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# Ordering Policy for Weibull Deteriorating Items for Quadratic Demand with Permissible Delay in Payments

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**Abstract**

This paper deals with the development of an inventory model for Weibull deteriorating items in quadratic demand market when delay in payments is allowed to the retailer to settle the account against the purchases made by him. Shortages are not allowed. In this paper we have considered two cases; those are for the case payment within the permissible time and for payment after the expiry of permissible time. Numerical examples are provided to illustrate our results. Sensitivity analysis has been carried out to analyze the changes in the optimal solution with respect to deterioration rate of units in inventory and the rate of change of demand.

**Mathematics Subject Classification**: 90B05

**Keywords:** Weibull deterioration, trade credit, Quadratic demand

## 1. Introduction

The main objective of inventory management deals with minimization of the inventory carrying cost for which it is required to determine the optimal stock and optimal time of replenishment of inventory to meet the future demand. When the inventories are subject to deterioration, delay in payment is permissible and the demand is either increasing or decreasing the situation becomes more complicated.

An EOQ model with permissible delay in payments was developed by Goyal [14] where he did not consider the difference between the selling price and purchase cost. Goyal's model was improved by Dave [16] under the assumption that the selling price is higher than its purchase price. Inventory models for the optimal pricing and ordering policies for the retailer with trade credit was formulated by Hwang and Shinn [8] and Liao et al. [7]. Considering the difference between the unit sale price and unit purchase cost Jamal et al. [1] and [2] and Sarker et al. [4] suggested that the retailer should settle the account sooner as the unit selling price increases relative to the unit cost. Chang et al. [6] have suggested a model under the condition that supplier offers trade credit to the buyer if the order quantity is greater than or equal to a pre-determined quantity. Further studies in this line are due to Ouyang et al. [12], Chang et al. [9], Chung and Huang [11], etc. Teng et al. [10] has suggested the strategy of granting credit items adds not only an additional cost to the supplier but also default risk to the supplier. Ouyang et al [13] have considered tread credit linked to order quantity for deteriorating items. More discussion in this line are given in the notes by Mitra et al. [3], Giri et al. [5] and Khanra and Choudhuri[15].

In developing the present model demand of a product is assumed to be quadratic function of time. We have considered the case of no shortages and infinite replenishment rate. Here the case of the retailer's generating revenue on unit selling price which is necessarily higher than the unit purchase cost has been considered. We have found the optimal total cost, optimal ordering quantity optimal cycle length for our model. Numerical examples have been given to illustrate the model. Sensitivity analysis has also been carried out to observe the effects on the optimal solution.

## 2. Notations and assumptions

We need the following notations and assumptions to develop the proposed mathematical model.

**Notations**

$R(t) = a + bt + ct^2$ ; the annual demand as a function of time where $a > 0$ , $b > 0$ and $c > 0$ .

$C$ : the unit purchase cost.

$P$ : the unit selling price with $(P > C)$ .

$h$: the inventory holding cost per unit per year excluding interest charges.

$A$ : the ordering cost per order.

$M$ : the permissible credit period offered by the supplier to the retailer for settling the account.

$I_c$ : the interest charged per monetary unit in stock per annum by the supplier.

$I_e$ : the interest earned per monetary unit per year, where $I_e < I_c$ .

$Q$ : the order quantity.

$\theta = \alpha\beta t^{\beta-1}$ : where $0 < \alpha << 1$ is the scale parameter and $\beta > 0$ is the shape parameter.

$I(t)$ : the inventory level at any instant of time $t, 0 \le t \le T$ .

$T$ : the replenishment cycle time.

$IC_1$ : the interest charged per time unit.

$IC_2$ : the interest charged per time unit.

$IE_1$ : the interest earned per time unit.

$IE_2$ : the interest earned per time unit.

Total cost of inventory includes (a) ordering cost, (b) cost due to deterioration, (c) inventory holding cost (excluding interest charges), (d) interest charged on unsold item after the permissible trade credit when $M < T$ , and (e) interest earned from sales revenue during the allowable permissible delay in period.

**Assumptions**

(i) The inventory system under consideration deals with single item.
(ii) The planning horizon is infinite.
(iii) The demand of the product is declining exponential function of the time.
(iv) Shortages are not allowed and lead-time is zero.
(v) The deteriorated units can neither be repaired nor replaced during the cycle time.
(vi) The retailer can deposit generated sales revenue in an interest bearing account during the permissible credit period. At the end of this period, the retailer settles the account for all the units sold keeping the difference for day-to-day expenditure, and paying the interest charges on the unsold items in the stock.

