the friction force F increases as N, M and β increase. However, above this critical value of Θ, the friction force F decreases as N, M and β increase.

Trapping is an interesting phenomenon in peristaltic motion in which an internally circulating bolus of fluid is formed by closed streamlines and this trapped bolus is pushed ahead along with the peristaltic wave. the effects of coupling number N, micropolar parameter M and slip parameter β can be seen through Figs.7- 9 . A general observation regarding the effects of N and M is that the trapped bolus increases in size as N and M increase. However, the size of the trapped bolus decreases in size as β increases.

6. Conclusions

In this study the effect of slip condition on peristaltic flow of a micropolar fluid in a two dimensional channel have been analyzed under long wave length and low Reynolds number approximations. The effects of the coupling number N, the micropolar parameter M and the slip parameter β on the pumping region, the friction force at the wall and the trapping phenomenon have been discussed in detail. The main findings of the present study are given in the following points:

- The peristaltic pumping region narrows down by increasing the micropolar parameter M and the slip parameter β .However, it widens by increasing the coupling number N .
- The friction force F increases by increasing N , M and β below the critical value of Θ . However, above this critical value of Θ , F decreases by increasing N , M and β .
- The size of trapped bolus increases by increasing N and M while it decreases by increasing β .
- The results for Newtonian fluid without slip can be obtained as the special cases of our analysis by choosing $N = 0,\ \beta = 0$.

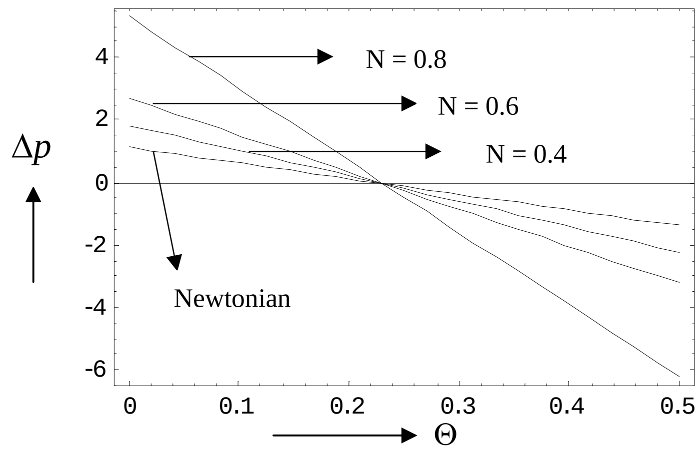

Fig.1 Pressure rise versus flow rate for $\phi = 0.4, M = 2.0$ and $\beta = 0$.

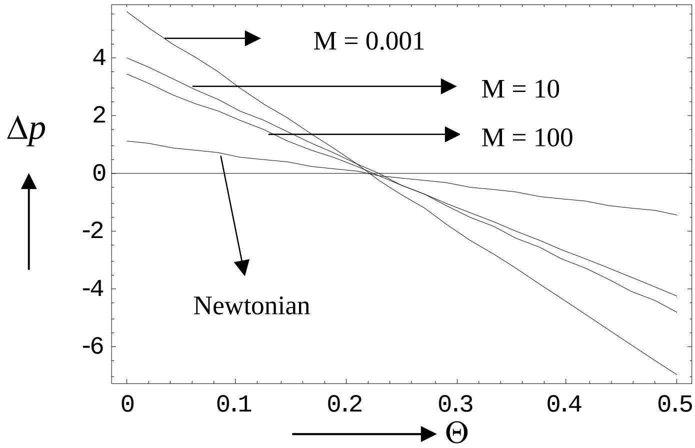

Fig.2 Pressure rise versus flow rate for $\phi = 0.4, N = 0.8$ and $\beta = 0$.

