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On Antipodal Fuzzy Graph

A. Nagoor Gani

PG and Research Department of Mathematics
Jamal Mohamed College (Autonomous)
Tiruchirappalli–620020, Tamilnadu, India
E-mail: ganijmc@yahoo.co.in

J. Malarvizhi

PG and Research Department of Mathematics
K.N. Govt. Arts College (Autonomous) for Women
Thanjavur-07, Tamilnadu, India
mathmalar@yahoo.co.in

Abstract

In this paper the isometry between two fuzzy graphs is defined. Nature of the isometry relation and concepts regarding isomorphism and isometry is discussed. Antipodal fuzzy graph of the given fuzzy graph is defined. When the given fuzzy graph is either complete or strong, the nature of its antipodal fuzzy graph is discussed. Isomorphism concept for the antipodal fuzzy graphs is also studied.

Mathematics Subject Classification: 03E72, 05C12, 05C99

Keywords: Fuzzy relation, Antipodal, μ - distance regular fuzzy graph, connected strong fuzzy graph

1. Introduction

In crisp graph theory the concept of antipodal graph of a given graph G was introduced by Smith [9]. The condition on the graph G, for A(G) = G and A(G)= $\overline{G}$ are discussed by Aravamudhan and Rajendran [1]. As a fuzzy analog to this, in this paper antipodal fuzzy graph is defined and its nature is discussed. Basic definitions are taken from [2,3,5,8,10]

2. Preliminaries

2.1. Definition

A fuzzy graph with a non-empty finite set S as the underlying set is a pair G:(σ,μ) where σ : S$\rightarrow$[0,1] is a fuzzy subset of S, μ :S$\times$S $\rightarrow$[0,1] is a symmetric fuzzy relation on the fuzzy subset σ, such that μ(x ,y) $\leq \sigma$(x)$\wedge\sigma$(y) for all x, y $\in$S , where $\wedge$ stands for minimum. The underlying crisp graph of the fuzzy graph G: (σ,μ) is denoted as G*: (σ^*,μ^*) where σ^*={u$\in$S/σ(u)>0},
μ^* = {(u,v) $\in$S$\times$S/μ(x ,y)>0}. If μ(x,y)> 0 then x and y are called neighbors , x and y are said to lie on e=(x ,y).
Throughout this paper G is a fuzzy graph unless it is mentioned.

2.2. Definition

A path ρ in a fuzzy graph G:(σ,μ) is a sequence of distinct nodes $v_0, v_1, v_2, \ldots, v_n$ such that μ(v_{i-1}, v_i) > 0, 1$\leq$i$\leq$n. Here 'n' is called the length of the path. The consecutive pairs (v_{i-1}, v_i) are called arcs of the path.

2.3. Definition

If u, v are nodes in G and if they are connected by means of a path then the strength of that path is defined as $\overset{n}{\underset{i=1}{\wedge}} \mu(v_{i-1}, v_i)$; i.e it is the strength of the weakest arc. If u, v are connected by means of paths of length 'k' then μ^k (u,v) is defined as μ^k (u,v) = sup{μ(u, v_1) $\wedge\mu$(v_1, v_2)$\wedge\mu$(v_2, v_3)….. $\wedge\mu$(v_{k-1}, v) / u,v_1 ,$v_2, \ldots, v_{k-1}$, v $\in$S}. If u, v $\in$S the strength of connectedness between u and v is denoted as μ^∞(u,v) = sup {μ^k (u,v)/ k= 1,2,3,…}.
A fuzzy graph G is said to be connected if μ^∞(u,v) > 0 for all u ,v $\in$S

2.4. Definition

A fuzzy graph G is said to be a strong fuzzy graph if μ(x ,y) = σ(x) $\wedge\sigma$(y)
for all (x,y) in μ^*.