## 3. Mathematical model

The rate of change of inventory level is governed by the following differential equation:

$$\frac{dI(t)}{dt} + \theta I(t) = -R(t) \qquad 0 \le t \le T \qquad (1)$$

Subject to the boundary conditions $I(0) = Q$ and $I(T) = 0$ .

Since $\alpha$ is very small using series expansion ignoring second and higher powers of $\alpha$ , From equation (1) we get

$$I(t) = a\left[T - t + \frac{\alpha T^{\beta+1}}{\beta+1} - \frac{\alpha t^{\beta+1}}{\beta+1} + \alpha t^{\beta}(t-T)\right] + b\left[\frac{T^2 - t^2}{2} + \frac{\alpha T^{\beta+2}}{\beta+2} - \frac{\alpha t^{\beta+2}}{\beta+1} + \frac{\alpha t^{\beta}(t^2 - T^2)}{2}\right]$$
$$+ c\left[\frac{T^3 - t^3}{3} + \frac{\alpha T^{\beta+3}}{\beta+3} - \frac{\alpha t^{\beta+3}}{\beta+3} + \frac{\alpha t^{\beta}(t^3 - T^3)}{3}\right] \qquad 0 \le t \le T \qquad (2)$$

and the order quantity is

$$Q = a\left[T + \frac{\alpha T^{\beta+1}}{\beta+1}\right] + b\left[\frac{T^2}{2} + \frac{\alpha T^{\beta+2}}{\beta+2}\right] + c\left[\frac{T^3}{3} + \frac{\alpha T^{\beta+3}}{\beta+3}\right] \tag{3}$$

a. Ordering cost; $OC = \frac{A}{T}$ (4)

b. Cost due to deterioration per unit time;

$$DC = \frac{C}{T}\left[Q - \int_0^T R(t)dt\right] = \frac{C}{T}\left[\frac{a\alpha T^{\beta+1}}{\beta+1} + \frac{b\alpha T^{\beta+2}}{\beta+2}\frac{c\alpha T^{\beta+3}}{\beta+3}\right] \tag{5}$$

c. Inventory holding cost per unit time;

$$IHC = \frac{h}{T}\int_0^T I(t)dt$$

$$= \frac{h}{T}\left[\frac{aT^2}{2} + \frac{bT^3}{3} + \frac{cT^4}{4} + \frac{\alpha bT^{\beta+3}}{\beta+2} + \frac{c\alpha T^{\beta+4}}{\beta+3} - \frac{\alpha bT^{\beta+3}}{2} - \frac{c\alpha T^{\beta+4}}{3} - \frac{a\alpha T^{\beta+2}}{(\beta+1)(\beta+2)}\right.$$
$$\left. - \frac{b\alpha T^{\beta+3}}{(\beta+2)(\beta+3)} - \frac{c\alpha T^{\beta+4}}{(\beta+3)(\beta+4)} + \frac{a\alpha T^{\beta+2}}{\beta+2} + \frac{b\alpha T^{\beta+3}}{2(\beta+3)} - \frac{c\alpha T^{\beta+4}}{3(\beta+4)}\right] \tag{6}$$

Now we consider the two cases based on the length of $T$ and $M$, using the fact that interest charged or earned (i.e., costs (d) and (e) in section 2.2),

**Case -I:** $M < T$

Under the assumption (b) above, the retailer sells $R(M)M$ units by the end of the permissible tread credit $M$ and has $CR(M)M$ to pay the supplier. The supplier charges an interest rate $I_c$ from time $M$ onwards for the unsold items in the stock. Hence, the interest charged, $IC_1$ per time unit is

