Applied Mathematical Sciences, Vol. 4, 2010, no. 44, 2157 - 2170

Propagation of SH Waves in an Irregular Non Homogeneous Monoclinic Crustal Layer over a Semi-Infinite Monoclinic Medium

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Abstract

The present paper discuss the dispersion equation for SH waves in a non homogeneous monoclinic layer over a semi infinite isotropic monoclinic medium with an irregularity. The dispersion equation has been obtained. In the isotropic case, when non homogeneity and irregularity are absent, the dispersion equation reduces to standard SH wave equation. The dispersion curves are depicted by means of graphs for different size of irregularity and different values of non-homogeneity parameters. The influence of depth of irregularity and non-homogeneity parameters on phase velocity has been studied.

Keywords: SH waves, Monoclinic, Irregular boundaries, Non homogeneity, Fourier transform

Introduction

Many results of theoretical and experimental studies revealed that a real earth is considerably more complicated than the models presented earlier. This has led to a need for more realistic representation as a medium through which seismic waves propagate. The study of wave propagation in elastic medium with different boundaries is of great importance to seismologists as well as to geophysicists to understand and predict the seismic behaviour at different margins of earth. The propagation of Love waves has been studied by many authors with assuming different forms of irregularities at the interface. Bhattacharya [7] discussed the dispersion curves for Love wave propagation in a transversely isotropic crustal layer with an irregularity in the interface. Chattopadhyay et al [3] studied the propagation of SH guided wave in an internal stratum with parabolic irregularity in the lower interface. The wave propagation in crystalline media plays a very interesting role in geophysics as also in ultrasonic and signal processing. Keeping in the mind the fact that the non-homogeneity characteristic is one of the most generalised elastic conditions inside the earth, many authors have studied the propagation of different waves in different media with non-homogeneity. Chattopadhyay [1] studied the Love wave propagation due to irregularity in the thickness of non-homogeneous crustal layer. Propagation of Love waves in a non-homogeneous stratum

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of finite depth sandwiched between two semi infinite isotropic media has been studied earlier by Sinha [11].Recently, the propagation of SH waves in an irregular monoclinic crustal layer has been studied by Chattopadhyay et al [4]. In this paper we have discussed the propagation of SH waves in a non-homogeneous monoclinic layer overlying a monoclinic half space. The irregularity is in the form of rectangle. The perturbation technique indicated by Erigen and Samuels [5] has been used. The dispersion curves are depicted by means of graphs for different size of irregularity and different values of non-homogeneity parameters. The influence of depth of irregularity and non-homogeneity parameters on phase velocity has been studied.

Formulation of the problem

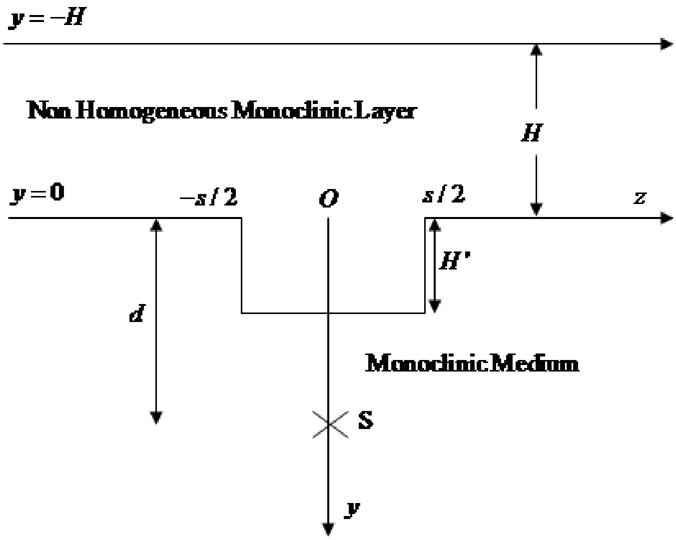

Fig.1: Geometry of the problem.

We have taken z-axis along the interface between the lower semi infinite medium and the non-homogeneous monoclinic layer, y-axis is taken downwards as shown in fig.1.The form of irregularity is assumed rectangular with length s and depth H'. Layer is taken of H thickness and origin is placed at the middle point of the lower interface irregularity. The distance of source of disturbance from origin is d with $d > H'$.

The interface between layer and half space is defined as

$$y = \varepsilon h(z) \tag{1}$$

where $h(z) = \begin{cases} 0 & \text{for } z \le \frac{-s}{2},\ z \ge \frac{s}{2} \\ f(z) & \text{for } \frac{-s}{2} \le z \le \frac{s}{2} \end{cases}$

where $\varepsilon = \dfrac{H'}{s}$ and $\varepsilon << 1$.

We have the following stain-displacement relations for monoclinic layer

$$S_1 = \frac{\partial u}{\partial x},\ S_2 = \frac{\partial v}{\partial y},\ S_3 = \frac{\partial w}{\partial z},\ S_4 = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z},\ S_5 = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x},\ S_6 = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \tag{2}$$

where u, v, w are displacement components in the direction x, y, z respectively, and

$S_i\ (i = 1, 2, ..., 6)$ are the strain components.

Also, the stress-strain relation for a rotated y-cut quartz plate, which exhibits monoclinic symmetry with x being the diagonal axis are

$$\begin{aligned}
T_1 &= C_{11}S_1 + C_{12}S_2 + C_{13}S_3 + C_{14}S_4, \\
T_2 &= C_{12}S_1 + C_{22}S_2 + C_{23}S_3 + C_{24}S_4, \\
T_3 &= C_{13}S_1 + C_{23}S_2 + C_{33}S_3 + C_{34}S_4, \\
T_4 &= C_{14}S_1 + C_{24}S_2 + C_{34}S_3 + C_{44}S_4, \\
T_5 &= C_{55}S_5 + C_{56}S_6, \\
T_6 &= C_{56}S_5 + C_{66}S_6
\end{aligned} \tag{3}$$

where $T_i\left(i = 1, 2, ..., 6\right)$ are the stress components and $C_{ij} = C_{ji}\left(i = 1, 2, ..., 6\right)$ are the elastic constants.

