Applied Mathematical Sciences, Vol. 4, 2010, no. 47, 2341 - 2354

Estimating the Population Mean Using Stratified Median Ranked Set Sampling

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Abstract

In this paper, stratified median ranked set sampling (SMRSS) method is suggested for estimating the population mean. The SMRSS is compared with simple random sampling (SRS), stratified simple random sampling (SSRS) and stratified ranked set sampling (SRSS). It is shown that SMRSS estimator is an unbiased of the population mean of symmetric distributions and is more efficient than its counterparts using SRS, SSRS and SRSS.

Keywords: Simple random sampling; ranked set sampling; median ranked set sampling; efficiency; stratified

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1. Introduction

Ranked set sampling method was proposed by McIntyre (1952) to estimate mean pasture yields. Recently, RSS was developed and modified by many authors to estimate the population parameters. Dell and Clutter (1972) showed that the mean of the RSS is an unbiased estimator of the population mean, whether or not there are errors in ranking. Muttlak (1997) suggested using median ranked set sampling (MRSS) to estimate the population mean. Al-Saleh and Al-Omari (2002) suggested multistage ranked set sampling (MSRSS) in order to increase the efficiency of estimating the population mean for specific value of the sample size. Jemain and Al-Omari (2006) suggested double quartile ranked set sampling (DQRSS) for estimating the population mean. Jemain and Al-Omari (2007) have extended DQRSS to multistage quartile ranked set sampling (MQRSS) for estimating the population mean and they showed that the efficiency of the mean estimator using MQRSS can be increased for specific value of the sample size *m* by increase the number of stages. Also see Al-Omari and Jaber (2008), Bouza (2002), Al-Nasser (2007) and Ohyama et al. (2008) for further information on ranked set sampling.

In this paper, stratified median ranked set sampling (SMRSS) is suggested for estimating the population mean. The paper is organized as follows: In Section 2 we describe several sampling methods considered in this study. Estimation of the population mean is given in Section 3. In Section 4, a simulation study is conducted to investigate the performance of the suggested methods. Finally, conclusions are given in Section 5.

2. Sampling methods

2.1: Simple Random Sampling

Simple random sampling (SRS) is a method of selecting n units out of N units such that every one of the ${}_NC_n$ distinct samples has an equal chance of being drawn. In practice, a simple random sample is drawn unit by unit.

2.2: Stratified Simple Random Sampling

In stratified sampling the population of N units is first divided into L subpopulations of $N_1, N_2, \cdots, N_L$ units. These subpopulations, or also known as strata, are not overlapping and when combined together they form the whole population, i.e. $N_1 + N_2 + \cdots + N_L = N$. To obtain the full benefit from stratification, the values of the N_h, $h = 1, 2, \cdots, L$ must be known. After the strata have been determined, a sample is drawn from each stratum. The sample sizes within the strata are denoted by $n_1, n_2, \cdots, n_L$, respectively and $n = \sum_{h=1}^{L} n_h$. If a simple random sample is taken in each stratum, the whole procedure is described as stratified simple random sampling (SSRS).

2.3: Ranked Set Sampling

The ranked set sampling (RSS), as suggested by McIntyre (1952), is conducted by selecting *n* random samples from the population of size *n* units each, and ranking each unit within each set with respect to the variable of interest. Then an actual measurement is taken of the unit with the smallest rank from the first sample. From the second sample an actual measurement is taken from the second smallest rank, and the procedure is continued until the unit with the largest rank is chosen for actual measurement from the *n*-th sample. Thus we obtain a total of *n* measured units, one from each ordered sample of size *n* and this completed one cycle. The cycle may be repeated m times until *nm* units have been measured

2.4: Median Ranked Set Sampling:

The MRSS procedure as proposed by Muttlak (1997) can be formed by selecting *n* random samples of size *n* units from the population and rank the units within each sample with respect to a variable of interest. If the sample size *n* is odd, then from each sample select for the measurement the $\left(\frac{n+1}{2}\right)th$ smallest ranked unit, i.e., the median of the sample. If the sample size *n* is even, then select for the measurement from the first $\frac{n}{2}$ samples the $\left(\frac{n}{2}\right)th$ smallest ranked unit and from the second $\frac{n}{2}$ samples the $\left(\frac{n}{2}+1\right)th$ smallest ranked. The cycle can be repeated *m* times if needed to get a sample of size *nm* units.