d. $IC_1 = \frac{CI_c}{T}\int_M^T I(t)dt$

$$= \frac{CI_c}{T}\left[\frac{aT^2}{2} + \frac{bT^3}{3} + \frac{cT^4}{4} + \frac{\alpha bT^{\beta+3}}{\beta+2} + \frac{c\alpha T^{\beta+4}}{\beta+3} - \frac{\alpha bT^{\beta+3}}{2} - \frac{c\alpha T^{\beta+4}}{3} - \frac{a\alpha T^{\beta+2}}{(\beta+1)(\beta+2)}\right.$$
$$- \frac{b\alpha T^{\beta+3}}{(\beta+2)(\beta+3)} - \frac{c\alpha T^{\beta+4}}{(\beta+3)(\beta+4)} + \frac{a\alpha T^{\beta+2}}{\beta+2} + \frac{b\alpha T^{\beta+3}}{2(\beta+3)} - \frac{c\alpha T^{\beta+4}}{3(\beta+4)} - aTM$$
$$- \frac{a\alpha T^{\beta+1}M}{\beta+1} - \frac{bT^2M}{2} - \frac{\alpha bT^{\beta+2}M}{\beta+2} - \frac{cT^3M}{3} - \frac{c\alpha T^{\beta+3}M}{\beta+3} + \frac{a\alpha M^{\beta+4}}{\beta+1} + \frac{b\alpha M^{\beta+1}T^2}{2}$$
$$- \frac{cT^3M}{3} - \frac{c\alpha T^{\beta+3}M}{\beta+3} + \frac{a\alpha M^{\beta+4}}{\beta+1} + \frac{b\alpha M^{\beta+1}T^2}{2} + \frac{c\alpha M^{\beta+1}T^3}{3} + \frac{aM^2}{2} + \frac{a\alpha M^{\beta+2}}{(\beta+1)(\beta+2)}$$

$$+\frac{bM^3}{6}+\frac{b\alpha M^{\beta+3}}{(\beta+2)(\beta+3)}+\frac{cM^4}{12}+\frac{c\alpha M^{\beta+4}}{(\beta+3)(\beta+4)}-\frac{a\alpha M^{\beta+2}}{\beta+2}-\frac{b\alpha M^{\beta+3}}{2(\beta+3)}+\frac{c\alpha M^{\beta+4}}{3(\beta+4)}\Bigg] \tag{7}$$

During $[0, M]$ the retailer sells the product and deposits the revenue into an interest earning account at the rate $I_e$ per monetary unit per year. We get the interest earned, $IE_1$ per time unit

e. $$IE_1=\frac{PI_e}{T}\int_0^M R(t)tdt=\frac{PI_e}{T}\left[\frac{aM^2}{2}+\frac{bM^3}{3}+\frac{cM^4}{4}\right] \tag{8}$$

Hence, the total cost; $TC_1(T)$ of an inventory system per time unit is

$$TC_1(T)=OC+DC+IHC+IC_1-IE_1$$

$$=\frac{A}{T}+\frac{C}{T}\left[\frac{a\alpha T^{\beta+1}}{\beta+1}+\frac{b\alpha T^{\beta+2}}{\beta+2}\frac{c\alpha T^{\beta+3}}{\beta+3}\right]+\frac{h}{T}\Bigg[\frac{aT^2}{2}+\frac{bT^3}{3}+\frac{cT^4}{4}+\frac{\alpha bT^{\beta+3}}{\beta+2}+\frac{c\alpha T^{\beta+4}}{\beta+3}$$

$$-\frac{\alpha bT^{\beta+3}}{2}-\frac{c\alpha T^{\beta+4}}{3}-\frac{a\alpha T^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{b\alpha T^{\beta+3}}{(\beta+2)(\beta+3)}-\frac{c\alpha T^{\beta+4}}{(\beta+3)(\beta+4)}+\frac{a\alpha T^{\beta+2}}{\beta+2}$$

$$+\frac{b\alpha T^{\beta+3}}{2(\beta+3)}-\frac{c\alpha T^{\beta+4}}{3(\beta+4)}\Bigg]+\frac{CI_c}{T}\Bigg[\frac{aT^2}{2}+\frac{bT^3}{3}+\frac{cT^4}{4}+\frac{\alpha bT^{\beta+3}}{\beta+2}+\frac{c\alpha T^{\beta+4}}{\beta+3}-\frac{\alpha bT^{\beta+3}}{2}$$

$$-\frac{c\alpha T^{\beta+4}}{3}-\frac{a\alpha T^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{b\alpha T^{\beta+3}}{(\beta+2)(\beta+3)}-\frac{c\alpha T^{\beta+4}}{(\beta+3)(\beta+4)}+\frac{a\alpha T^{\beta+2}}{\beta+2}+\frac{b\alpha T^{\beta+3}}{2(\beta+3)}$$