The equation of motion in the absence of body forces are

$$\begin{aligned}
\frac{\partial T_1}{\partial x} + \frac{\partial T_6}{\partial y} + \frac{\partial T_5}{\partial z} &= \rho_1 \frac{\partial^2 u}{\partial t^2} \\
\frac{\partial T_6}{\partial x} + \frac{\partial T_2}{\partial y} + \frac{\partial T_4}{\partial z} &= \rho_1 \frac{\partial^2 v}{\partial t^2} \\
\frac{\partial T_5}{\partial x} + \frac{\partial T_4}{\partial y} + \frac{\partial T_3}{\partial z} &= \rho_1 \frac{\partial^2 w}{\partial t^2}
\end{aligned} \tag{4}$$

where ρ_1 is the density of the layer.

For the propagation of SH type wave in z-direction with the displacement in the x-direction, we have

$$T_1 = T_2 = T_3 = T_4 = 0,\ T_5 = C_{55}\frac{\partial u}{\partial z} + C_{56}\frac{\partial u}{\partial y} \text{ and } T_6 = C_{56}\frac{\partial u}{\partial z} + C_{66}\frac{\partial u}{\partial y}. \tag{5}$$

Let we take $\rho_r, u_r\,(r=1,2)$ as densities and displacements of the layer and lower media, respectively. The non homogeneity in the layer is considered as

$$C_{66}^{(1)} = C_{66}' e^{\nu z},\ C_{56}^{(1)} = C_{56}' e^{\nu z}, C_{55}^{(1)} = C_{55}' e^{\nu z}, \rho_1 = \rho_1' e^{\nu z}$$

and the elastic constants of the half-space as $C_{66}^{(2)} = C_{66}''$, $C_{56}^{(2)} = C_{56}''$, $C_{55}^{(2)} = C_{55}''$.

Using (5) and (4) we get equation of motion for the monoclinic layer in the form

$$C_{66}'\frac{\partial^2 u_1}{\partial y^2} + 2C_{56}'\frac{\partial^2 u_1}{\partial y \partial z} + C_{55}'\frac{\partial^2 u_1}{\partial z^2} + \nu C_{56}'\frac{\partial u_1}{\partial z} + \nu C_{66}'\frac{\partial u_1}{\partial y} = \rho_1'\frac{\partial^2 u_1}{\partial t^2} \tag{6}$$

and equation of motion for lower isotropic monoclinic semi-infinite medium is

$$C_{66}''\frac{\partial^2 u_2}{\partial y^2} + 2C_{56}''\frac{\partial^2 u_2}{\partial y \partial z} + C_{55}''\frac{\partial^2 u_2}{\partial z^2} = \rho_2\frac{\partial^2 u_2}{\partial t^2}\ . \tag{7}$$

Here in equation (6) and (7) suffix “1” and “2” are used for monoclinic layer and lower semi-infinite medium respectively.

The boundary conditions are as following

(i) The upper surface of monoclinic layer is stress free, i.e. $T_6 = 0$ at $y = -H$

$$C_{56}'\frac{\partial u_1}{\partial z} + C_{66}'\frac{\partial u_1}{\partial y} = 0 \qquad \text{at } y = -H \tag{8}$$

(ii) The stresses are continuous at the interface $y = \varepsilon h(z)$

$$\frac{\partial u_1}{\partial y}\left[C_{66}' - C_{56}'\varepsilon h'\right]e^{\nu\varepsilon h} + \frac{\partial u_1}{\partial z}\left[C_{56}' - C_{55}'\varepsilon h'\right]e^{\nu\varepsilon h} = \left[C_{66}'' - C_{56}''\varepsilon h'\right]\frac{\partial u_2}{\partial y} + \left[C_{56}'' - C_{55}''\varepsilon h'\right]\frac{\partial u_2}{\partial z}, \tag{9}$$

where $h' = \dfrac{dh(z)}{dz}$

(iii) The displacements are continuous at the interface $y = \varepsilon h(z)$

$$u_1 = u_2 \tag{10}$$

Solution of the problem

For $u_i(y,z,t) = U_i(y,z)e^{i\omega t}\ (i = 1,2...)$, equation (6) and (7) reduces to

$$C_{66}{}'\frac{\partial^2 U_1}{\partial y^2}+2C_{56}{}'\frac{\partial^2 U_1}{\partial y\partial z}+C_{55}{}'\frac{\partial^2 U_1}{\partial z^2}+\nu C_{56}{}'\frac{\partial U_1}{\partial z}+\nu C_{66}{}'\frac{\partial U_1}{\partial y}+\rho_1{}'\omega^2 U_1=0 \tag{11}$$

$$C_{66}{}''\frac{\partial^2 U_2}{\partial y^2}+2C_{56}{}''\frac{\partial^2 U_2}{\partial y\partial z}+C_{55}{}''\frac{\partial^2 U_2}{\partial z^2}+\rho_2\omega^2 U_2=0 \tag{12}$$

where ω is circular frequency.

