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An Inventory Model for Weibull Deteriorating Items with Price Dependent Demand and Time-Varying Holding Cost

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Abstract

This paper deals with development of an inventory model when the deterioration rate follows Weibull two parameter distributions. Here it is assumed that demand rate is a function of selling price and holding cost is time dependent. With shortage and without shortage both cases have been taken care of in developing the inventory models. Shortages are completely backlogged whenever they are allowed. The results are illustrated with the help of numerical examples. The sensitivity analysis for the model has been performed to study the effect changes of the values of the parameters associated with the model.

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Keywords: EOQ model; deteriorating items; Weibull distribution; shortage; price dependent demand; time varying holding cost

1. Introduction

The control and maintenance inventories for deteriorating items with shortages have received much attention of several researchers in the recent years because most of the physical goods deteriorate over time. In reality, some of the items are either damaged or decayed or affected by some other factors and is not in a perfect condition to satisfy the demand. Food items, drugs, pharmaceuticals, radioactive substances are examples of such items where deterioration can take place during the normal storage period of the commodity and consequently this loss must be taken into account when analyzing the system. So decay or deterioration of physical goods in stock is a very realistic feature and researchers felt the necessity to use this factor into consideration in developing inventory models.

Ghare and Schrader [10] developed a model for an exponentially decaying inventory. An order level inventory model for items deteriorating at a constant rate was proposed by Shah and Jaiswal, [20], Aggarwal [1], Dave and Patel [7]. Inventory models with a time dependent rate of deterioration were considered by Covert and Philip [6], Mishra [17] and Deb and Chaudhuri [8]. Some of the significant recent work in this field have been done by Chung and Ting [5], Fujiwara [9], Hariga [13], Hariga and Benkherouf[14], Wee [23], Jalan et al. [16], Su, et al. [21], Chakraborty and chaudhuri [4], Giri and Chaudhuri[11], Chakraborty, et al. [3] and Jalan and Chaudhuri, [15],etc.

At the beginning, demand rate were assumed to be constant which is in general likely to be time dependent and stock dependent. Burwell [2] developed economic lot size model for price-dependent demand under quantity and freight discounts. Inventory model for ameliorating items for price dependent demand rate was proposed by Mondal et.al [18] and inventory model with price and time dependent demand was developed by You [25]. In general holding cost is assumed to be known and constant. But in realistic condition holding cost may not always be constant. So several researchers like Van der Veen [22], Muhlemann and Valtis Spanopoulos [19], Weiss [24], and Goh [12] considered various functions to describe holding cost.

In this paper, we have developed generalized EOQ model for deteriorating items where deterioration rate follows two-parameter Weibull distribution and holding cost are expressed as linearly increasing functions of time and demand rate is considered to be a function of selling price. For the model where shortages are allowed they are completely backlogged. Here we have considered both the case of with shortage and without shortage in developing the model.

2. Assumptions and Notations

The fundamental assumptions of the model are as follows:

a) The demand tare is a function of selling price.
b) Shortages, whenever allowed, are completely backlogged.
c) The deterioration rate is proportional to time.
d) Holding cost $h(t)$ per item per time-unit is time dependent and is assumed to be $h(t) = h + \delta t$ where $\delta > 0$, $h > 0$
e) Replenishment is instantaneous and lead time is zero.
f) T is the length of the cycle.
g) The order quantity in one cycle is q.
h) A is the cost of placing an order.
i) The selling price per unit item is p.

j) C is the unit cost of an item.
k) The inventory holding cost per unit per unit time is $h(t)$.
l) C_1 is the shortage cost per unit per unit time.
m) The deterioration of units follows the two parameter Weibull distribution (say) $\theta(t) = \alpha\beta t^{\beta-1}$ where $0 < \alpha < 1$ is the scale parameter and $\beta > 0$ is the shape parameter.
n) During time t_1, inventory is depleted due to deterioration and demand of the item. At time t_1 the inventory becomes zero and shortages start occurring.
o) Selling price p follows an increasing trend, and demand rate possess the negative derivative through out its domain where demand rate is $f(p) = (a - p) > 0$