$$-\frac{c\alpha T^{\beta+4}}{3(\beta+4)}-aTM-\frac{a\alpha T^{\beta+1}M}{\beta+1}-\frac{bT^2M}{2}-\frac{\alpha bT^{\beta+2}M}{\beta+2}-\frac{cT^3M}{3}-\frac{c\alpha T^{\beta+3}M}{\beta+3}+\frac{a\alpha M^{\beta+4}}{\beta+1}$$

$$+\frac{b\alpha M^{\beta+1}T^2}{2}+\frac{c\alpha M^{\beta+1}T^3}{3}+\frac{aM^2}{2}+\frac{a\alpha M^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{bM^3}{6}+\frac{b\alpha M^{\beta+3}}{(\beta+2)(\beta+3)}+\frac{cM^4}{12}$$

$$+\frac{c\alpha M^{\beta+4}}{(\beta+3)(\beta+4)}-\frac{a\alpha M^{\beta+2}}{\beta+2}-\frac{b\alpha M^{\beta+3}}{2(\beta+3)}+\frac{c\alpha M^{\beta+4}}{3(\beta+4)}\Bigg]+\frac{PI_e}{T}\left[\frac{aM^2}{2}+\frac{bM^3}{3}+\frac{cM^4}{4}\right] \tag{9}$$

**Case -II:** $M \geq T$

Here, the retailer sells *R(T)T* units in all by the end of the cycle time and has *CR(T)T* to pay the supplier in full by the end of the credit period *M*. Hence, interest charges

d. $$IC_2=0 \tag{10}$$

and the interest earned per time unit is

e. $IE_2 = \frac{PI_e}{T}\left[\int_0^T R(t)tdt + R(T)T(M-T)\right]$

$$= \frac{PI_e}{T}\left[aTM + bT^2M + cT^3M - \frac{aT^2}{2} - \frac{2bT^3}{3} - \frac{3cT^3}{4}\right] \qquad (11)$$

The total cost; $TC_2(T)$ of an inventory system per time unit is

$TC_2(T) = OC + DC + IHC + IC_2 - IE_2$

$$= \frac{A}{T} + \frac{C}{T}\left[\frac{a\alpha T^{\beta+1}}{\beta+1} + \frac{b\alpha T^{\beta+2}}{\beta+2}\frac{c\alpha T^{\beta+3}}{\beta+3}\right] + \frac{h}{T}\left[\frac{aT^2}{2} + \frac{bT^3}{3} + \frac{cT^4}{4} + \frac{\alpha bT^{\beta+3}}{\beta+2} + \frac{c\alpha T^{\beta+4}}{\beta+3}\right.$$
$$- \frac{\alpha bT^{\beta+3}}{2} - \frac{c\alpha T^{\beta+4}}{3} - \frac{a\alpha T^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{b\alpha T^{\beta+3}}{(\beta+2)(\beta+3)} - \frac{c\alpha T^{\beta+4}}{(\beta+3)(\beta+4)} + \frac{a\alpha T^{\beta+2}}{\beta+2}$$
$$\left. + \frac{b\alpha T^{\beta+3}}{2(\beta+3)} - \frac{c\alpha T^{\beta+4}}{3(\beta+4)}\right] + \frac{PI_e}{T}\left[aTM + bT^2M + cT^3M - \frac{aT^2}{2} - \frac{2bT^3}{3} - \frac{3cT^3}{4}\right] \qquad (12)$$

Hence, the total cost; $TC(T)$ of an inventory system per time unit is

$$TC(T) = \begin{cases} TC_1(T), & M < T \\ TC_2(T), & M \geq T \end{cases} \qquad (13)$$

For $T = M$, in equation (12) we have

$$= \frac{A}{M} + \frac{C}{M}\left[\frac{a\alpha M^{\beta+1}}{\beta+1} + \frac{b\alpha M^{\beta+2}}{\beta+2}\frac{c\alpha M^{\beta+3}}{\beta+3}\right] + \frac{h}{T}\left[\frac{aM^2}{2} + \frac{bM^3}{3} + \frac{cM^4}{4} + \frac{\alpha bM^{\beta+3}}{\beta+2}\right.$$
$$+ \frac{c\alpha M^{\beta+4}}{\beta+3} - \frac{\alpha bM^{\beta+3}}{2} - \frac{c\alpha M^{\beta+4}}{3} - \frac{a\alpha M^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{b\alpha M^{\beta+3}}{(\beta+2)(\beta+3)} - \frac{c\alpha M^{\beta+4}}{(\beta+3)(\beta+4)}$$
$$\left. + \frac{a\alpha M^{\beta+2}}{\beta+2} + \frac{b\alpha M^{\beta+3}}{2(\beta+3)} - \frac{c\alpha M^{\beta+4}}{3(\beta+4)}\right] + \frac{PI_e}{M}\left[cM^4 + \frac{aM^2}{2} + \frac{bM^3}{3} - \frac{3cM^3}{4}\right] \qquad (14)$$

