Applied Mathematical Sciences, Vol. 4, 2010, no. 44, 2201 - 2205

A Note on Non-Existence of (8,4)-2 Burst Error Correcting Code

Navneet Singh Rana

Research Scholar, Department of Mathematics
University of Delhi, Delhi-110007, India
nsrana13@hotmail.com

Abstract

This paper presents the non-existence of (8,4)-2 burst error correcting perfect code over GF(2). *An (n,k) linear code is said to be perfect if for some positive integer b, it corrects all b and fewer errors and no more.*

Mathematics Subject Classification: 94B20

Keywords: Perfect codes, Burst-error, Parity-check matrix, Hamming weight

1. INTRODUCTION

Fire (1959) gave the lower bound on the number of parity check digits required in a linear code over GF(q) that corrects all bursts of length b or less as follows:

$$\mathbf{q^{n-k} \geq q^{b-1}[(q-1)(n-b+1)+1].} \qquad \textbf{(1.1)}$$

A code will be b-burst error correcting perfect, if it corrects all the burst errors of length b or less and satisfies the equality

$$\mathbf{q^{n-k} = q^{b-1}[(q-1)(n-b+1)+1].} \qquad \textbf{(1.2)}$$

The problem of non-existence of linear as well as perfect codes has been a good exercise for mathematicians for the last several years. The Non-Existence of Certain Binary Linear (n, k)-Codes with different values of the parameters n and k satisfying equation (1.2) was given by Hill and Traynor [2]. Also, the non-existence of ternary linear codes has been given by Eupen [4], Dasakalov and Metodia [8]. However, the non-existence of (n, k)-perfect codes have been proved in a series of papers by Eupen [5], Hamada and Watamori [6] and Landjev [3]. The non-existence of Ternary perfect codes were obtained in [7] and also it has been mentioned by Tuvi Etzion [9], that (8,4)-2 burst correcting perfect code does not exist.

In this small note, we prove by using parity check construction technique, that (8,4)-2 burst error correcting perfect code does not exist.

2. NON-EXISTENCE OF (8,4)-2 BURST ERROR CORRECTING CODE OVER GF(2)

To explore the possibility of existence of burst error correcting perfect codes, it is evident that equation (1.2) must have integral solutions for q, n, k and b. Though (8,4)-Code satisfies the equation (1.2) for b=2 and q=2, however, we illustrate in our next result that it is not possible to form a parity check matrix for this code.

Result: (8,4)-2 burst error correcting code over GF(2) does not exist.

Proof: We prove this theorem by assuming that such codes exists. So let us suppose that H denotes the parity check matrix for **(8, 4)-2 burst error-correcting code.**

Since $n - k = 4$, therefore, the most trivial way in which the first four columns of H can be constructed, is to take the binary representation of the numbers 1, 2, 2^2, 2^3 i.e. (0 0 0 1), (0 0 1 0), (0 1 0 0) and (1 0 0 0) columnwise.

Suppose we take H as

$$H = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 1 & 0 & x \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & y \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & z \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 & w \end{pmatrix}.$$

Let S denote the set of all the non-zero 4-tuples over GF(2). Then

$$S = GF(2)^4 \sim \{(0, 0, 0, 0)\}$$

$$\therefore \ |S| = 2^4 - 1 = 15$$

Let A denote the set of all the syndromes corresponding to the error pattern of first seven columns (i.e. 1^{st}-2^{nd}, 2^{nd}-3^{rd}, 3^{rd}-4^{th}, 4^{th}-5^{th}, 5^{th}-6^{th} and 6^{th}-7^{th}, including the vector (0 1 1 1)) of H. Then

$$A = \begin{Bmatrix} (0\ 0\ 0\ 1), & (0\ 0\ 1\ 0), & (0\ 0\ 1\ 1), & (0\ 1\ 0\ 0), \\ (0\ 1\ 0\ 1), & (0\ 1\ 1\ 0), & (0\ 1\ 1\ 1), & (1\ 0\ 0\ 0), \\ (1\ 0\ 0\ 1), & (1\ 0\ 1\ 1), & (1\ 1\ 0\ 0), & (1\ 1\ 0\ 1), \\ (1\ 1\ 1\ 0) & & & \end{Bmatrix}$$

Also if $B = S \sim A$, then

$$B = \{(1\ 0\ 1\ 0), \quad (1\ 1\ 1\ 1)\}$$

Therefore $|A| = 13$ and $|B| = 2$

Clearly $S = A \cup B$ and $A \cap B = \phi$

So that $|S| = |A| + |B|$.