3. Mathematical formulation and solution

Let $Q(t)$ be the inventory level at time $t(0 \le t \le T)$. The differential equations to describe instantaneous state over $(0,T)$ are given by

$$\frac{dQ(t)}{dt} + \alpha\beta t^{\beta-1} I(t) = -(a-p) \qquad 0 \le t \le t_1 \qquad (1)$$

$$\frac{dQ(t)}{dt} = -(a-p) \qquad t_1 \le t \le T \qquad (2)$$

With $Q(t) = 0$ at $t = t_1$

Solving equation (1) and equation (2) and neglecting higher powers of α, we get

$$Q(t) = (a-p)(1-\alpha t^{\beta})\left\{ t - t_1 + \frac{\alpha}{\beta+1}\left(t_1^{\beta+1} - t^{\beta+1}\right)\right\} \qquad 0 \le t \le t_1$$

and

$$Q(t) = (a-p)(t - t_1) \qquad 0 \le t_1 \le T$$

Now stock loss due to deterioration

$$D = (a-p)\int_0^{t_1} e^{\alpha t^{\beta}} dt - (a-p)\int_0^{t_1} dt$$

$$= (a-p)\left\{\frac{\alpha t_1^{\beta+1}}{\beta+1}\right\}$$

$$q = D + \int_0^T (a-p)dt$$

$$= (a-p)\left\{\frac{\alpha t_1^{\beta+1}}{\beta+1}\right\} + (a-p)T \qquad (3)$$

Holding cost is

$$H = \int_0^{t_1} (h + \delta t) e^{-\alpha t^\beta} \left\{ \int_t^{t_1} (a - p) e^{\alpha u^\beta} du \right\} dt$$

Neglecting higher powers of α, we get

$$H = (a-p)\left[\int_0^{t_1} (h+\delta t)(1-\alpha t^\beta) \left\{ \int_0^{t_1} (1+\alpha u^\beta) du \right\} dt \right]$$

$$H = (a-p)\left\{ ht_1^2 - \frac{ht_1^{\,2}}{2} - \frac{h\alpha t_1^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{h\alpha^2 t_1^{2\beta+2}}{(\beta+1)^2} - \frac{h\alpha t_1^{\beta+2}}{\beta+2} - \frac{h\alpha^2 t_1^{\beta+3}}{(\beta+1)(\beta+3)} + \frac{\delta t_1^3}{2} \right.$$
$$\left. + \frac{\delta\alpha t_1^{\,\beta+3}}{2\beta+2} - \frac{\delta t_1^3}{2} - \frac{\delta\alpha t_1^{\beta+3}}{(\beta+1)(\beta+3)} - \frac{\delta\alpha t_1^{\beta+3}}{\beta+2} - \frac{\delta\alpha^2 t_1^{2\beta+3}}{(\beta+1)(\beta+2)} + \frac{\delta\alpha t_1^{\beta+3}}{\beta+3} + \frac{\delta\alpha^2 t_1^{2\beta+3}}{(\beta+1)(2\beta+3)} \right\} \tag{4}$$

Now shortage cost during the cycle

$$S = -\int_{t_1}^{T} \{-(a-p)(t-t_1)\} dt$$
$$= \frac{1}{2}(a-p)(T-t_1)^2 \tag{5}$$

From (3), (4) and (5) Total profit per unit time is given by

$$P(T, t_1, p) = p(a-p) - \frac{1}{T}(A + Cq + H + C_1 S)$$

$$= p(a-p) - \frac{1}{T}\left[A + C\left[(a-p)\left\{ \frac{\alpha t_1^{\beta+1}}{\beta+1} \right\} + (a-p)T \right] + (a-p)\left\{ ht_1^2 - \frac{ht_1^{\,2}}{2} - \frac{h\alpha t_1^{\beta+2}}{(\beta+1)(\beta+2)} \right.\right.$$
$$- \frac{h\alpha^2 t_1^{2\beta+2}}{(\beta+1)^2} - \frac{h\alpha t_1^{\beta+2}}{\beta+2} - \frac{h\alpha^2 t_1^{\beta+3}}{(\beta+1)(\beta+3)} + \frac{\delta t_1^3}{2} + \frac{\delta\alpha t_1^{\,\beta+3}}{2\beta+2} - \frac{\delta t_1^3}{2} - \frac{\delta\alpha t_1^{\beta+3}}{(\beta+1)(\beta+3)} - \frac{\delta\alpha t_1^{\beta+3}}{\beta+2}$$
$$\left.\left. - \frac{\delta\alpha^2 t_1^{2\beta+3}}{(\beta+1)(\beta+2)} + \frac{\delta\alpha t_1^{\beta+3}}{\beta+3} + \frac{\delta\alpha^2 t_1^{2\beta+3}}{(\beta+1)(2\beta+3)} \right\} + C_1\left\{ \frac{1}{2}(a-p)(T-t_1)^2 \right\} \right. \tag{6}$$