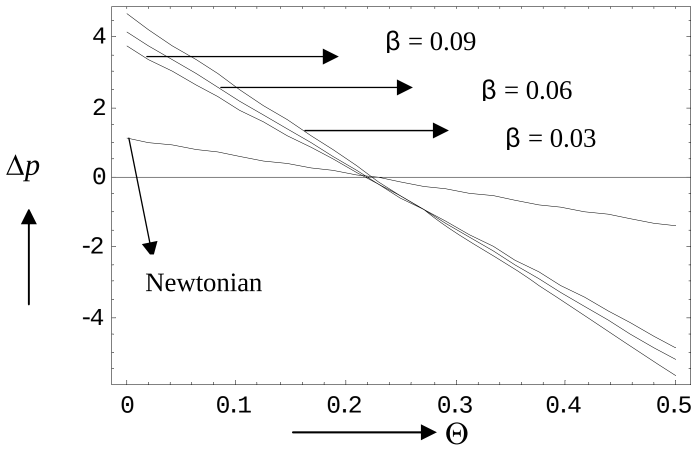

Fig.3 Pressure rise versus flow rate for $\phi = 0.4, M = 2.0$ and $N = 0.8$.

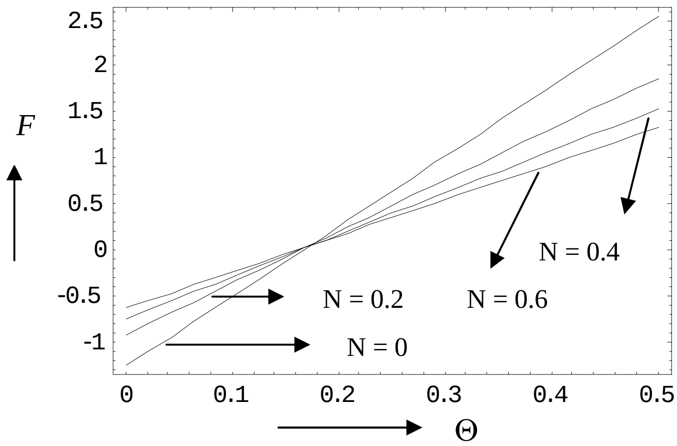

Fig.4 The friction force at the wall versus flow rate for $\phi = 0.4, M = 10$ and $\beta = 0$.

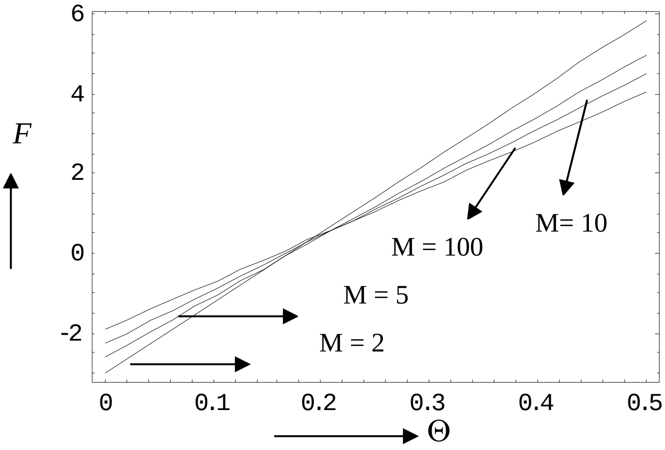

Fig.5 The friction force at the wall versus flow rate for $\phi = 0.4, N = 0.8$ and $\beta = 0$.

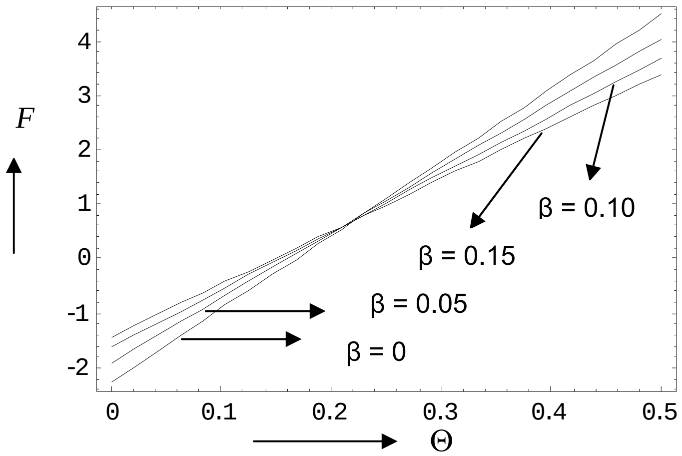

Fig.6 The friction force at the wall versus flow rate for $\phi = 0.4, m = 10$ and $N = 0.8$.