2.5. Definition
A fuzzy graph G is said to be a complete fuzzy graph if μ(x ,y) = σ(x) $\wedge$ σ(y) for all x, y in σ^*, it is denoted as K_σ : (σ , μ)

2.6. Definition
Given a fuzzy graph G: (σ , μ), with the underlying set S, the order of G is defined as order(G)= $\sum_{x\in S}\sigma(x)$ and size of G is defined as size(G) = $\sum_{x,y\in S}\mu(x,y)$

2.7. Definition
The degree of a vertex 'u' in G is defined as $d_G(u) = \sum_{\substack{v\neq u\\ v\in S}}\mu(u,v)$. If every node in G is of same degree 'k' then G is a regular fuzzy graph of degree k

2.8. Definition
The μ - distance $\delta(u,v)$ is the smallest μ - length of any u-v path, where the μ - length of a path ρ : u_0 , u_1 , $u_2, \ldots,$ u_n is $\ell(\rho) = \sum_{i=1}^{n}\frac{1}{\mu(u_{i-1},u_i)}$. The eccentricity of a node v is defined as e(v) = $\max_u(\delta(u,v)$ .The diameter diam(G) = $\vee$ {e(v)/ v $\in$ S}, radius r(G) = $\wedge$ {e(v)/ v $\in$ S}.A node whose eccentricity is minimum in a connected fuzzy graph is called a central node. A connected fuzzy graph is called self-centered if each node is a central node.

2.9. Definition
Let G: (σ , μ) be a fuzzy graph. The complement of G is defined as $\overline{G}$: (σ , $\overline{\mu}$) where $\overline{\mu}$(x , y) = σ(x) $\wedge$ σ(y) - μ(x ,y) $\forall$ x, y $\in$ S.

2.10. Definition
Let G and G' be fuzzy graphs with underlying sets S and S' respectively. A homomorphism of fuzzy graphs, h: G $\rightarrow$ G' is a map h: S $\rightarrow$ S' which satisfies
σ(x) $\leq$ σ'(h(x)) for all x $\in$ S
μ(x ,y) $\leq$ μ'(h(x), h(y)) for all x,y $\in$ S

2.11. Definition
An isomorphism h: G $\rightarrow$ G' is a bijective map h:S $\rightarrow$ S' which satisfies
σ(x) = σ'(h(x)) for all x $\in$ S
μ(x ,y) = μ'(h(x), h(y)) for all x,y $\in$ S
Then G is said to be isomorphic with G'

2.12. Definition
A weak isomorphism h: G $\rightarrow$ G' is a map, h: S $\rightarrow$ S' which is a bijective homomorphism that satisfies σ(x) = σ'(h(x)) for all x $\in$ S

2.13. Definition
A co-weak isomorphism h:G $\rightarrow$ G' is a map, h:S $\rightarrow$ S' which is a bijective homomorphism that satisfies μ(x ,y) = μ'(h(x) ,h(y)) for all x ,y $\in$ S

3. Isometric Fuzzy Graph

3.1. Definition

Let G_i: (σ_i, μ_i) be the fuzzy graphs with underlying sets V_i, for i=1,2. G_2 is said to be isometric from G_1 if for each $v \in G_1$ there is a bijection $\phi_v : V_1 \to V_2$ such that $\delta_1(u,v) = \delta_2(\phi_v(u), \phi_v(v))$, for every u $\in G_1$. If they are isometric from each other they are said to be isometric.

3.2. Example

Let G_1 be the fuzzy graph on the set V_1= {a,b,c,d}such that σ_1(a)= 1/7, σ_1(b)= 1, σ_1(c)=1/2, σ_1(d)=1/5, with μ_1(a,b)=1/16, μ_1(b,c)=1/3, μ_1(a,d)=1/8, μ_1(b,d)=1/5. In this fuzzy graph, $\delta_1(a,b)$=13, $\delta_1(a,c)$=16, $\delta_1(a,d)$=8, $\delta_1(b,c)$=3, $\delta_1(b,d)$=5, $\delta_1(c,d)$=8. Let G_2 be the fuzzy graph on the set V_2={u,v,w,x} such that σ_2(u)=1, σ_2(v)= 1, σ_2(w)=1, σ_2(x)=1.with μ_2(u,v)=1/13, μ_2(u,x)=1/8, μ_2(v,x)=1/5, μ_2(w,x)=1/8, μ_2(v,w)=1/3,Defining ϕ: $V_1 \to V_2$ as ϕ(a)=u, ϕ(b)=v, ϕ(c)=w, ϕ(d)=x it is one-one ,onto that preserves the distance between every pair of vertices in G_1 and G_2. Hence G_2 is isometric from G_1.