Now $TC_1(T)$ will be minimum, the optimum values of $T$ for the minimum average total cost $TC_1(T)$ is the solution of equation

$$\frac{dTC_1(T)}{dT} = 0 \qquad (15)$$

Provided

$$\frac{d^2TC_1(T)}{dT^2} > 0 \qquad (16)$$

Using equation (15) we get

$$-\frac{A}{T^2} + C\left[\frac{a\alpha\beta T^{\beta-1}}{\beta+1} + \frac{(\beta+1)b\alpha T^{\beta}}{\beta+2}\frac{(\beta+2)c\alpha T^{\beta+1}}{\beta+3}\right] + h\left[\frac{a}{2} + \frac{2bT}{3} + \frac{3cT^2}{4} + \alpha bT^{\beta+1}\right.$$

$$+\alpha cT^{\beta+2}-\frac{(\beta+2)\alpha bT^{\beta+1}}{2}-\frac{(\beta+3)c\alpha T^{\beta+2}}{3}-\frac{a\alpha T^{\beta}}{\beta+2}-\frac{b\alpha T^{\beta+1}}{\beta+3}-\frac{c\alpha T^{\beta+2}}{\beta+4}+\frac{(\beta+2)a\alpha T^{\beta}}{\beta+2}$$
$$+\frac{(\beta+2)b\alpha T^{\beta+1}}{2(\beta+3)}-\frac{(\beta+3)c\alpha T^{\beta+2}}{3(\beta+4)}\Bigg]+CI_c\Bigg[\frac{a}{2}+\frac{2bT}{3}+\frac{3cT^2}{4}+\alpha bT^{\beta+1}+c\alpha T^{\beta+2}$$
$$-\frac{\alpha b(\beta+2)T^{\beta+1}}{2}-\frac{c\alpha(\beta+3)T^{\beta+2}}{3}-\frac{a\alpha T^{\beta}}{\beta+2}-\frac{b\alpha T^{\beta+1}}{\beta+3}-\frac{c\alpha T^{\beta+2}}{\beta+4}+\frac{(\beta+1)a\alpha T^{\beta}}{\beta+2}$$
$$+\frac{(\beta+2)b\alpha T^{\beta+1}}{2(\beta+3)}-\frac{(\beta+3)c\alpha T^{\beta+2}}{3(\beta+4)}-\frac{a\alpha\beta T^{\beta-1}M}{\beta+1}-\frac{bM}{2}-\frac{(\beta+1)\alpha bT^{\beta}M}{\beta+2}-\frac{2cTM}{3}$$
$$-\frac{(\beta+2)c\alpha T^{\beta+1}M}{\beta+3}+\frac{b\alpha M^{\beta+1}}{2}+\frac{2c\alpha M^{\beta+1}T}{3}-\frac{aM^2}{2T^2}-\frac{a\alpha M^{\beta+2}}{T^2(\beta+1)(\beta+2)}-\frac{bM^3}{6T^2}$$
$$-\frac{b\alpha M^{\beta+3}}{T^2(\beta+2)(\beta+3)}-\frac{cM^4}{12T^2}-\frac{c\alpha M^{\beta+4}}{(\beta+3)(\beta+4)T^2}+\frac{a\alpha M^{\beta+2}}{T^2\beta+2}+\frac{b\alpha M^{\beta+3}}{2(\beta+3)T^2}-\frac{c\alpha M^{\beta+4}}{3(\beta+4)T^2}\Bigg]$$
$$-\frac{PI_e}{T^2}\left[\frac{aM^2}{2}+\frac{bM^3}{3}+\frac{cM^4}{4}\right]=0 \qquad (17)$$