Defining the Fourier transform $\bar{U}_r(y,\eta)$ of $U_r(y,z)$ $(r=1,2)$ as

$$\bar{U}_r(y,\eta)=\int_{-\infty}^{\infty}U_r(y,z)e^{i\eta z}dz,$$

We find the inverse Fourier transform as

$$U_r(y,z)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\bar{U}_r(y,\eta)e^{-i\eta z}d\eta.$$

So the Fourier transform of Eqs. (11) and (12) are

$$\frac{d^2\bar{U}_1}{dy^2}+\left(\nu+a_1\right)\frac{d\bar{U}_1}{dy}+\left(p_1{}^2+\frac{a_1\nu}{2}\right)\bar{U}_1=0 \tag{13}$$

$$\frac{d^2\bar{U}_2}{dy^2}+a_2\frac{d\bar{U}_2}{dy}-(p_2{}^2)\bar{U}_2=0 \tag{14}$$

where

$$\beta_1{}^2=\frac{C_{66}{}'}{\rho_1{}'},\ p_1{}^2=\frac{\omega^2}{\beta_1^2}-\frac{C_{55}{}'}{C_{66}{}'}\eta^2,\ a_1=-2i\eta\frac{C_{56}{}'}{C_{66}{}'},\ a_2=-2i\eta\frac{C_{56}{}''}{C_{66}{}''},\ p_2{}^2=-\frac{\omega^2}{\beta_2{}^2}+\frac{C_{55}{}''}{C_{66}{}''}\eta^2,\ \beta_2{}^2=\frac{C_{66}{}''}{\rho_2}.$$

Now the solutions of Eqs. (13) and (14) are

$$\bar{U}_1=e^{-\left(\frac{\nu+a_1}{2}\right)y}\left(A\cos p_3y+B\sin p_3y\right) \tag{15}$$

$$\bar{U}_2=De^{-\left(\frac{a_2}{2}\right)y}e^{-p_4y} \tag{16}$$

where $p_3{}^2=p_1{}^2-\left(\frac{\nu^2+a_1{}^2}{4}\right)=\left\{\frac{\omega^2}{\beta_1^2}-\frac{C_{55}{}'}{C_{66}{}'}\eta^2+\left(\frac{C_{56}{}'}{C_{66}{}'}\right)^2\eta^2-\frac{\nu^2}{4}\right\},$

$$p_4{}^2=p_2{}^2+\frac{a_2{}^2}{4}=\left\{\frac{C_{55}{}''}{C_{66}{}''}\eta^2-\frac{\omega^2}{\beta_2{}^2}-\left(\frac{C_{56}{}''}{C_{66}{}''}\right)^2\eta^2\right\}$$

and A, B, D are functions of η.

The displacements in the two media are

$$U_1 = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-\left(\frac{\nu+a_1}{2}\right)y}\left(A\cos p_3 y + B\sin p_3 y\right)e^{-i\eta z}d\eta \tag{17}$$

$$U_2 = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left\{De^{-\left(\frac{a_2}{2}\right)y}e^{-p_4 y} + \frac{2}{p_4+\frac{a_2}{2}}e^{\left(p_4+\frac{a_2}{2}\right)y}e^{-\left(p_4+\frac{a_2}{2}\right)d}\right\}e^{-i\eta z}d\eta \tag{18}$$

where the second term in the integrand of U_2 is introduced due to the effect of source at S in M_2 [10]. We set the following approximations due to small value of ε

$$A \cong A_0 + A_1\varepsilon,\ B \cong B_0 + B_1\varepsilon,\ D \cong D_0 + D_1\varepsilon. \tag{19}$$

Since the boundary is not uniform at the interface of M_1 and M_2 so the term A, B and D appearing in Eq.(19) are also function of ε, expanding these terms in ascending powers of ε and keeping in view that ε is small and so retaining the term up to the first order of ε, hence approximated as in Eq.(19). These assumptions are justified in the real earth model where the depth H' of the irregular boundary is too small with respect to the length of the boundary s. Also for small ε, following approximations can be accepted

$$e^{\pm\gamma\varepsilon h} \cong 1 \pm \gamma\varepsilon h,\ \cos p_3\varepsilon h \cong 1,\ \sin p_3\varepsilon h \cong p_3\varepsilon h.$$

where γ is any quantity.

Using the boundary conditions (i), (ii), (iii) and after simplification, we obtain

$$A_0 = \frac{-8C_{66}''e^{-\left(p_4+\frac{a_2}{2}\right)d}\left\{2ikC_{56}'\tan p_3H + 2p_3C_{66}' + (\nu+a_1)C_{66}'\tan p_3H\right\}}{G(k)},$$

$$B_0 = \frac{-8C_{66}''e^{-\left(p_4+\frac{a_2}{2}\right)d}\left\{2ikC_{56}' - 2p_3C_{66}'\tan p_3H + (\nu+a_1)C_{66}'\right\}}{G(k)},$$

$$D_0 = \frac{-4e^{-\left(p_4+\frac{a_2}{2}\right)d}}{(2p_4+a_2)G(k)}\left\{E_1+E_2+E_3+E_4\right\},$$

$$A_1 = \frac{-\left\{R_1\left(2p_4C_{66}'' + a_2C_{66}'' + 2ikC_{56}''\right)+2R_2\right\}\left\{2ikC_{56}'\tan p_3H + 2p_3C_{66}' + (\nu+a_1)C_{66}'\tan p_3H\right\}}{G(k)},$$

$$B_1 = \frac{-\left\{R_1\left(2p_4C_{66}'' + a_2C_{66}'' + 2ikC_{56}''\right)+2R_2\right\}\left\{2ikC_{56}' - 2p_3C_{66}'\tan p_3H + (\nu+a_1)C_{66}'\right\}}{G(k)},$$