2.5 Stratified Median Ranked Set Sampling:

If the MRSS is performed in each stratum instead off SRSS described in Section 2.2, the method is known as stratified median ranked set sampling. To illustrate the method, let us consider the following two cases, if the subpopulations involve odd number of elements in each set, and the second example if the subpopulations involve even number of elements in each set. Note that the number of subpopulations (strata) is immaterial, either odd or even.

Example:

1) Assume that the sample size is odd and we have two strata, i.e., $L=2,\ h=1,2$.
Let $X_{ih(\frac{n_h+1}{2})}$ be the median of the *i*th sample in the *h*th stratum.

<u>Stratum (1):</u> Five samples each of size 5 are obtained and ranked as follows:

$$X_{11(1)}, X_{11(2)}, \boxed{X_{11(3)}}, X_{11(4)}, X_{11(5)}$$
$$X_{21(1)}, X_{21(2)}, \boxed{X_{21(3)}}, X_{21(4)}, X_{21(5)}$$

$$X_{31(1)}, X_{31(2)}, \boxed{X_{31(3)}}, X_{31(4)}, X_{31(5)}$$
$$X_{41(1)}, X_{41(2)}, \boxed{X_{41(3)}}, X_{41(4)}, X_{41(5)}$$
$$X_{51(1)}, X_{51(2)}, \boxed{X_{51(3)},} X_{51(4)}, X_{51(5)}$$

For h=1, select $X_{i1(\frac{n_1+1}{2})} = X_{i1(3)}$ for $i = 1,\cdots,5$. Thus the following units are obtained from the first stratum:

$$X_{11(3)}, X_{21(3)}, X_{31(3)}, X_{41(3)}, X_{51(3)}$$

Stratum (2): Seven samples, each of 7 units, are selected and measured as follows:

$$X_{12(1)}, X_{12(2)}, X_{12(3)}, \boxed{X_{12(4)}}, X_{12(5)}, X_{12(6)}, X_{12(7)}$$
$$X_{22(1)}, X_{22(2)}, X_{22(3)}, \boxed{X_{22(4)}}, X_{22(5)}, X_{22(6)}, X_{22(7)}$$
$$X_{32(1)}, X_{32(2)}, X_{32(3)}, \boxed{X_{32(4)}}, X_{32(5)}, X_{32(6)}, X_{32(7)}$$
$$X_{42(1)}, X_{42(2)}, X_{42(3)}, \boxed{X_{42(4)}}, X_{42(5)}, X_{42(6)}, X_{42(7)}$$
$$X_{52(1)}, X_{52(2)}, X_{52(3)}, \boxed{X_{52(4)}}, X_{52(5)}, X_{52(6)}, X_{52(7)}$$
$$X_{62(1)}, X_{62(2)}, X_{62(3)}, \boxed{X_{62(4)}}, X_{62(5)}, X_{62(6)}, X_{62(7)}$$
$$X_{72(1)}, X_{72(2)}, X_{72(3)}, \boxed{X_{72(4)}}, X_{72(5)}, X_{72(6)}, X_{72(7)}$$

Therefore, the SMRSS units consist of the selected units in the first and second stratums, given by

$$X_{11(3)}, X_{21(3)}, X_{31(3)}, X_{41(3)}, X_{51(3)}, X_{12(4)}, X_{22(4)}, X_{32(4)}, X_{42(4)}, X_{52(4)}, X_{62(4)}, X_{72(4)}$$

2) Assume that the sample size is even and we have two strata, i.e., $L = 2, h = 1, 2$. Let $X_{ih(\frac{n_h}{2})}$ and $X_{ih(\frac{n_h+2}{2})}$ be the $\left(\frac{n}{2}\right)th$ and $\left(\frac{n+2}{2}\right)th$ ranked units of the ith sample in the hth stratum, respectively.