3.3. Theorem

Isometry is an equivalence relation.

Proof:

Let G_i : (σ_i, μ_i) be the fuzzy graphs with underlying sets V_i, for i=1,2,3.

Case 1: To prove isometry is reflexive.

Considering the identity map i: $V_1 \to V_1$,G_1 is isometric to G_1; $\Rightarrow$ isometry is a reflexive relation.

Case 2: To prove isometry is symmetric.

Assume that G_1 is isometric to G_2; To prove G_2 is isometric to G_1;

G_1 is isometric to $G_2 \Rightarrow G_2$ is isometric from G_1, and G_1 is isometric from G_2. By rearranging, G_2 is isometric to G_1

Case 3: To prove isometry is transitive.

Let G_1 be isometric to G_2 and G_2 be isometric to G_3 i.e G_2 is isometric from G_1 and G_3 is isometric from G_2 and vice-versa. So, for each $v \in V_1$, there exists a bijective map f_v: $V_1 \to V_2$ such that δ_1(v,u) = δ_2(f_v(v), f_v(u)) $\forall u \in V_1$ ……………………………..........................(1)

Let $f_v(v)$ = v' ………………….............................(2)

Similarly for each v' $\in V_2$ there exists a bijective map $g_{v'}$: $V_2 \to V_3$ such that

δ_2(v',u') = δ_3($g_{v'}$(v'), $g_{v'}$(u')) $\forall u' \in V_2$ …………………………………………..(3)

From (1) (2),&(3) δ_1(v,u) = δ_2(f_v(v), f_v(u)) $\forall$ $u \in V_1$

= δ_2(v',u')

= δ_3($g_{v'}$(v'), $g_{v'}$(u'))

= δ_3($g_{v'}$(f_v(v)), $g_{v'}$(f_v(u))) $\forall$ $u \in V_1$

Hence G_3 is isometric from G_1 using the composite map $g_{v'} \circ f_v$: $V_1 \to V_2$ as $g_{v'} \circ f_v(v)$= $g_{v'}(f_v(v))$

Similarly it can be proved that G_1 is isometric from G_3. Hence G_1 is isometric to G_3. So isometry is an equivalence relation.

3.4. Theorem

G_1 is isomorphic to G_2 implies G_1 is isometric to G_2.

Proof:

As G_1 is isomorphic to G_2 there is a bijection h: $V_1 \rightarrow V_2$ such that

$\sigma_1(x) = \sigma_2(h(x))$ for all $x \in V_1$

$\mu_1(x,y) = \mu_2(h(x), h(y))$ for all $x, y \in V_1$

For each $u \in V_1$ consider $\delta_1(u,v) = \wedge\left\{\sum_{i=1}^{n} \frac{1}{\mu_1(u_{i-1}, u_i)}\right\}$ where $u_0 = u$, $u_n = v$;

$$= \wedge\left\{\sum_{i=1}^{n} \frac{1}{\mu_2(h(u_{i-1}), h(u_i))}\right\}$$

$$= \delta_2(f(u), f(v)) \quad \text{for all } v \in V_1$$

So, G_2 is isometric from G_1 and similarly it may be proved that G_1 is isometric from G_2 also.