$$D_1 = \frac{E_5+E_6}{G(k)},$$

$$E_1 = 4ikp_4C_{66}''C_{56}'\tan p_3H + 2ika_2C_{66}''C_{56}'\tan p_3H + 4p_3p_4C_{66}''C_{66}' + 2p_3a_2C_{66}''C_{66}',$$

$$E_2 = 2p_4(\nu+a_1)C_{66}''C_{66}'\tan p_3H + a_2(\nu+a_1)C_{66}''C_{66}'\tan p_3H - 4ikp_3C_{66}'C_{56}' + 4p_3^2C_{66}'^2\tan p_3H,$$

$$E_3 = 4ik(\nu+a_1)C_{66}'C_{56}'\tan p_3H + (\nu+a_1)^2C_{66}'^2\tan p_3H - 4k^2C_{56}'^2\tan p_3H + 4ik(\nu+a_1)C_{66}'C_{56}'\tan p_3H,$$

$$E_4 = 4k^2C_{56}''C_{56}'\tan p_3H - 4ikp_3C_{56}''C_{66}' - 2ik(\nu+a_1)C_{56}''C_{66}'\tan p_3H,$$

$$E_5 = R_2\left(-4ikC_{56}'\tan p_3H - 4p_3C_{66}' - 2(\nu+a_1)C_{66}'\tan p_3H\right),$$

$$E_6 = R_1\left(4k^2C_{56}'^2\tan p_3H - 4p_3^2C_{66}'^2\tan p_3H - 4ik(\nu+a_1)C_{66}'C_{56}'\tan p_3H - (\nu+a_1)^2C_{66}'^2\tan p_3H\right),$$

$$\begin{aligned} G(k) = &-4k^2C_{56}'^2\tan p_3H - 4ikp_4C_{56}'C_{66}''\tan p_3H + 4k^2C_{56}''C_{56}'\tan p_3H \\ &4p_3^2C_{66}'^2\tan p_3H + 4ik(\nu+a_1)C_{66}'C_{56}'\tan p_3H + (\nu+a_1)^2C_{66}'^2\tan p_3H \\ &-4p_3p_4C_{66}''C_{66}' - 4ikp_3C_{56}''C_{66}' - 2ik(\nu+a_1)C_{56}''C_{66}'\tan p_3H - 2p_3a_2C_{66}''C_{66}' \\ &-2p_4(\nu+a_1)C_{66}''C_{66}'\tan p_3H - a_2(\nu+a_1)C_{66}''C_{66}'\tan p_3H - 2ika_2C_{56}'C_{66}''\tan p_3H. \end{aligned}$$

With the help of above obtained values the displacement in the monoclinic layer is given by

$$\begin{aligned} U_1 = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{4C_{66}''e^{-\left(p_4+\frac{a_2}{2}\right)d}e^{-\frac{a_1}{2}y}}{G(k)}&\left[1+\frac{\varepsilon\left\{R_2+R_1\left(C_{66}''\left(p_4+\dfrac{a_2}{2}\right)+ikC_{56}''\right)\right\}}{4C_{66}''e^{-\left(p_4+\frac{a_2}{2}\right)d}}\right] \\ &\times\left\{\left(2ikC_{56}'\tan p_3H + 2p_3C_{66}' + (\nu+a_1)C_{66}'\tan p_3H\right)\cos p_3y\right. \\ &\left.+\left(2ikC_{56}' - 2p_3C_{66}'\tan p_3H + (\nu+a_1)C_{66}'\right)\sin p_3y\right\}e^{-ikz}dk. \end{aligned} \tag{20}$$

From Eq. (1), the interface for the rectangular irregularity gives

$$\bar{h}(\lambda) = \frac{2s}{\lambda}\sin\left(\frac{\lambda s}{2}\right). \tag{21}$$

Further on simplification, we get

$$R_2 + R_1\left\{C_{66}''\left(p_4+\frac{a_2}{2}\right)+ikC_{56}''\right\}=\frac{2sC_{66}''}{\pi}\int_{-\infty}^{\infty}\{\psi(k-\lambda)+\psi(k+\lambda)\}\frac{1}{\lambda}\sin\left(\frac{\lambda s}{2}\right)d\lambda, \tag{22}$$

where
$$\psi(k-\lambda)=\left[\left(B_2+B_3+B_4+B_5+B_6+B_7+B_8+B_9+B_{10}+B_{11}\right)\frac{e^{-\left(p_4+\frac{a_2}{2}\right)d}}{\left(p_4+\frac{a_2}{2}\right)G(k)}\right]^{\eta=k-\lambda} \tag{23}$$

$$\begin{aligned}B_2 &= \left\{\left(p_4+\frac{a_2}{2}\right)^2+ik\frac{C_{56}''}{C_{66}''}\left(p_4+\frac{a_2}{2}\right)\right\}\left\{-4ik\left(\nu+a_1\right)C_{56}'C_{66}''\tan p_3H\right.\\
&\quad -2\left(\nu+a_1\right)^2C_{66}'C_{66}''\tan p_3H+8ikp_3C_{56}'C_{66}''-8p_3^2C_{66}'C_{66}''\tan p_3H+8p_3^2C_{66}'^2\tan p_3H\\
&\quad +8ik\left(\nu+a_1\right)C_{66}'C_{56}'\tan p_3H+2\left(\nu+a_1\right)^2C_{66}'^2\tan p_3H-8k^2C_{56}'^2\tan p_3H\\
&\quad \left.+8k^2C_{56}'C_{56}''\tan p_3H-8ikp_3C_{66}'C_{56}''-4ik\left(\nu+a_1\right)C_{66}'C_{56}''\tan p_3H\right\},\end{aligned}$$