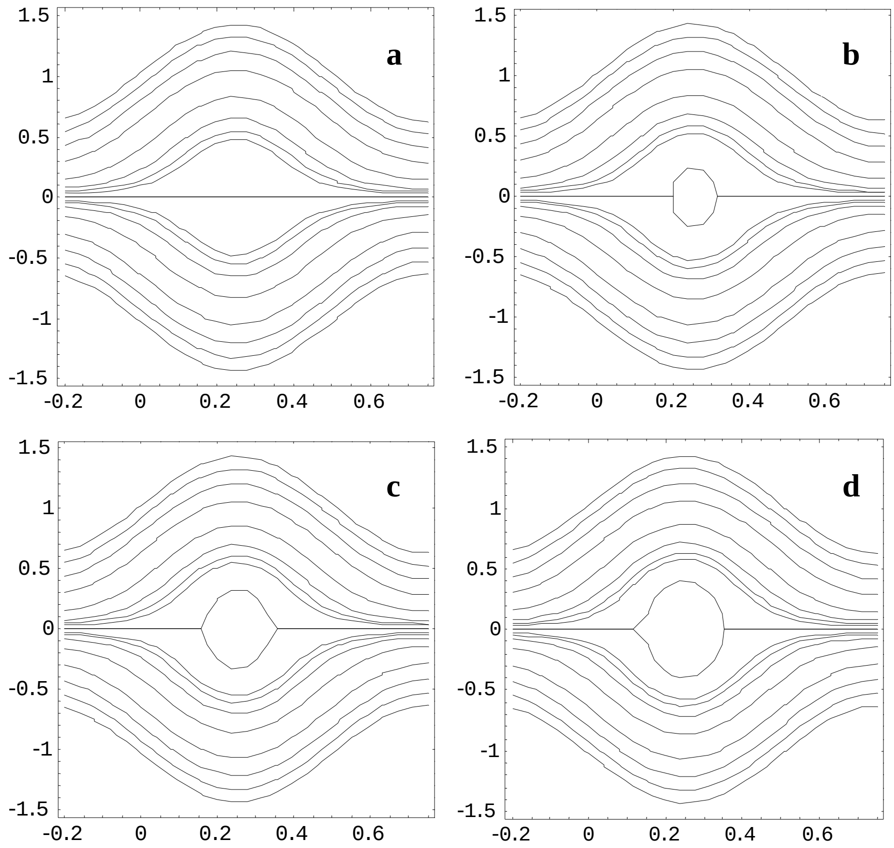

Fig.7 Streamlines for $M = 2.0, \phi = 0.4, \beta = 0$ and $\overline{Q} = 0.53$: **a** $N = 0$. **b** $N = 0.4$. **c** $N = 0.6$. **d** $N = 0.8$.