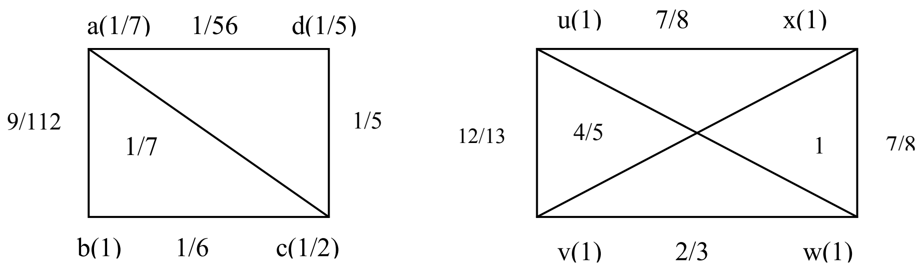
3.5. Note

(i)The above result is true even when G_1 is co-weak isomorphic to G_2 also.

(ii)It is proved in theorem 4.2 of [6], that G_1 is isomorphic to G_2 implies $\overline{G_1}$ is is isomorphic to $\overline{G_2}$.

But this is not so in the case of isometry.

3.6. Example For the isometric fuzzy graphs G_1 and G_2 in example 3.2,



$\overline{G_1}$ Fig1 $\overline{G_2}$

In $\overline{G_1}$, $\delta_1(a,b) = 112/9 = 12.44$, $\delta_1(a,c) = 7$, $\delta_1(a,d) = 12$, $\delta_1(b,c) = 6$, $\delta_1(b,d) = 11$, $\delta_1(c,d) = 5$.

In $\overline{G_2}$, $\delta_2(u,v) = 1.08$, $\delta_2(u,w) = 1$, $\delta_2(u,x) = 1.14$, $\delta_2(v,w) = 1.5$, $\delta_2(v,x) = 1.25$, $\delta_2(x,w) = 1.14$,

So there does not exist a bijection between $\overline{G_1}$ and $\overline{G_2}$ preserving the μ - distance.

4. Antipodal Fuzzy Graphs

4.1. Definition

Let G: (σ, μ) be a fuzzy graph with the underlying set V. A(G) : $(\sigma_{A(G)}, \mu_{A(G)})$ is defined as follows:

The node set of G is taken as the node set of A(G) also. Two nodes in A(G) are made as neighbors if the μ - distance between them is diam(G).i.e.,

$\sigma_{A(G)}(u) = \sigma(u)$ for all $u \in V$ and if $\delta(u,v)$ = diam(G) then

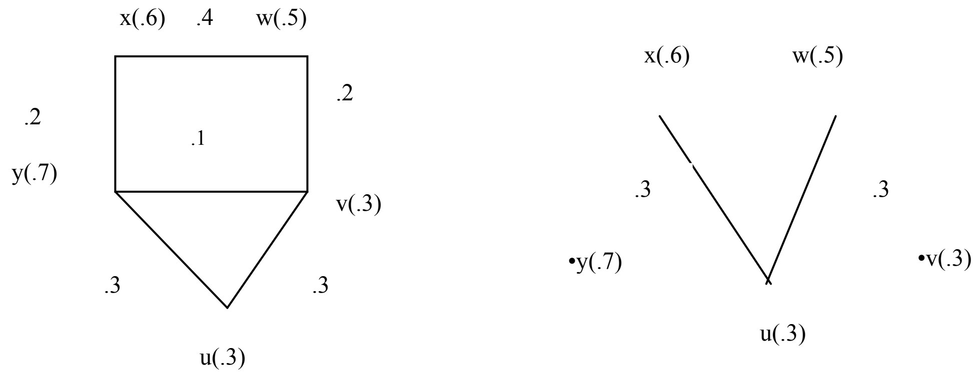
$\mu_{A(G)}(u,v) = \mu(u,v)$ if u and v are neighbors in G

$= \sigma(u) \wedge \sigma(v)$ if u and v are not neighbors in G

$\mu_{A(G)}(u,v) = 0$ otherwise.

