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On Non-Existence of Some (1,2)-Optimal Codes

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Abstract

This paper presents the non-existence of certain blockwise burst error correcting (1,2)-Optimal linear codes over GF(q) (q is a prime) where the code length n is divided into two sub-blocks of lengths n_1 and n_2 ; ($n = n_1 + n_2$). *An* ($n = n_1 + n_{2,}$ *k*) *linear code that corrects all bursts of length* $b_1=1$ *in the first sub-block of length* n_1 *and all bursts of length* $b_2=2$ *(fixed) in the second sub-block of length* n_2, *and no other error pattern, will be called as (1,2)-Optimal linear code*.

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1. INTRODUCTION

In many communication systems, the information is stored in various parts (sub-blocks) of the code length. Therefore we partition the code length into various sub-blocks in such a way that the pattern of errors in each sub-block is known. So, when we consider error correction in such a system, we correct errors which occur in the same sub-block. This results in fast and accurate error detection and correction. Hence there is need to study blockwise error correcting codes in detail. Blockwise burst error correcting (1,2)-Optimal linear codes were first introduced by Dass and Tyagi [2] in 1982. The parameters of such codes were obtained as a particular case of a bound given by the same authors [3] in 1980 on the number of parity check digits required for an ($n = n_1 + n_2$, k) linear code. The bound proved by Dass and Tyagi is as follows:

Theorem: The number of parity-check digits in an (n, k) linear code correcting all bursts of length b_1 (fixed) in the first sub-block of length n_1 and all bursts of length b_2 (fixed) in the second sub-block of length n_2 ($n = n_1 + n_2$), is at least

$$\log_q[1+\{(n_1-b_1+1)q^{b_1-1}+(n_2-b_2+1)q^{b_2-1}\}(q-1)].$$

The bound may also be put as follows:

$$q^{n-k} \geq 1+(n_1-b_1+1)(q-1)q^{b_1-1}+(n_2-b_2+1)(q-1)q^{b_2-1} \qquad (1.1)$$

The definition of burst used here is CTD-burst [4] according to which "*A burst of length b (fixed) is a vector whose all the non-zero components are confined to some b-consecutive positions, the first of which is non-zero and the number of its starting positions in an n-tuple is first n-b+1 positions*". This definition is a modification by Dass [4] on the definition of a burst due to Chien and Tang [12] and has been found very useful in error analysis experiments on telephone lines [7], in the study of convolutional codes where the code word is in the form of semi-infinite sequence and in communication channels where errors do not occur near the end of a vector.

Dass and Tyagi [2] considered (1.1) as equality, i.e.

$$q^{n-k} = 1+\left[(n_1-b_1+1)(q-1)q^{b_1-1}\right]+\left[(n_2-b_2+1)(q-1)\ q^{b_2-1}\right] \qquad (1.2)$$

and obtained a class of optimal codes for $b_1 = 1$ and $b_2 = 2$(fixed) for the binary case. These codes were named as (1,2)-Optimal codes and were obtained for two sub-blocks for total code length upto 50; ($n_1 + n_2 \leq 50$). The codes are optimal in a specific sense as they are capable of correcting all single errors in the first sub-block of length n_1 and all bursts of length 2 (fixed) in the second sub-block of length n_2 and no other error pattern.

2. NON-BINARY (1,2)-OPTIMAL CODES

Some possibilities for the existence of non-binary (1,2)-optimal codes have recently been explored by Buccimazza, Dass and Jain [1], Dass, Iembo and Jain [5], [6], Tyagi and Rana [15], [14], Rana [8], [9]. In fact, Tyagi and Rana [14] have also given a table for values of $q \leq 19$ that works as a ready reckoner to check if a particular ($n = n_1 + n_2$, k) optimal code exist or not for given values of the parameters.

Example: For q = 3, we find that (4+4, 5) code should exist (from Table in [14]). If we consider the following matrix

$$H = \begin{pmatrix} 0 & 0 & 1 & 1 & & 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & 1 & & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 2 & & 1 & 2 & 0 & 1 \end{pmatrix}$$

as the parity check matrix for the desired code, then it can be verified that the code which is the null space of H corrects all single errors in the first sub-block of length 4 and all burst of lengths 2 (fixed) in the second sub-block of length 4 and no other error. Thus (4+4, 5) code is (1, 2)-Optimal.

After closely observing the table (in [14]) for different values of q, it was found that in many cases (1,2)-Optimal codes do not exist even though the values of the parameters n_1, n_2 and k satisfy equation (1.2). These values of the parameters are listed below in Table-1.

Table 1

Values of n_1, n_2 (≥ 5) and k for n – k = 3 and possible codes

q	n_1	n_2	k	Possible Codes
3	1	5	3	(1 + 5, 3)
5	1	7	5	(1 + 7, 5)
	6	6	9	(6 + 6, 9)
	11	5	13	(11 + 5, 13)
7	1	9	7	(1 + 9, 7)
	8	8	13	(8 + 8, 13)
	15	7	19	(15 + 7, 19)
	22	6	25	(22 + 6, 25)
	29	5	31	(29 + 5, 31)
11	1	13	11	(1 + 13, 11)
	12	12	21	(12 + 12, 21)
	23	11	31	(23 + 11, 31)
	34	10	41	(34 + 10, 41)
	45	9	51	(45 + 9, 51)
	56	8	61	(56 + 8, 61)
	67	7	71	(67 + 7, 71)
	78	6	81	(78 + 6, 81)
	89	5	91	(89 + 5, 91)
13	1	15	13	(1 + 15, 13)
	14	14	25	(14 + 14, 25)
	27	13	37	(27 + 13, 37)
	40	12	49	(40 + 12, 49)
	53	11	61	(53 + 11, 61)
	66	10	73	(66 + 10, 73)
	79	9	85	(79 + 9, 85)
	92	8	97	(92 + 8, 97)
	105	7	109	(105 + 7, 109)
	118	6	121	(118 + 6, 121)
	131	5	133	(131 + 5, 133)
⋮				
q	1	q+2	q	(q+3, q)
	q + 1	q+1	2q–1	(2q + 2, 2q –1)
	2q + 1	q	3q – 2	(3q + 1, 3q –2)
	3q + 1	q–1	4q – 3	(4q, 4q – 3)
	...	...	...	...
	$q^2 - 4q + 1$	6	$q^2 - 4q + 4$	$(q^2 - 4q + 7, q^2 - 4q + 4)$
	$\mathbf{q^2 - 3q + 1}$	**5**	$\mathbf{q^2 - 3q + 3}$	$\mathbf{((q^2 - 3q + 1) + 5, q^2 - 3q + 3)}$