Let $t_1 = \gamma T \quad ,0 < \gamma < 1$

Hence we get the profit function

$$P(T, p) = p(a-p) - \frac{1}{T}\left[A + C\left[(a-p)\left\{ \frac{\alpha T^{\beta+1}\gamma^{\beta+1}}{\beta+1} \right\} + (a-p)T \right] + (a-p)\left\{ h\gamma^2 T^2 \right.\right.$$
$$- \frac{h\gamma^2 T^2}{2} - \frac{h\gamma^{\beta+2} T^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{h\alpha^2\gamma^{2\beta+2} T^{2\beta+2}}{(\beta+1)^2} - \frac{h\alpha\gamma^{\beta+2} T^{\beta+2}}{\beta+2} - \frac{h\alpha^2\gamma^{\beta+3} T^{\beta+3}}{(\beta+1)(\beta+3)} + \frac{\delta\gamma^3 T^3}{2}$$
$$+ \frac{\delta\alpha\gamma^{\beta+2} T^{\beta+2}}{2\beta+2} - \frac{\delta\gamma^3 T^3}{3} - \frac{\delta\alpha\gamma^{\beta+3} T^{\beta+3}}{(\beta+1)(\beta+3)} - \frac{\delta\alpha\gamma^{\beta+3} T^{\beta+3}}{\beta+2} - \frac{\delta\alpha^2\gamma^{2\beta+3} T^{2\beta+3}}{(\beta+1)(\beta+2)}$$
$$\left.\left. + \frac{\delta\alpha\gamma^{\beta+3} T^{\beta+3}}{\beta+3} + \frac{\delta\alpha^2\gamma^{2\beta+3} T^{2\beta+3}}{(\beta+1)(2\beta+3)} \right\} + \frac{C_1}{2}(a-p)T^2(1-\gamma)^2 \right] \tag{7}$$

Our objective is to maximize the profit function $P(T,p)$.The necessary conditions for maximizing the profit are

$$\frac{\partial P(T,p)}{\partial T}=0 \text{ and } \frac{\partial P(T,p)}{\partial p}=0$$

We get

$$\frac{A}{T^2}-C(a-p)\left(\frac{\beta\alpha\gamma^{\beta+1}T^{\beta+1}}{\beta+1}\right)-(a-p)h\left\{\ \gamma^2+\frac{\beta\alpha\gamma^{\beta+1}T^{\beta-1}}{\beta+1}-\frac{\gamma^2}{2}-\frac{\alpha\gamma^{\beta+2}T^{\beta}}{\beta+2}-\alpha\gamma^{\beta+2}T^{\beta}\right.$$
$$\left.-\frac{\alpha^2\gamma^{2\beta+2}T^{2\beta}(2\beta+1)}{(\beta+1)^2}-\frac{\alpha\gamma^{\beta+2}T^{\beta}(\beta+1)}{\beta+2}-\frac{\alpha^2\gamma^{\beta+3}T^{\beta+1}(\beta+2)}{(\beta+1)(\beta+3)}\ \right\}-(a-p)\delta\left\{\ \gamma^3T\right.$$
$$+\frac{\alpha\gamma^{\beta+2}T^{\beta}(\beta+1)}{2\beta+2}-\frac{2\gamma^3T}{3}-\frac{\alpha\gamma^{\beta+3}T^{\beta+1}(\beta+2)}{(\beta+1)(\beta+3)}-\alpha\gamma^{\beta+3}T^{\beta+1}-\frac{2\alpha^2\gamma^{2\beta+3}T^{2\beta+1}}{(\beta+1)(\beta+3)}$$
$$\left.+\frac{\alpha\gamma^{\beta+3}T^{\beta+1}(\beta+2)}{\beta+3}+\frac{2\alpha^2\gamma^{2\beta+3}T^{2\beta+1}}{2\beta+3}\ \right\}-\frac{C_1}{2}(a-p)(1-\gamma)^2=0 \qquad (8)$$