This pair A(G) is a fuzzy graph, because $\sigma_{A(G)}(u)=\sigma(u)$ for all $u \in V$ implies $\sigma_{A(G)}$ is a fuzzy subset on V and by the definition of $\mu_{A(G)}$, it is a fuzzy relation on $\sigma_{A(G)}$, such that, $\mu_{A(G)}(u,v) \leq \sigma(u) \wedge \sigma(v) = \sigma_{A(G)}(u) \wedge \sigma_{A(G)}(v)$ for all u,v in V . This fuzzy graph A(G) is termed as Antipodal fuzzy graph of G.

4.2. Example



G: (σ, μ) Fig2 A(G) : $(\sigma_{A(G)}, \mu_{A(G)})$

In G diam(G)= 8.3 $= \delta(u,x) = \delta(u,w)$; As (u,x) and $(u,w) \notin \mu^*$,

$\mu_{A(G)}(u,x) = \sigma(u) \wedge \sigma(x) = .3$ and similarly $\mu_{A(G)}(u,w) = 0.3$

4.3. Theorem

Let G: (σ, μ) be a μ- distance regular fuzzy graph Then G is a spanning fuzzy subgraph of A(G).

Proof:

Given G: (σ, μ) be a μ- distance regular fuzzy graph. i.e $\delta(u,v) = k$ for all u,v in V(G)Hence the eccentricity e(u)= k for all u in V $\Rightarrow$ diam(G) = k. Hence every pair of vertices are made as neighbours in A(G) such that,

$\mu_{A(G)}(u,v) = \mu(u,v)$ if u and v are neighbours in G

$= \sigma(u) \wedge \sigma(v)$ if u and v are not neighbours in G. Hence every edge in G is also an edge in A(G) .So G is a spanning fuzzy sub graph of A(G).

Moreover, the underlying graph of A(G) is also a complete graph in this case.

4.4. Theorem [10]

A complete fuzzy graph is self-centered and r(G)= $\dfrac{1}{\sigma(u)}$ where $\sigma(u)$ is least.

4.5. Theorem

Let G: (σ, μ) be a complete fuzzy graph. Then A(G) is a spanning fuzzy subgraph of G, such that weight of each edge in A(G) is $\wedge\{\ \sigma(v)/\ v \in V\}$

Proof:
By theorem 4.4, G: (σ, μ) being complete is self centered.
Self-centered$\Rightarrow$Each node is a central node.$\Rightarrow$ Each node is having the minimum eccentricity. $\Rightarrow \wedge\{e(v)/\ v \in V\} = \vee\{e(v)/\ v \in V\}$

$\Rightarrow$ r(G) = diam (G)

$\Rightarrow \frac{1}{\sigma(u)}$ = diam(G),where σ(u) is least…..……………………………...(4)

G being a complete fuzzy graph, $\mu(x\ ,y) = \sigma(x) \wedge \sigma(y)$ for all x , y in σ^*. As each node is a central node, the node u, with σ(u) least is also a central node.
Case 1: Let (x,y) in μ^* be such that either x=u or y=u. Then $\mu(x\ ,y) = \sigma(u)$.

$\delta(\ x,y)=\frac{1}{\sigma(u)}$ since every path other than (x,y) will have its μ- length greater than $\frac{1}{\sigma(u)}$ .So $\delta(\ x,y)$= diam(G) if (x,y) is in μ^* such that either x=u or y=u. Hence in this case x & y are made as neighbours such that $\mu_{A(G)}(x,\ y) = \sigma(u)$.

Case 2: Let (x,y) in μ^* be such that neither x nor y have their weights to be σ(u). Assume without loss of generality that $\sigma(x) \le \sigma(y)$.

Hence $\mu(x\ ,y) = \sigma(x) > \sigma(u) \Rightarrow \delta(\ x,y) = \frac{1}{\sigma(x)} < \frac{1}{\sigma(u)}$ = diam (G) .So nodes x ,y in which neither x nor y have their weights to be σ(u) are not neighbors in A(G).
Hence by case 1 &2 a node with minimum weight is made as a neighbor of every other node in A(G) with $\mu_{A(G)}(x\ ,y) = \wedge\{\ \sigma(v)/\ v \in V\}$ $\therefore$A(G) is a spanning fuzzy subgraph of G, such that weight of each edge in A(G) is $\wedge\{\ \sigma(v)/\ v \in V\}$.