$$\begin{aligned}B_3 &= \left(p_4+\frac{a_2}{2}\right)^2\left\{-8ikp_4C_{56}'C_{66}''\tan p_3H-4ika_2C_{56}'C_{66}''\tan p_3H-8p_3p_4C_{66}'C_{66}''\right.\\
&\quad \left.-4p_3a_2C_{66}'C_{66}''-4p_4\left(\nu+a_1\right)C_{66}'C_{66}''\tan p_3H-2a_2\left(\nu+a_1\right)C_{66}'C_{66}''\tan p_3H\right\},\end{aligned}$$

$$\begin{aligned}B_4 &= ik\frac{C_{56}''}{C_{66}''}\left(p_4+\frac{a_2}{2}\right)\left\{8p_3^2C_{66}'^2\tan p_3H-8ikp_3C_{56}''C_{66}'-8k^2C_{56}'^2\tan p_3H+8k^2C_{56}'C_{56}''\tan p_3H\right.\\
&\quad \left.+2\left(\nu+a_1\right)^2C_{66}'^2\tan p_3H+8ik\left(\nu+a_1\right)C_{56}'C_{66}'\tan p_3H-4ik\left(\nu+a_1\right)C_{56}''C_{66}'\tan p_3H\right\},\end{aligned}$$

$$\begin{aligned}B_5 &= -\left(p_4+\frac{a_2}{2}\right)C_{66}'\left\{8ikp_3\left(\nu+a_1\right)C_{56}'+4p_3\left(\nu+a_1\right)^2C_{66}'-4p_3^2\left(\nu+a_1\right)C_{66}'\tan p_3H\right.\\
&\quad \left.-2ik\left(\nu+a_1\right)^2C_{56}'\tan p_3H-\left(\nu+a_1\right)^3C_{66}'\tan p_3H+8ikp_3^2C_{56}'\tan p_3H+8p_3^3C_{66}'\right\},\end{aligned}$$

$$\begin{aligned}B_6 &= -\nu\left(p_4+\frac{a_2}{2}\right)C_{66}'\left\{4ik\left(\nu+a_1\right)C_{56}'\tan p_3H+2\left(\nu+a_1\right)^2C_{66}'\tan p_3H\right.\\
&\quad \left.-8ikp_3C_{56}'+8p_3^2C_{66}'\tan p_3H\right\},\end{aligned}$$

$$\begin{aligned}B_7 &= \left(p_4+\frac{a_2}{2}\right)\left\{8k^2p_3C_{56}'^2+8ikp_3^2C_{56}'C_{66}'\tan p_3H-4k^2\left(\nu+a_1\right)C_{56}'^2\tan p_3H\right.\\
&\quad \left.+2ik\left(\nu+a_1\right)^2C_{56}'C_{66}'\tan p_3H\right\},\end{aligned}$$

$$B_8 = \nu\left(p_4+\frac{a_2}{2}\right)\left\{8k^2C_{56}'^2\tan p_3H-8ikp_3C_{56}'C_{66}'-4ik\left(\nu+a_1\right)C_{56}'C_{66}'\tan p_3H\right\},$$

$$B_9 = \left(p_4+\frac{a_2}{2}\right)^2\left\{-8\lambda kC_{56}'C_{56}''\tan p_3H+8i\lambda p_3C_{66}'C_{56}''+4i\lambda\left(\nu+a_1\right)C_{66}'C_{56}''\tan p_3H\right\},$$

$$B_{10} = \left(p_4+\frac{a_2}{2}\right)\left\{-8i\lambda k^2C_{56}'C_{55}''\tan p_3H-8\lambda kp_3C_{66}'C_{55}''-4i\lambda k\left(\nu+a_1\right)C_{66}'C_{55}''\tan p_3H\right\},$$

$$\begin{aligned}B_{11} &= \left(p_4+\frac{a_2}{2}\right)\left\{4\lambda k\left(\nu+a_1\right)C_{56}'^2\tan p_3H-2i\lambda\left(\nu+a_1\right)^2C_{56}'C_{66}'\tan p_3H\right.\\
&\quad -8\lambda kp_3C_{56}'^2-8i\lambda p_3^2C_{56}'C_{66}'\tan p_3H+8i\lambda kC_{55}'C_{56}'\tan p_3H+8\lambda kp_3C_{55}'C_{66}'\\
&\quad \left.+4\lambda k\left(\nu+a_1\right)C_{66}'C_{55}'\tan p_3H\right\}.\end{aligned}$$

Here the argument of $\psi(k-\lambda)$ is because of $\eta+\lambda=k$. Following the asymptotic formula of Willis [10] and neglecting the terms containing $2/s$ and higher powers of $2/s$ for large s, we get

$$\int_{-\infty}^{\infty}\{\psi(k-\lambda)+\psi(k+\lambda)\}\frac{1}{\lambda}\sin\left(\frac{\lambda s}{2}\right)d\lambda \cong \frac{\pi}{2}2\psi(k)=\pi\psi(k), \text{ (cf. Tranter [6]).} \tag{24}$$

Hence on using Eq. (24) on Eq. (22) one can have

$$R_2+R_1\left\{C_{66}''\left(p_4+\frac{a_2}{2}\right)+ikC_{56}''\right\}=2sC_{66}''\psi(k)=2C_{66}''\frac{H'}{\varepsilon}\psi(k). \tag{25}$$