and

$$-a+2p+\frac{1}{T}\left[\ -\left\{T+\frac{\alpha T^{\beta+1}\gamma^{\beta+1}}{\beta+1}\right\}-h\left\{\ \gamma^2T^2-\frac{\gamma^2T^2}{2}-\frac{\alpha\gamma^{\beta+2}T^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{\alpha^2\gamma^{2\beta+2}T^{2\beta+2}}{(\beta+1)^2}\right.\right.$$
$$\left.-\frac{\alpha\gamma^{\beta+2}T^{\beta+2}}{\beta+2}-\frac{\alpha^2\gamma^{\beta+3}T^{\beta+3}}{(\beta+1)(\beta+3)}\ \right\}-\delta\left\{\ \frac{\gamma^3T^3}{2}+\frac{\alpha\gamma^{\beta+2}T^{\beta+2}}{2\beta+2}-\frac{\gamma^3T^3}{3}-\frac{\alpha\gamma^{\beta+3}T^{\beta+3}}{(\beta+1)(\beta+3)}\right.$$
$$\left.-\frac{\alpha\gamma^{\beta+3}T^{\beta+3}}{\beta+2}-\frac{\alpha^2\gamma^{2\beta+3}T^{2\beta+3}}{(\beta+1)(\beta+2)}+\frac{\alpha\gamma^{\beta+3}T^{\beta+3}}{\beta+3}+\frac{\alpha^2\gamma^{2\beta+3}T^{2\beta+3}}{(\beta+1)(2\beta+3)}\ \right\}$$
$$\left.-\frac{C_1}{2}T^2(1-\gamma)^2\ \right]=0 \qquad (9)$$

Using the software Mathematica-5.1, from equation (8) and equation (9) we can calculate the optimum values of T^* and p^* simultaneously and the optimal value $P^*(T,p)$ of the average net profit is determined by (7) provided they satisfy the sufficiency conditions for maximizing $P^*(T,p)$ are

$$\frac{\partial^2 P(T,p)}{\partial T^2}<0,\frac{\partial^2 P(T,p)}{\partial p^2}<0 \qquad (10)$$

and

$$\frac{\partial^2 P(T,p)}{\partial T^2}\frac{\partial^2 P(T,p)}{\partial p^2}-\left(\frac{\partial^2 P(T,p)}{\partial T\partial p}\right)^2>0 \text{ at } p=p^* \text{ and } T=T^* \qquad (11)$$

If the solutions obtained from equations (8) and (9) do not satisfy the sufficiency conditions (10) and (11), we conclude that no feasible solution will be optimal for the set of parameter values taken to solve equations (8) and (9). Such a situation will imply that the parameter values are inconsistent and there is some error in their estimation.

4. Numerical example

Case -I (with shortages)
Example -1:

Let $A = 200$, $a = 100$, $C = 20$, $h = 1.2$, $C_1 = 1.2$, $\alpha = 0.1$, $\beta = 0.3$, $\gamma = 0.5$ and $i = 0.9$.

Based on these input data, the computed outputs are as follows:

$P^*(T, p) = 1393.61$, $T^* = 3.14165$, $p^* = 50.7834$, $t_1^* = 1.570825$ and $q^* = 160.568$

Case -II (without shortages)
Example -1:

Let $A = 200$, $a = 100$, $C = 20$, $h = 1.2$, $\alpha = 0.1$, $\beta = 0.3$, $\gamma = 0.5$ and $i = 0.9$.

Based on these input data, the computed outputs are as follows:

$P^*(T, p) = 1415.34$, $T^* = 3.92398$, $p^* = 50.2758$, $t_1^* = 1.96199$ and $q^* = 202.621$

5. Sensitivity analysis

To study the effects of changes of the parameters on the optimal profit derived by proposed method, a sensitivity analysis is performed considering the numerical example given above. Sensitivity analysis is performed by changing (increasing or decreasing) the parameters by 20% and 50% and taking one parameter at a time, keeping the remaining parameters at their original values. The results are shown in table-1 and table-2 for with shortage case and without shortage case respectively.