Stratum (1): Six samples, each of 6 units, are ranked as given below:

$$X_{11(1)}, X_{11(2)}, \boxed{X_{11(3)}}, X_{11(4)}, X_{11(5)}, X_{11(6)}$$
$$X_{21(1)}, X_{21(2)}, \boxed{X_{21(3)}}, X_{21(4)}, X_{21(5)}, X_{21(6)}$$
$$X_{31(1)}, X_{31(2)}, \boxed{X_{31(3)}}, X_{31(4)}, X_{31(5)}, X_{31(6)}$$

$$X_{41(1)}, X_{41(2)}, X_{41(3)}, \boxed{X_{41(4)}}, X_{41(5)}, X_{41(6)}$$

$$X_{51(1)}, X_{51(2)}, X_{51(3)}, \boxed{X_{51(4)}}, X_{51(5)}, X_{51(6)}$$

$$X_{61(1)}, X_{61(2)}, X_{61(3)}, \boxed{X_{61(4)}}, X_{61(5)}, X_{61(6)}$$

Stratum (2):

Eight samples, each with 8 ranked units, are as given below:

$$X_{12(1)}, X_{12(2)}, X_{12(3)}, \boxed{X_{12(4)}}, X_{12(5)}, X_{12(6)}, X_{12(7)}, X_{12(8)}$$

$$X_{22(1)}, X_{22(2)}, X_{22(3)}, \boxed{X_{22(4)}}, X_{22(5)}, X_{22(6)}, X_{22(7)}, X_{22(8)}$$

$$X_{32(1)}, X_{32(2)}, X_{32(3)}, \boxed{X_{32(4)}}, X_{32(5)}, X_{32(6)}, X_{32(7)}, X_{32(8)}$$

$$X_{42(1)}, X_{42(2)}, X_{42(3)}, \boxed{X_{42(4)}}, X_{42(5)}, X_{42(6)}, X_{42(7)}, X_{42(8)}$$

$$X_{52(1)}, X_{52(2)}, X_{52(3)}, X_{52(4)}, \boxed{X_{52(5)}}, X_{52(6)}, X_{52(7)}, X_{52(8)}$$

$$X_{62(1)}, X_{62(2)}, X_{62(3)}, X_{62(4)}, \boxed{X_{62(5)}}, X_{62(6)}, X_{62(7)}, X_{62(8)}$$

$$X_{72(1)}, X_{72(2)}, X_{72(3)}, X_{72(4)}, \boxed{X_{72(5)}}, X_{72(6)}, X_{72(7)}, X_{72(8)}$$

$$X_{82(1)}, X_{82(2)}, X_{82(3)}, X_{82(4)}, \boxed{X_{82(5)}}, X_{82(6)}, X_{82(7)}, X_{82(8)}$$

Therefore, the SMRSS units consist of $X_{11(3)}, X_{21(3)}, X_{31(3)}, X_{41(4)}, X_{51(4)}, X_{61(4)}$, from the first stratum and $X_{12(4)}, X_{22(4)}, X_{32(4)}, X_{42(4)}, X_{52(5)}, X_{62(5)}, X_{72(5)}, X_{82(5)}$ from the second stratum.