Therefore, in view of Eq. (25) the displacement in the monoclinic layer is

$$U_1=-\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{4C_{66}''e^{-\left(p_4+\frac{a_2}{2}\right)d}e^{-\frac{a_1}{2}y}}{G(k)\left[1-\frac{H'}{2}\psi(k)e^{\left(p_4+\frac{a_2}{2}\right)d}\right]}\Big\{\Big(2ikC_{56}'\tan p_3H+2p_3C_{66}'$$
$$+\left(\nu+a_1\right)C_{66}'\tan p_3H\Big)\cos p_3y+\Big(2ikC_{56}'-2p_3C_{66}'\tan p_3H+\left(\nu+a_1\right)C_{66}'\Big)\sin p_3y\Big\}e^{-ikz}dk.$$

Since the value of this integral depends entirely on the contribution of the poles of the Integrand, the dispersion equation for SH waves is given by

$$G(k)\left[1-\frac{H'}{2}\psi(k)e^{\left(p_4+\frac{a_2}{2}\right)d}\right]=0. \tag{26}$$

The poles are located at the roots of the Eq.(26) which has been examined in our study of shear waves (Achenbach [8]). Solving Eq. (26) with the help of Eq. (23) and setting

$p_3=kP_3,\ p_4=kP_4,\ a_1=kA_1$ and $a_2=kA_2$

where $P_3=\left(\frac{c^2}{\beta_1^2}-\frac{C_{55}'}{C_{66}'}-\frac{\nu^2}{4k^2}+\frac{C_{56}'^2}{C_{66}'^2}\right)^{1/2}$, $P_4=\left(\frac{C_{55}''}{C_{66}''}-\frac{c^2}{\beta_2^2}-\frac{C_{56}''^2}{C_{66}''^2}\right)^{1/2}$,

$A_1=-2i\frac{C_{56}'}{C_{66}'}$ and $A_2=-2i\frac{C_{56}''}{C_{66}''}$, we get

$$\tan\left\{\sqrt{\left(\frac{c^2}{\beta_1^2}-\frac{C_{55}'}{C_{66}'}-\frac{\nu^2}{4k^2}+\frac{C_{56}'^2}{C_{66}'^2}\right)}kH\right\}=J_1+iJ_2, \tag{27}$$

$$J_1 = \frac{L_1L_3 + L_2L_4}{L_3^{\,2} + L_4^{\,2}},\ J_2 = \frac{L_2L_3 - L_1L_4}{L_3^{\,2} + L_4^{\,2}}$$

and $L_1 = 8H'k^3P_3P_4^2C_{66}'C_{66}'' - 8k^2P_3P_4C_{66}'C_{66}'' - 8H'P_3C_{66}'C_{56}'' + 8H'k^3P_3\frac{C_{66}'}{C_{66}''}C_{56}''^2$

$$+8H'k^3P_3^3C_{66}'^2 + 2H'k\nu^2P_3C_{66}'^2,$$

$$L_2 = -8H'k^3P_3P_4C_{56}'C_{66}'' - 8H'k^3P_3P_4C_{66}'C_{56}'' - 8H'\nu k^2P_3C_{56}'C_{66}',$$

$$L_3 = 4k\nu P_4C_{66}'C_{66}'' - 8k^2P_3^2C_{66}'^2 - 2\nu^2C_{66}'^2 - 2H'k\nu^2P_4C_{66}'C_{66}'' - 8H'k^3P_4P_3^2C_{66}'C_{66}''$$

$$+8H'k^3P_4P_3^2C_{66}'^2 + 2H'k\nu^2P_4C_{66}'^2 - 4H'\nu k^2P_4^2C_{66}'C_{66}'' + 4H'\nu k^2C_{66}'\frac{C_{56}''^2}{C_{66}''}$$

$$-4H'\nu k^2C_{56}''^2\frac{C_{66}'}{C_{66}''} - \nu^3H'C_{66}'^2 - 4H'\nu k^2P_3^2C_{66}'^2 + 8H'\nu k^2C_{56}'^2,$$

$$L_4 = 4H'\nu k^2P_4C_{56}'C_{66}'' + 4H'\nu k^2P_4C_{66}'C_{56}'' - 8H'k^3P_3^2C_{66}'^2\frac{C_{56}''}{C_{66}''} - 2H'\nu^2kC_{66}'^2\frac{C_{56}''}{C_{66}''}$$

$$-8H'k^3P_3^2C_{56}'C_{66}' - 2H'\nu^2kC_{56}'C_{66}'.$$

The real part of the Eq. (28) gives the dispersion equation of SH waves.

The dispersion relation for the SH waves is

$$\tan\left\{\sqrt{\left(\frac{c^2}{\beta_1^2} - \frac{C_{55}'}{C_{66}'} - \frac{\nu^2}{4k^2} + \frac{C_{56}'^2}{C_{66}'^2}\right)}\ kH\right\} = J_1\ . \tag{28}$$

Particular cases

Case 1: When $\nu = 0,\ C_{66}'' = C_{55}'' = \mu_2$ and $C_{56}'' = 0$ the dispersion relation (28) reduces to

$$\tan\left\{\sqrt{\left(\frac{c^2}{\beta_1^2} - \frac{C_{55}'}{C_{66}'} + \frac{C_{56}'^2}{C_{66}'^2}\right)}\ kH\right\} = \frac{J_3}{J_4}$$

where $J_3 = 2P_5\left(4H'kP_6^2C_{66}'\mu_2 - 4P_6C_{66}'\mu_2 + 4H'kP_5^2C_{66}'^2\right)\left(8H'kP_6P_5^2C_{66}'^2\right.$

$$\left.-8P_5^2C_{66}'^2 - 8H'kP_6P_5^2C_{66}'\mu_2\right) + 64H'^2k^2P_5^3P_6C_{56}'^2C_{66}'\mu_2,$$

$$J_4 = \left(8H'kP_6P_5^2C_{66}'^2 - 8P_5^2C_{66}'^2 - 8H'kP_6P_5^2C_{66}'\mu_2\right)^2 + 64H'^2k^2P_5^4C_{56}'^2C_{66}'^2,$$

$$P_5 = \sqrt{\frac{c^2}{\beta_1^2} - \frac{C_{55}'}{C_{66}'} + \frac{C_{56}'^2}{C_{66}'^2}},\ P_6 = \sqrt{1 - \frac{c^2}{\beta_2^2}}$$

which is the result obtained by Chattopadhyay et al [4] for SH waves, propagating in an irregular monoclinic layer lying over an isotropic half space.