Table-1

Changing Parameter	% Change in system	Change in T^*	Change in p^*	Change in q^*	Change in t_1^*	Change in $P^*(t_1, T)$
	-50	4.48377	25.9159	112.141	2.241885	61.1037
a	-20	3.5246	40.8196	143.407	1.7623	709.163
	+20	2.85808	60.7574	175.832	1.42904	2279.21
	+50	2.54325	75.7292	196.154	1.271625	3984.16
	-50	3.12661	50.8047	156.772	1.563305	1418.31
α	-20	3.13411	50.792	158.969	1.567055	1403.52
	+20	3.15155	50.7747	162.296	1.575775	1383.64
	+50	3.17161	50.7613	165.176	1.585805	1368.57
	-50	3.29327	50.7942	169.094	1.646635	1392.49
β	-20	3.19954	50.7876	163.806	1.59977	1393.20
	+20	3.10498	50.7808	158.443	1.55249	1394.14
	+50	3.01151	50.7739	153.357	1.505755	1394.47

We study from above table-1 reveals the following

(i) Increase in the values of either of the parameters a, will result in increase of $P^*(t_1,T)$, p^*, and q^* but decrease T^*, t_1^*.

(ii) Decrease in the values of either of the parameters a, will result in decrease of $P^*(t_1,T)$, p^*, and q^* but increase T^*, t_1^*.

(iii) Increase in the values of either of the parameter α, will result in decrease of $P^*(t_1,T)$ and p^*, but increase T^*, t_1^* and q^*.

(iv) Decrease in the values of either of the parameter α, will result in increase of $P^*(t_1,T)$ and p^*, but decrease T^*, t_1^* and q^*.

(v) Increase in the values of either of the parameter β, will result in increase of $P^*(t_1,T)$ but increase T^*, t_1^*, p^* and q^*.

(vi) Decrease in the values of either of the parameter β, will result in decrease of $P^*(t_1,T)$ but increase T^*, t_1^*, p^* and q^*.

Table-2

Changing Parameter	% Change in system	Change in T^*	Change in p^*	Change in q^*	Change in t_1^*	Change $P^*(t_1,T)$
	-50	5.46188	25.0534	141.496	2.73094	46.382
a	-20	4.37542	40.2175	180.76	2.18771	725.149
	+20	3.58393	60.3157	222.132	1.791965	2305.70
	+50	3.20095	75.3568	248.119	1.600475	4016.61
	-50	3.80208	50.3282	192.355	1.831205	1442.27
α	-20	3.86747	50.2985	198.133	1.933735	1426.23
	+20	3.99474	50.2496	207.913	1.99737	1404.21
	+50	4.14088	50.2009	218.096	2.07044	1386.89
	-50	4.1421	50.2958	215.047	2.07105	1415.52
β	-20	4.0097	50.2642	207.467	2.00485	1415.44
	+20	3.89049	50.2868	197.939	1.92022	1415.21
	+50	3.71965	50.3023	191.232	1.859825	1414.98

We study from above table-2 reveals the following

(i) Increase in the values of either of the parameters a, will result in increase of $P^*(t_1,T)$, p^*, and q^* but decrease T^*, t_1^*.

(ii) Decrease in the values of either of the parameters a, will result in decrease of $P^*(t_1,T)$, p^*, and q^* but increase T^*, t_1^*.

(iii) Increase in the values of either of the parameter α, will result in decrease of $P^*(t_1,T)$ and p^*, but increase T^*, t_1^* and q^*.

(iv) Decrease in the values of either of the parameter α, will result in increase of $P^*(t_1,T)$ and p^*, but decrease T^*, t_1^* and q^*.

(v) Increase in the values of either of the parameter β, will result in decrease of $P^*(t_1,T)$, T^*, t_1^* and p^* but increase q^*.

(vi) Decrease in the values of either of the parameter β, will result in increase of $P^*(t_1,T)$ T^*, t_1^* and p^* but decrease q^*.

6. Conclusion

In this present paper we have developed deterministic inventory model for deteriorating items for with shortage and without shortage cases. The deterministic demand rate is assumed to be a function of selling price. Whenever shortages are allowed and they are completely backlogged and holding cost is assumed here to be time varying. We can make a good comparative study between the results of the with-shortage case and without-shortage case. In the numerical examples, it is found that the optimum average profit in without-shortage case is more than that of the shortage case. From the above model one can calculate the optimum average profit margins for with shortage case and without shortage case for the deterministic inventory model with varying demand rate and holding cost subject to the conditions.

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