This situation arises mainly for codes when n_2 ≥ 5 and n – k = 3. In this paper, we prove that such (1,2)-Optimal codes do not exist.

3. NON-EXISTENCE OF (1,2)-OPTIMAL CODES

Proving non-existence of certain category of perfect codes has been an interesting exercise [13], [11], [10]. It may be noted that $\mathbf{((q^2-3q+1)+5,\ q^2-3q+3)}$ code over GF(q) exist theoretically (Table 2). However, we illustrate in our next theorem that it is not possible to form a parity check matrix for this code, so that it does not become (1,2)-Optimal code.

Theorem: (1,2)-Optimal $\mathbf{((q^2-3q+1)+5,\ q^2-3q+3)}$ code over GF(q), does not exist.

Proof: We prove this theorem by assuming that such codes exist. So, let us suppose that H denotes the parity check matrix for $\mathbf{((q^2-3q+1)+5,\ q^2-3q+3))}$ code. The matrix H is formed by first constructing 2^{nd} sub-block. Once 2^{nd} sub-block is constructed, we will be left with exactly (q^2-3q+1) number of 3-tuples (non-zero), which can be placed as the columns in the first sub-block of H in any order. Therefore our main problem is to construct the second sub-block of H. Since n–k = 3, therefore, the most trivial way in which the first three columns of 2^{nd} sub-block can be constructed, is to take the q-ary representation of the numbers 1, q, q^2 i.e. (0 0 1), (0 1 0) and (1 0 0) columnwise.

Therefore, we get the matrix form of first three columns of 2^{nd} sub-block of H as

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Let us consider $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ as first three columns of second sub-block of H and let fourth column in the 2^{nd} sub-block of H be $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$. Fifth column can be taken as one of the first three columns. Let this be (1 0 0). Then the complete 2^{nd} sub-block of H becomes:

$$\begin{pmatrix} 1 & 0 & 0 & x & 1 \\ 0 & 1 & 0 & y & 0 \\ \underbrace{0 \quad 0 \quad 1 \quad z \quad 0}_{\text{2nd Sub-block}} \end{pmatrix}$$

Suppose S denote the set of all the non-zero 3-tuples over GF(q). Then

$$\mathbf{S = GF(q)^3 \sim \{(0, 0, 0)\}.}$$

$\therefore \quad |\,S\,| = (q^3-1).$

Let A denote the set of all the syndromes corresponding to the error pattern of first three columns (i.e. 1^{st}-2^{nd}, 2^{nd}-3^{rd}, including the vectors (0 0 1), (0 0 2),...,(0 0 q−1)) of second sub-block in H. Then

$$A = \{(0, 0, 0) \neq (a, b, c) \in GF(q)^3 \mid a = 0 \text{ or } c = 0\}$$

Also, if B = S ~ A, then

$$B = \{(a, b, c) \in GF(q)^3 \mid a \neq 0 \text{ and } c \neq 0\}$$

Therefore $|A| = (q-1)(2q+1)$ and $|B| = q(q-1)^2$.

Clearly $S = A \cup B$ and $A \cap B = \phi$

So that $|S| = |A| + |B|$.

Then for H to be the parity check matrix of **$((q^2 - 3q + 1) + 5, q^2 - 3q + 3))$** code, each of the (q − 1) elements in the set C given by

$$C = \{(ax, ay, az + 1) \mid 0 \neq a \in GF(q)\}$$

must not belong to A.

But there is an element $\left(\frac{q-1}{z}x, \frac{q-1}{z}y, 0\right)$ of C which belongs to A,

($\because$ for each $0 \neq z \in GF(q)$, $\frac{q-1}{z}z + 1 = 0$ in GF(q))

which is a contradiction to the above assumption.

Therefore it is not possible to add the fourth column in the second sub-block of H and hence we are unable to find the second sub-block in H which may yield different syndromes in the respective error pattern-syndrome table. Hence the result.

4. CONCLUSION AND REMARKS

In this paper, we have shown the non-existence of (1,2)-Optimal $(n_1 + n_2, k)$ codes over GF(q) for $n - k = 3$ and $5 \leq n_2 \leq q + 2$.

However, the problem needs further investigation to **find the possibility of the non-existence of (1,2)-Optimal $(n = n_1 + n_2, k)$ codes over GF(q), for $n - k = 4$ and $m \leq n_2 \leq q(q + 1) + 2$, where**

$$\mathbf{m} = \begin{cases} \mathbf{12} & \textbf{if } \mathbf{q = 3} \\ \mathbf{16} & \textbf{if } \mathbf{q = 5} \\ \mathbf{28} & \textbf{if } \mathbf{q = 7} \\ \dots & \dots \quad \dots \end{cases}$$

and for higher values of n − k.

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