Case 2: When $\nu = 0,\ H' = 0,\ C_{66}'' = C_{55}'' = \mu_2$ and $C_{56}'' = 0$ the dispersion relation (28) reduces to

$$\tan\left\{\sqrt{\left(\frac{c^2}{\beta_1^2} - \frac{C_{55}'}{C_{66}'} + \frac{C_{56}'^2}{C_{66}'^2}\right)}\,kH\right\} = \frac{\mu_2\sqrt{1-\dfrac{c^2}{\beta_2^2}}}{C_{66}'\sqrt{\dfrac{c^2}{\beta_1^2} - \dfrac{C_{55}'}{C_{66}'} + \dfrac{C_{56}'^2}{C_{66}'^2}}}$$

which is the result obtained by Chattopadhyay and Pal [2] for SH waves, propagating in a regular monoclinic layer lying over an isotropic half space.

Case 3: When $\nu = 0, C_{66}' = C_{55}' = \mu_1,\ C_{56}' = 0,\ C_{66}'' = C_{55}'' = \mu_2$ and $C_{56}'' = 0$ the dispersion relation (28) reduces to

$$\tan\left\{\sqrt{\left(\frac{c^2}{\beta_1^2} - 1\right)}\,kH\right\} = \frac{J_5}{J_6}$$

where $J_5 = 2P_7\left(4H'kP_8^2\mu_1\mu_2 - 4P_8\mu_1\mu_2 + 4H'kP_7^2\mu_1^2\right)\left(8H'kP_8P_7^2\mu_1^2 - 8P_7^2\mu_1^2 - 8H'kP_8P_7^2\mu_1\mu_2\right),$

$$J_4 = \left(8H'kP_8P_7^2\mu_1^2 - 8P_7^2\mu_1^2 - 8H'kP_8P_7^2\mu_1\mu_2\right)^2,$$

$$P_7 = \sqrt{\frac{c^2}{\beta_1^2} - 1},\ P_8 = \sqrt{1 - \frac{c^2}{\beta_2^2}}$$

which is the result obtained by Chattopadhyay [1] for SH waves, propagating in an irregular isotropic layer lying over an isotropic half space.

Case 4: When $\nu = 0,\ H' = 0, C_{66}' = C_{55}' = \mu_1,\ C_{56}' = 0,\ C_{66}'' = C_{55}'' = \mu_2$ and $C_{56}'' = 0$ the dispersion relation (28) reduces to

$$\tan\left\{\sqrt{\left(\frac{c^2}{\beta_1^2} - 1\right)}\,kH\right\} = \frac{\mu_2\sqrt{1-\dfrac{c^2}{\beta_2^2}}}{\mu_1\sqrt{\dfrac{c^2}{\beta_1^2} - 1}},$$ which is the classical SH wave equation.

Numerical examples

For the case of an irregular non-homogeneous monoclinic layer lying over monoclinic half space, we take the following data

(i) For monoclinic layer, [9]

$C_{55}' = 94\times10^9\,\text{N/m}^2,\qquad C_{56}' = -11\times10^9\,\text{N/m}^2,$

$C_{66}' = 93\times10^9\,\text{N/m}^2,\qquad \rho_1 = 7,450\,\text{Kg/m}^3.$

(ii) For monoclinic half space, [9]

$C_{55}'' = 57.94 \times 10^9 \, \text{N/m}^2$, $C_{56}'' = -17.91 \times 10^9 \, \text{N/m}^2$,
$C_{66}'' = 39.88 \times 10^9 \, \text{N/m}^2$, $\rho_2 = 2,649 \, \text{Kg/m}^3$.

The effect of non-homogeneity and irregular boundary on the propagation of plane SH waves propagating in non-homogeneous monoclinic layer with rectangular irregularity lying over a monoclinic half spaces has been depicted by means of graphs, which are shown in fig. 2 and 3. These graphs give the variation of non-dimensional phase velocity (c/β_1) with respect to non-dimensional wave number kH for different values of non-homogeneity parameter (νH) and different size of irregularity. It is observed that as the size of irregularity (H'/H) increases, the phase velocity decreases. Also, the presence of non-homogeneity increases the phase velocity for the corresponding wave number. Evidently, the dispersion curve gets steeper for the higher values of irregularity.