3. Estimation of the population mean

Let $X_1, X_2, ..., X_n$ be a random sample of size *n* form a population of size *N* units, with pdf $f(x)$, mean μ and variance σ^2. The commonly simple random sample estimator of the population mean μ from a sample of size *n* is given by

$$\overline{X}_{SRS} = \frac{1}{n}\sum_{i=1}^{n} X_i ,$$

with variance

$$Var(\overline{X}_{SRS}) = \frac{\sigma^2}{n} .$$

The RSS estimator of the population mean is

$$\overline{X}_{RSS} = \frac{1}{n}\sum_{i=1}^{n} X_{i(i)} ,$$

with variance

$$Var(\overline{X}_{RSS}) = \frac{\sigma^2}{n} - \frac{1}{n^2}\sum_{i=1}^{n}\left(\mu_{(i)} - \mu\right)^2 ,$$

where $\mu_{(i)}$ is the mean of the ith order statistics, $X_{(i)}$ of a sample of size n.

When n_h is odd, the stratified median ranked set sampling (SMRSS) estimator of the population mean is given by

$$\overline{X}_{SMRSS1} = \sum_{h=1}^{L}\frac{W_h}{n_h}\left(\sum_{i=1}^{n_h} X_{ih((n_h+1)/2)}\right), \quad (1)$$

where $W_h = \frac{N_h}{N}$, N_h is the stratum size. The variance of SMRSS1 is given by

$$\begin{aligned} Var(\overline{X}_{SMRSS1}) &= Var\left[\sum_{h=1}^{L}\frac{W_h}{n_h}\left(\sum_{i=1}^{n_h} X_{ih((n_h+1)/2)}\right)\right] \\ &= \sum_{h=1}^{L}\frac{W_h^2}{n_h^2}\left(\sum_{i=1}^{n_h} Var(X_{ih((n_h+1)/2)})\right) \\ &= \sum_{h=1}^{L}\frac{W_h^2}{n_h^2}\left(\sum_{i=1}^{n_h} \sigma^2_{h(i:\frac{n_h+1}{2})}\right) \\ &= \sum_{h=1}^{L}\frac{W_h^2}{n_h}\sigma^2_{h(i:\frac{n_h+1}{2})} . \quad (2) \end{aligned}$$

When n_h is even, the estimator of the population mean using stratified median ranked set sampling is given by

$$\overline{X}_{SMRSS2} = \sum_{h=1}^{L}\frac{W_h}{n_h}\left(\sum_{i=1}^{\frac{n_h}{2}} X_{ih(n_h/2)} + \sum_{i=\frac{n_h}{2}+1}^{n_h} X_{ih((n_h+2)/2)}\right), \quad (3)$$

with variance given by

$$\begin{aligned} Var(\overline{X}_{SMRSS2}) &= Var\left[\sum_{h=1}^{L}\frac{W_h}{n_h}\left(\sum_{i=1}^{\frac{n_h}{2}} X_{ih(n_h/2)} + \sum_{i=\frac{n_h}{2}+1}^{n_h} X_{ih((n_h+2)/2)}\right)\right] \\ &= \sum_{h=1}^{L}\frac{W_h^2}{n_h^2}\left(\sum_{i=1}^{\frac{n_h}{2}} Var(X_{ih(n_h/2)}) + \sum_{i=\frac{n_h}{2}+1}^{n_h} Var(X_{ih((n_h+2)/2)})\right) \\ &= \sum_{h=1}^{L}\frac{W_h^2}{n_h^2}\left(\sum_{i=1}^{\frac{n_h}{2}} \sigma^2_{h(i:\frac{n_h}{2})} + \sum_{i=\frac{n_h}{2}+1}^{n_h} \sigma^2_{h(i:\frac{n_h}{2}+1)}\right). \quad (4) \end{aligned}$$

Lemma 1: If the distribution is symmetric about μ, then $E\left(\overline{X}_{SMRSS}\right) = \mu$.