Similarly $TC_2(T)$ will be minimum, the optimum values of $T$ for the minimum average total cost $TC_2(T)$ is the solution of equation

$$\frac{dTC_2(T)}{dT}=0 \qquad (18)$$

Provided

$$\frac{d^2TC_2(T)}{dT^2}>0 \qquad (19)$$

From equation (18) we get

$$-\frac{A}{T^2}+C\left[\frac{a\alpha\beta T^{\beta-1}}{\beta+1}+\frac{(\beta+1)b\alpha T^{\beta}}{\beta+2}\frac{(\beta+2)c\alpha T^{\beta+1}}{\beta+3}\right]+h\Bigg[\frac{a}{2}+\frac{2bT}{3}+\frac{3cT^2}{4}+\alpha bT^{\beta+1}$$
$$+\alpha cT^{\beta+2}-\frac{(\beta+2)\alpha bT^{\beta+1}}{2}-\frac{(\beta+3)c\alpha T^{\beta+2}}{3}-\frac{a\alpha T^{\beta}}{\beta+2}-\frac{b\alpha T^{\beta+1}}{\beta+3}-\frac{c\alpha T^{\beta+2}}{\beta+4}+\frac{(\beta+2)a\alpha T^{\beta}}{\beta+2}$$
$$+\frac{(\beta+2)b\alpha T^{\beta+1}}{2(\beta+3)}-\frac{(\beta+3)c\alpha T^{\beta+2}}{3(\beta+4)}\Bigg]+PI_e\left[bM+2cTM-\frac{a}{2}+\frac{4bT}{3}-\frac{9cT^4}{4}\right]=0 \quad (20)$$

## 4. Numerical Examples

**Example-1:** Let $a=100$ units/year, $b=200$, $c=400$, $A=\$30$ per order, $C=\$20$/unit, $P=\$40$/unit, $h=\$6$/unit /annum, $I_c=\$0.12$/year, $I_e=\$0.09$/year,

$M = 30/365$ years $\alpha = 0.04$/annum and $\beta = 0.04$ in appropriate units. By the help of Mathematica 5.1 software, we obtain the optimum solution for $T$ of Equation (17) of case-I, as $T^* = 0.194007$ year which is greater than $M = 0.082$ year. Putting $T^*$ in (9) and (3) we get the optimum average cost and ordering quantity as $TC_1(T)^* = 286.219$ and $Q^* = 24.5088$ respectively.

**Example-2:** Let $a = 100$ units/year, $b = 200$, $c = 400$, $A = \$30$ per order, $C = \$20$ /unit, $P = \$40$ /unit, $h = \$30$ /unit/annum $I_e = \$0.09$ /year, $M = 90/365$ years $\alpha = 0.04$/annum and $\beta = 0.04$ in appropriate units. By the help of Mathematica 5.1 software, we obtain the optimum solution for $T$ of Equation (20) of case-II, as $T^* = 0.117871$ year which is less than $M = 0.246$ year. Putting $T^*$ in (12) and (3) we get the optimum average cost and ordering quantity as $TC_2(T)^* = 577.169$ and $Q^* = 13.561$ respectively.

## 5. Sensitivity Analysis

We have performed sensitivity analysis by changing parameters $a$, $b$, $c$, $\alpha$, $\beta$ and $M$ as 20%, 50%, - 20% and –50% and keeping the remaining parameters at their original values. The corresponding changes in the cycle time, purchase quantities and the total cost are exhibited in table-1 and table-2.

**Table-1: Sensitivity analysis for Case -I** ($M < T$)

| Parameters | % Change | $T^*$ | $Q^*$ | $TC_1(T)^*$ |
|---|---|---|---|---|
| $a$ | +20 | 0.186576 | 27.1246 | 306.241 |
| | +50 | 0.176765 | 30.8196 | 335.108 |
| | -20 | 0.202257 | 21.711 | 265.483 |
| | -50 | 0.216384 | 17.1303 | 232.832 |
| $b$ | +20 | 0.189700 | 24.5679 | 291.733 |
| | +50 | 0.183913 | 24.662 | 299.535 |
| | -20 | 0.19875 | 24.454 | 280.415 |
| | -50 | 0.206869 | 24.3819 | 271.069 |
| $c$ | +20 | 0.19191 | 24.3722 | 287.165 |
| | +50 | 0.189008 | 24.1866 | 288.558 |
| | -20 | 0.196255 | 24.6575 | 285.258 |
| | -50 | 0.199959 | 24.9072 | 283.792 |
| $\alpha$ | +20 | 0.19163 | 24.2091 | 293.92 |
| | +50 | 0.188195 | 23.7784 | 305.346 |
| | -20 | 0.196457 | 24.819 | 278.448 |
| | -50 | 0.200276 | 25.3052 | 266.655 |
| $\beta$ | +20 | 0.194721 | 24.5593 | 279.894 |
| | +50 | 0.195921 | 24.6745 | 272.388 |
| | -20 | 0.193435 | 24.4936 | 290.90 |
| | -50 | 0.193053 | 24.5781 | 308.734 |
| $M$ | +20 | 0.196547 | 24.9089 | 286.138 |
| | +50 | 0.20091 | 25.6018 | 286.149 |
| | -20 | 0.19179 | 24.1615 | 286.345 |
| | -50 | 0.189115 | 23.7449 | 286.567 |