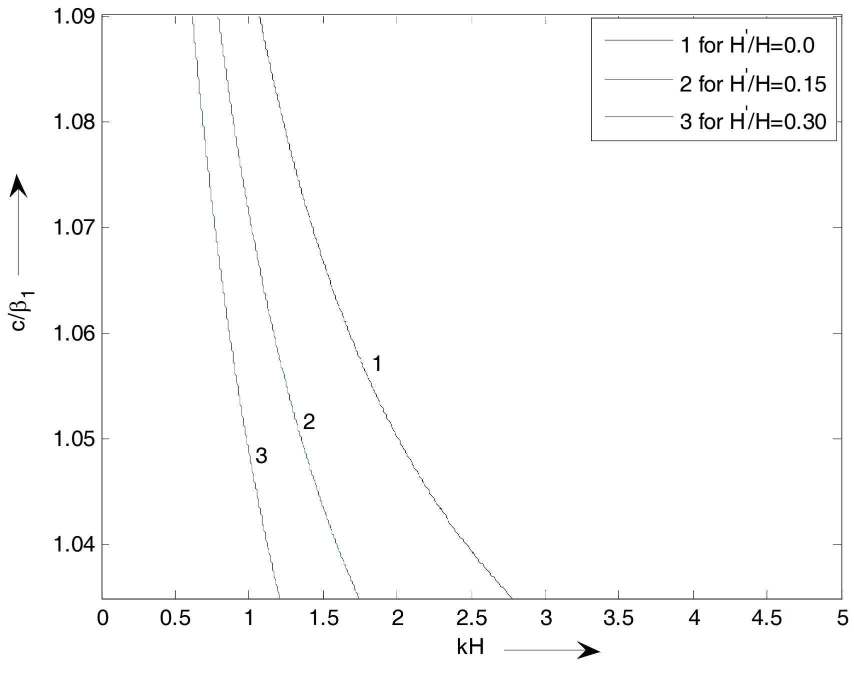

Fig. 2: Curve of c/β_1 versus kH for $\nu H = 0.0$.

Acknowledgement. The authors convey their sincere thanks to Indian School of Mines, Dhanbad for providing JRF to Mr. Abhishek Kr. Singh and also facilitating us with its best facility. Acknowledgement is also due to DST, New Delhi for the providing financial support through Project No. SR/S4/MS: 436/07, Project title: “Wave propagation in anisotropic media”.

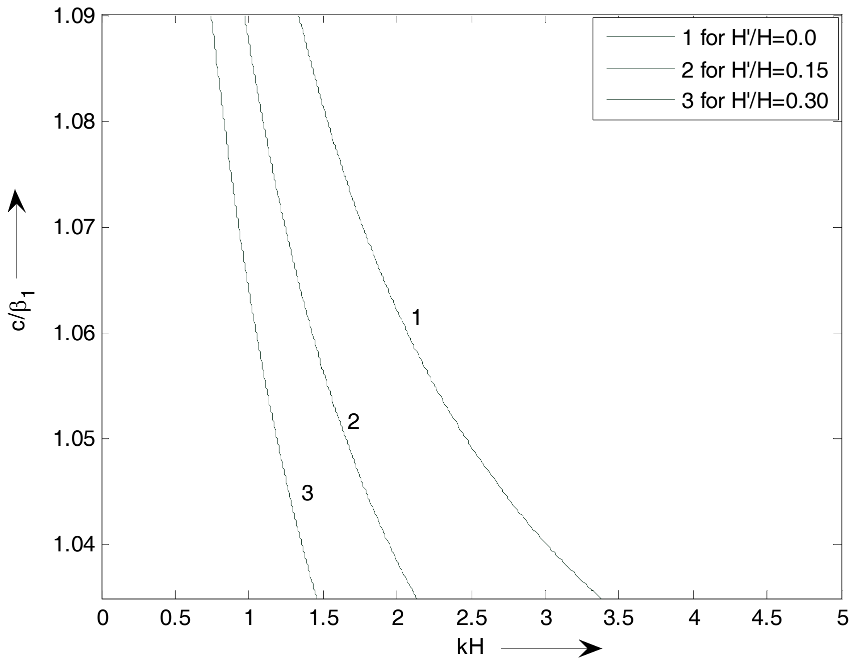

Fig. 3: Curve of c/β_1 versus kH for $\nu H = 0.5$.

References

[1] A. Chattopadhyay, On the dispersion equation for Love wave due to irregularity in the thickness of non-homogeneous crustal layer, Acta Geophysica Polonica, 23 (1975), 307-317.

[2] A. Chattopadhyay and A.K. Pal, Dispersion curves of SH waves caused by irregularity in the prestressed internal stratum, Acta Geophysica Polonica, Volume-31 No.1 (1983), 37-49.

[3] A. Chattopadhyay, M. Chakraborty and A.K. Pal, Effects of irregularity on the propagation of guided SH waves, Jr. de Mecanique Theo. et appl, Volume-2 No. 2 (1983), 215-225.

[4] A. Chattopadhyay, S. Gupta, V.K. Sharma and Pato Kumari, Propagation of SH waves in an irregular monoclinic crustal layer, Archive of Applied Mechanics, Volume-78 No. 12 (2008), 989-999.

[5] A.C. Eringen and C.J. Samuels, Impact and moving loads on a slightly curved elastic half space, J. Appl. Mech., 26 (1959), 491-498.

[6] C.J. Tranter, Integral Transform in Mathematical Physics, Methuen and Co. Ltd. London, 1966.

[7] J. Bhattacharya, On the dispersion curve for Love wave due to irregularity in the thickness of the transversely isotropic crustal layer, Gerland Beit. Zur Geoph., 6 (1962), 324-334.

[8] J.D. Achenbach, Wave propagation in elastic solids, North Holland pub. Comp., New York, 1975.

[9] H.F. Tiersten, Linear Piezoelectric plate Vibrations, Plenum Press, New York, 1969.

[10] H.F. Willis, A formula for expanding an integral as series, Philosophical Magazine, 39 (1948), 455-459.

[11] N. Sinha, Propagation of Love waves in a non-homogeneous stratum of finite depth sandwiched between two semi-infinite isotropic media, Pure Applied Geophysics, 67 (1967), 65-70.

Received: November, 2009