4.5. Theorem
Let G: (σ, μ) be a connected strong fuzzy graph. Then A(G) is the edge induced fuzzy subgraph of $\overline{G}$, induced by the edges of $\overline{G}$, whose end vertices are with maximum eccentricity in G .

Proof:
Given G is a strong fuzzy graph. If G has n-nodes arrange the vertices of G in such a way that $\sigma(u_1) \le \sigma(u_2) \le \sigma(u_3) \le \ldots\ldots \le \sigma(u_n)$. Let u_i and u_j be in σ^* such that $\sigma(u_i) \le \sigma(u_j)$, and if $e_i = (u_i, u_j) \in \mu^*$, then $\mu(e_i) = \sigma(u_i)$

Hence $\delta(\ u_i, u_j) = \frac{1}{\mu(e_i)} = \frac{1}{\sigma(u_i)} \le \frac{1}{\sigma(u_1)}$ ……………………………………………(5)

Claim 1 : Neighbours in G are not neighbours in A(G). Consider an arbitrary path connecting u_k, u_t such that $(u_k, u_t) \notin \mu^*$. If ρ is a path of length at least 2 between u_k, u_t then μ- length of $\rho \ge \frac{2}{\sigma(u_1)}$ ……………………………………………………..(6)

Hence $\delta(u_k, u_t) \geq \frac{2}{\sigma(u_1)} > \frac{1}{\sigma(u_1)} \geq \delta(u_i, u_j)$ (by using (5) & (6).)

$\Rightarrow$ $\delta(u_i, u_j) < \delta(u_k, u_t) \leq$ diam (G) where $(u_k, u_t) \notin \mu^*$ and $(u_i, u_j) \in \mu^*$

$\Rightarrow$ $\delta(u_i, u_j) <$ diam (G) if $(u_i, u_j) \in \mu^*$

$\Rightarrow$ If $(u_i, u_j) \in \mu^*$ then u_i and u_j are not neighbors in A(G).

Claim 2 : Edges in A(G) are edges in $\overline{G}$.

If $(u_m, u_n) \in \mu^*_{A(G)}$ then by claim 1, $(u_m, u_n) \notin \mu^*$.

So, $\mu_{A(G)}(u_m, u_n) = \sigma(u_m) \wedge \sigma(u_n)$ by the definition of A(G)

$\Rightarrow$ Edges in A(G) are edges in $\overline{G}$ .Hence A(G) is a fuzzy subgraph of $\overline{G}$, induced by the edges of $\overline{G}$, whose end vertices are with maximum eccentricity in G .

4.6. Theorem

Let G: (σ, μ) be a connected fuzzy graph such that $\mu(x, y) = c \ \forall (x,y) \in \mu^*$. Then A(G) is the spanning fuzzy subgraph of $\overline{G}$, induced by the edges of $\overline{G}$, whose end vertices are with maximum eccentricity in G .

Proof:

For any $(x,y) \in \mu^*$, $\delta(x,y) = \frac{1}{c}$ But for any $(u,v) \notin \mu^*$, $\delta(u,v) \geq \frac{2}{c}$

$\delta(x,y) = \frac{1}{c} < \frac{2}{c} \leq \delta(u,v)$, where $(x,y) \in \mu^*$ & $(u,v) \notin \mu^*$

$\Rightarrow$ $\delta(x,y) <$ diam (G) if $(x,y) \in \mu^*$

Hence x, y are nodes in A(G) , but are not neighbours in A(G).The remaining proof is similar to claim 2 of Theorem 4.5.

4.7 Remark:

From theorem 4.5 &4.6 whether G is a strong fuzzy graph or a constant fuzzy graph A(G) is a strong fuzzy graph.

4.8. Theorem

If G_1 and G_2 are isomorphic to each other then $A(G_1)$ and $A(G_2)$ are also isomorphic.