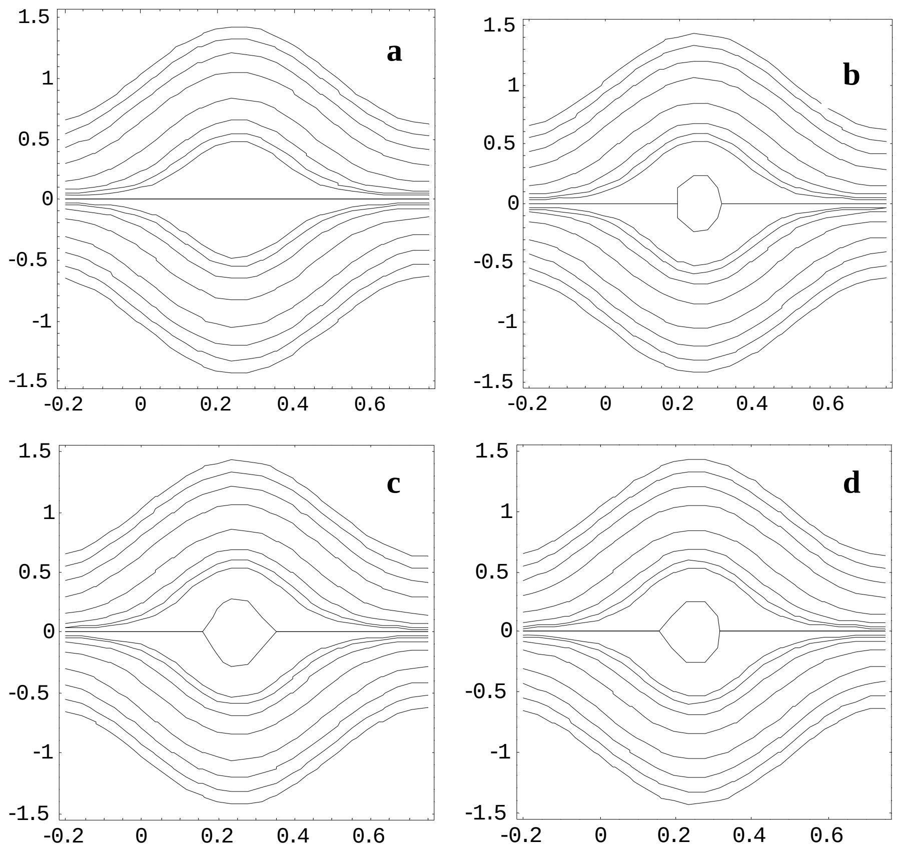

Fig.8 Streamlines for $N = 0.4, \phi = 0.4, \beta = 0$ and $\overline{Q} = 0.53$: **a** $M = 0.1$ **.** **b** $M = 2$ **.** **c** $M = 4$ **.** **d** $M = 6$ **.**

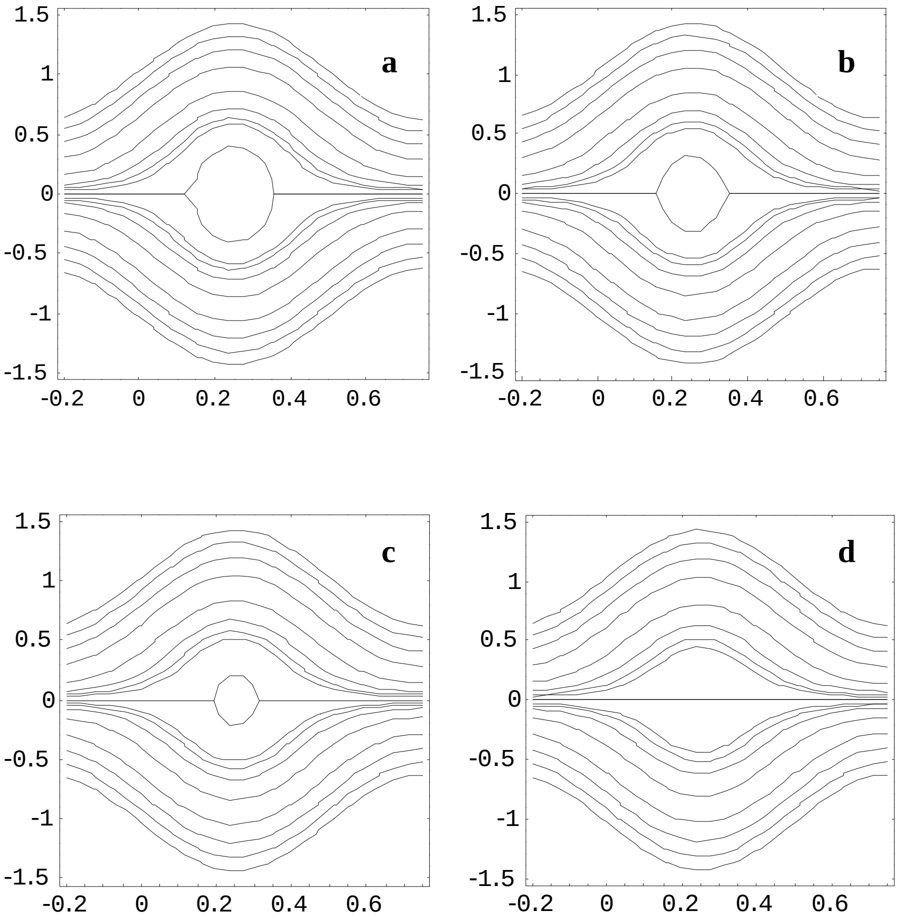

Fig.9 Streamlines for $N = 0.7, \phi = 0.4, M = 4$ and $\overline{Q} = 0.53$: **a** $\beta = 0$ **. b** $\beta = 0.02$ **. c** $\beta = 0.04$ **. d** $\beta = 0.08$ **.**

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Received: October, 2009