**Table-2: Sensitivity analysis for Case -II** ( $M > T$ )

| Parameters | % Change | $T^*$ | $Q^*$ | $TC_2(T)^*$ |
|---|---|---|---|---|
| $a$ | +20 | 0.111548 | 14.9941 | 630.773 |
| | +50 | 0.103535 | 16.9458 | 706.746 |
| | -20 | 0.125187 | 11.9951 | 522.639 |
| | -50 | 0.138543 | 9.32559 | 436.065 |
| $b$ | +20 | 0.115225 | 13.4838 | 586.393 |
| | +50 | 0.111652 | 13.3821 | 599.219 |
| | -20 | 0.120772 | 13.6473 | 568.271 |
| | -50 | 0.125706 | 13.7978 | 523.763 |
| $c$ | +20 | 0.117055 | 13.4973 | 579.190 |
| | +50 | 0.115886 | 13.4062 | 581.688 |
| | -20 | 0.118721 | 13.6274 | 575.763 |
| | -50 | 0.120061 | 13.7322 | 573.112 |
| $\alpha$ | +20 | 0.117201 | 13.506 | 583.236 |
| | +50 | 0.116231 | 13.4273 | 591.819 |
| | -20 | 0.11855 | 13.6168 | 571.728 |
| | -50 | 0.119586 | 13.7017 | 563.048 |
| $\beta$ | +20 | 0.118137 | 13.5626 | 591.805 |
| | +50 | 0.118573 | 13.5821 | 565.378 |
| | -20 | 0.117659 | 13.5752 | 584.671 |
| | -50 | 0.117545 | 13.6461 | 599.232 |
| $M$ | +20 | 0.116639 | 13.3994 | 577.525 |
| | +50 | 0.114866 | 13.1677 | 577.688 |
| | -20 | 0.119147 | 13.7288 | 577.522 |
| | -50 | 0.121147 | 13.9928 | 577.703 |

From table-1, we observed that as parameters $a$ and $b$ increases, ordering quantity $Q^*$ and average total cost $TC_1(T)^*$ increases and increase in demand parameter $c$ decrease ordering quantity $Q^*$ while the average total cost $TC_1(T)^*$ of an inventory system increases. It is interesting to observe that increases in deterioration parameter $\alpha$ decrease ordering quantity $Q^*$ and increase total cost $TC_1(T)^*$ of an inventory system and also increases in deterioration parameter $\beta$ increase ordering quantity $Q^*$ and decrease total cost $TC_1(T)^*$ of an inventory system. Increase in delay period results in increase ordering quantity $Q^*$ and total cost of inventory system.

From table-2, we observed that as parameter $a$ increases, ordering quantity $Q^*$ and average total costs $TC_2(T)^*$ increase and increase in parameters $b$ and $c$ results in decreasing ordering quantity $Q^*$ while the average total costs $TC_2(T)^*$ of an inventory system increase. It is interesting to observe that increase in deterioration parameter $\alpha$ decrease ordering quantity $Q^*$ and increase total cost $TC_2(T)^*$ of an inventory system. Also increase in deterioration parameter $\beta$ increase ordering quantity $Q^*$ and decrease total cost $TC_2(T)^*$. Increase in delay period decrease ordering quantity $Q^*$ and increases total cost $TC_2(T)^*$ of inventory system.

## 6. Conclusion

The model developed in this paper assumes demand of a product to be quadratic with respect to time. Shortages are not allowed and replenishment rate is infinite. It is assumed that the retailer generates revenue on unit selling price which is necessarily higher than the unit purchase cost. The effect of delay period offered by the supplier to retailer is analyzed when the demand of the product is quadratic in nature. The units in inventory are assumed to be subject to time dependent Weibull deterioration. It is observed that increase in delay period results in increase in total inventory cost.

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