Proof:

As G_1 and G_2 are isomorphic, the isomorphism h, between them preserves the edge weights , so the μ- length and μ- distance will also be preserved Hence if the vertex 'v' has the maximum eccentricity, in G_1, then h(v) has the maximum eccentricity, in G_2. So, G_1 & G_2 will have the same diameter (say k).

If the μ- distance between u,v is 'k' in G_1 then h(u), and h(v) will also have their μ- distance as 'k'. The same mapping h itself is a bijection between $A(G_1)$ and $A(G_2)$ satisfying the isomorphism condition.

(i) $\sigma_{A(G_1)}(u) = \sigma_{G_1}(u) = \sigma_{G_2}(h(u)) = \sigma_{A(G_2)}(h(u))$ for all u in G_1

(ii) $\mu_{A(G_1)}(u, v) = \mu_{G_1}(u, v)$ if u and v are neighbours in G_1 ....(7)

$=\sigma_{G_1}(u) \wedge \sigma_{G_1}(v)$ if u and v are not neighbours in G_1…….…………(8)

As h: $G_1 \to G_2$ is an isomorphism,

$\mu_{A(G_1)}(u,v) = \mu_{G_2}(h(u),h(v))$ if u and v are neighbours in G_1 (by (7)) ……………..(9)

$=\sigma_{G_2} h(u) \wedge \sigma_{G_2} h(v)$ if u and v are not neighbours in G_1 ……………..(10)

Hence $\mu_{A(G_1)}(u,v) = \mu_{A(G_2)}(h(u),h(v))$

So, the same h is an isomorphism between $A(G_1)$ and $A(G_2)$

4.8. Theorem

If G_1 and G_2 are complete fuzzy graphs such that G_1 is co-weak isomorphic to G_2 then $A(G_1)$ is co-weak isomorphic to $A(G_2)$.

Proof:

As G_1 is co-weak isomorphic G_2, there exists a bijection $h:G_1 \to G_2$ satisfying, $\sigma_1(v_i) \le \sigma_2(h(v_i))$, $\mu_1(v_i,v_j) = \mu_2(h(v_i),h(v_j))$ for v_i, v_j in V_1 If G_1 has n-nodes arrange the vertices of G_1 in such a way that $\sigma_1(v_1) \le \sigma_1(v_2) \le \sigma_1(v_3) ..... \le \sigma_1(v_n)$ As G_1 and G_2 are complete ,co-weak isomorphic fuzzy graphs, $\sigma_1(v_i) = \sigma_2(h(v_i))$, for i= 1,2…n-1and $\sigma_1(v_n) \le \sigma_2(h(v_n))$, with the condition that, $\mu_1(v_i,v_j) = \mu_2(h(v_i),h(v_j))$ for v_i, v_j in V_1.By theorem 4.4,$A(G_i)$ is a spanning fuzzy graph of G_i, i=1,2 with weights of each edge in $A(G_i)$ as $\wedge \{\sigma_i(v) / v \in V_i\}$ i=1,2

So the same bijection h is a co-weak isomorphism between $A(G_1)$ and $A(G_2)$

4.9. Theorem

If G_1 and G_2 are connected fuzzy graphs such that G_1 is co-weak isomorphic to G_2 then $A(G_1)$ is homomorphic to $A(G_2)$.

Proof:

As G_1 is co-weak isomorphic G_2, there exists a bijection $h:G_1 \to G_2$ satisfying, $\sigma_1(v_i) \le \sigma_2(h(v_i))$, $\mu_1(v_i,v_j) = \mu_2(h(v_i),h(v_j))$ for v_i,v_j in V_1. So the μ- distance and hence, the diameter will be preserved. Let diam(G_1)= diam(G_2)=k.