Then for H to be the parity check matrix of (8,4)-2 burst error correcting code, each of the two elements in the set C (set of all the syndromes corresponding to 7^{th} and 8^{th} columns of H including the vector (x y z w)) given by

$$C = \{(x\ \ y\ \ z\ \ w), (x\ \ y+1\ \ z+1\ \ w+1)\},$$

must not belong to A. So in order to prove that the above code exist, we must have

$$C \subset B \qquad (\because S = A \cup B \text{ and } A \cap B = \phi)$$

But if we take the element (x y + 1 z + 1 w + 1) of C in B, then we always arrive at a contradiction. For example, if we take (x y + 1 z + 1 w +1) in B as (1 0 1 0), then we have

$(x\ \ y+1\ \ z+1\ \ w+1) = (1\ \ 0\ \ 1\ \ 0)$

$\Rightarrow \quad x = 1,\ y = 1,\ z = 0,\ w = 1$

$\Rightarrow \quad (x\ \ y\ \ z\ \ w) = (1\ \ 1\ \ 0\ \ 1)$

$\quad (x\ \ y\ \ z\ \ w) \in A$

This is a contradiction to our assumption that $(x\ \ y\ \ z\ \ w) \notin A$.

Similarly if we take (x y+1 z+1 w+1) in B as (1 1 1 1), then again we get a contradiction.

Therefore it is not possible to add the eighth column in H and hence we are unable to find H which may yield different syndromes in the respective error-pattern syndrome table. Hence the result.

3. OBSERVATIONS

By considering equation (1.2) i.e.

$$q^{n-k} = q^{b-1}[(q-1)(n-b+1)+1],$$

we get

$$n = \frac{q^{(n-k)-(b-1)} + (q-1)b - q}{(q-1)} \qquad (3)$$

For determining n, the code length, and k, the number of information digits for a code over GF(q), we assign different values to b as follows:

(i) b = 1: Equation (3) reduces to

$$n = \frac{q^{(n-k)} - 1}{q-1} \qquad (3.1)$$

It is well known that this equation has infinitely many solutions and corresponding to every solution there is a single error correcting perfect code viz. Hamming code.

(ii) b = 2 : Equation (3) reduces to

$$n = \frac{q^{(n-k)-1} + (q-2)}{(q-1)} \tag{3.2}$$

Clearly equation (3.2) has infinitely many integral solutions, which shows the possibility of the existence of 2-burst error correcting perfect codes.

(iii) b = 3: Equation(3) reduces to

$$n = \frac{q^{(n-k)-2} + (2q-3)}{(q-1)} \tag{3.3}$$

which leads to the similar assertion as in the above two cases, leading to the possibility of the existence of 3-burst error correcting perfect codes.

Continuing in this manner it is evident that for higher values of b viz. b = 4,5,... equation (3) has infinitely many solutions.

Based on this study it may be observed that there does exist a class of perfect codes for a fixed value of b. Such codes correct only b-adjacent errors for fixed b and have been studied by Sharma & Dass [1].

4. CONCLUSION AND REMARKS

We have investigated the solutions of equation (1.2) for b = 1, 2, 3, ... (Section 3) and it is hoped that equation (1.2) shall have solutions for higher values of b also, leading to the possibilities of the existence of perfect codes.

Acknowledgement

The author is thankful to Dr. V.K. Tyagi (Department of Mathematics, Shyam Lal College (E), University of Delhi), for his suggestions and illuminating discussions on this paper.

REFERENCES

1. B.D. Sharma and B.K. Dass, 'Adjacent-Error Correcting Binary Perfect Codes, Journal of Cybernetics', 7 (1977), 9-13.
2. H. Raymond and T. Karen, ‘The Non-existence of Certain Binary Linear Codes’; IEEE Transactions on Information Theory, 36(4) (1990), 917-922.
3. I. Landjev, ‘The Non-existence of Some Optimal Ternary Codes of Dimension Five'; Designs Codes Cryptography, 15 (1998), 245-258.

4. M.V. Eupen, 'Four Non-Existence Results for Ternary Linear Codes'; IEEE Transactions on Information Theory, 41(3) (1995), 800-805.

5. M.V. Eupen, 'Some New Results for Ternary Linear Codes of Dimension 5 and 6'; IEEE Transactions on Information Theory, 41 (1995), 2048-2051.

6. N. Hamada and Y. Watamori, 'The Non-existence of $[71, 5, 46]_3$ Codes'; J. Statistical Planning Inference. 52 (1996), 379-394.

7. R. Dasakalov and E. Metodia, 'The Non-existence of Ternary [105, 6, 68] and [230, 6, 152] codes'; Discrete Mathematics, 286 (2004), 225-232.

8. R. Dasakalov and E. Metodia, 'The Non-existence of Ternary [284, 6, 188] Codes'; Problems of Information Transmission 40(2) (2004), 135-146.

9. T. Etzion, 'Constructions for Perfect 2-Burst Correcting Codes'; IEEE Transactions on Information Theory, 47(6) (2001), 2553-2555.

Received: November, 2009