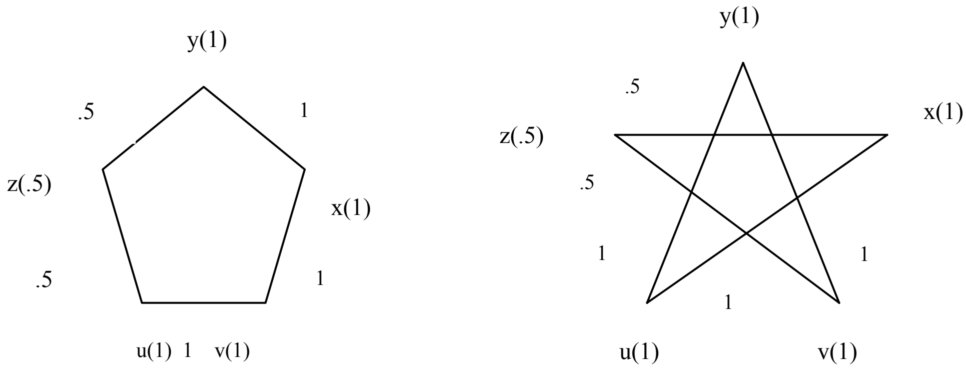
If $u,v \in V_1$ are at a distance k in G_1 then they are made as neighbours in $A(G_1)$. So, h(u), h(v) in G_2 are also at a distance k in G_2 and h(u), h(v) are made as neighbours in $A(G_2)$. If u and v are neighbours in G_1 then $\mu_{A(G_1)}(u,v) = \mu_1(u,v) = \mu_2(h(u),h(v)) = \mu_{A(G_2)}(h(u),h(v))$. If u and v are not neighbours in G_1 then $\mu_{A(G_1)}(u,v) = \sigma_1(u) \wedge \sigma_1(v) \le \sigma_2(h(u)) \wedge \sigma_2(h(v)) = \mu_{A(G_2)}(h(u),h(v))$

Hence $A(G_1)$ is homomorphic to $A(G_2)$.

4.10. Remark

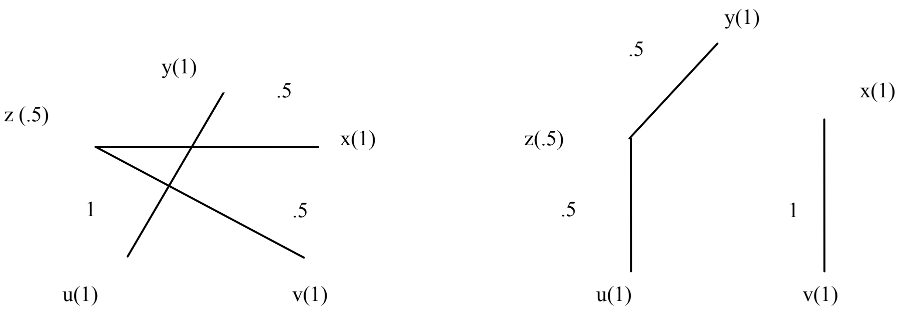
If G is a self complementary fuzzy graph then its antipodal fuzzy graph need not be self complementary. i.e., $G \cong \overline{G} \nRightarrow A(G) \cong \overline{A(G)}$

4.11Example



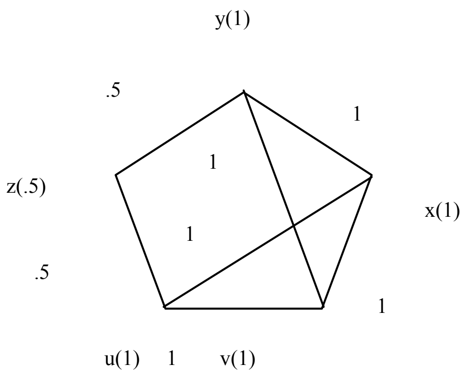
G: $(\sigma,\ \mu)$ Fig 3 a $\overline{G}\ :(\sigma,\overline{\mu})$

Here G $\cong \overline{G}$



A(G) Fig 3b A($\overline{G}$)

Here A(G) $\cong$ A($\overline{G}$)



$\overline{A(G)}$ Fig3c

Fig 3b and 3c show A(G) $\ncong$ $\overline{A(G)}$, though G $\cong \overline{G}$.

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