Proof: If the sample size within the strata is odd, we have

$$E(\overline{X}_{SMRSS1}) = E\left[\sum_{h=1}^{L}\frac{W_h}{n_h}\left(\sum_{i=1}^{n_h} X_{ih((n_h+1)/2)}\right)\right]$$
$$= \sum_{h=1}^{L}\frac{W_h}{n_h}\left(\sum_{i=1}^{n_h} E(X_{ih((n_h+1)/2)})\right)$$
$$= \sum_{h=1}^{L}\frac{W_h}{n_h}\left(\sum_{i=1}^{n_h} \mu_{h(\frac{n_h+1}{2})}\right)$$

Since the distribution is symmetric about μ, then $\mu_{h(\frac{n_h+1}{2})} = \mu_h$. Therefore, we have

$$E(\overline{X}_{SMRSS1}) = \sum_{h=1}^{L}\frac{W_h}{n_h}\left(\sum_{i=1}^{n_h}\mu_h\right)$$
$$= \sum_{h=1}^{L}\frac{W_h}{n_h}(n_h\mu_h)$$
$$= \sum_{h=1}^{L} W_h \cdot \mu_h = \mu .$$

If n_h $(h = 1, 2, \cdots, L)$ are even, then

$$E\left(\overline{X}_{SMRSS2}\right) = E\left[\sum_{h=1}^{L}\frac{W_h}{n_h}\left(\sum_{i=1}^{\frac{n_h}{2}} X_{ih(n_h/2)} + \sum_{i=\frac{n_h}{2}+1}^{n_h} X_{ih((n_h+2)/2)}\right)\right]$$
$$= \sum_{h=1}^{L}\frac{W_h}{n_h}\left(\sum_{i=1}^{\frac{n_h}{2}} E(X_{ih(n_h/2)}) + \sum_{i=\frac{n_h}{2}+1}^{n_h} E(X_{ih((n_h+2)/2)})\right)$$
$$= \sum_{h=1}^{L}\frac{W_h}{n_h}\left(\sum_{i=1}^{\frac{n_h}{2}} \mu_{h(\frac{n_h}{2})} + \sum_{i=\frac{n_h}{2}+1}^{n_h} \mu_{h(\frac{n_h}{2}+1)}\right)$$

Since the distribution is symmetric about μ, then we have $\mu_{h(\frac{n_h}{2})} + \mu_{h(\frac{n_h}{2}+1)} = 2\mu_h$.

Therefore,

$$E\left(\overline{X}_{SMRSS2}\right) = \sum_{h=1}^{L}\frac{W_h}{n_h}\left[\frac{n_h}{2}\left(\mu_{h(\frac{n_h}{2})} + \mu_{h(\frac{n_h}{2}+1)}\right)\right]$$
$$= \sum_{h=1}^{L}\frac{W_h}{n_h}\left(\frac{n_h}{2}(2\mu_h)\right)$$
$$= \sum_{h=1}^{L} W_h \cdot \mu_h = \mu .$$

Lemma 2: If the distribution is symmetric about μ, then

$$Var\left(\overline{X}_{SMRSS1}\right) < Var\left(\overline{X}_{SRS}\right) \quad \text{and} \quad Var\left(\overline{X}_{SMRSS2}\right) < Var\left(\overline{X}_{SRS}\right)$$

Proof: If the sample size is odd, the variance of $\overline{X}_{SMRSS1}$ is given by

$$Var(\overline{X}_{SMRSS1}) = \sum_{h=1}^{L} \frac{W_h^2}{n_h} \sigma^2_{h(i:\frac{n_h+1}{2})}$$

But $\sigma^2_{h(i:\frac{n_h+1}{2})} < \sigma_h^2$ for each stratum $h = 1,2,\cdots,L$, which implies that

$$Var(\overline{X}_{SMRSS1}) = \sum_{h=1}^{L} \frac{W_h^2}{n_h} \sigma^2_{h(i:\frac{n_h+1}{2})} < \sum_{h=1}^{L} \frac{W_h^2}{n_h} \sigma_h^2 = Var(\overline{X}_{SSRS}) < Var\left(\overline{X}_{SRS}\right),$$

and the proof is the same for even sample size.

4. Simulation study

In this section, a simulation study is conducted to investigate the performance of SMRSS for estimating the population mean. Symmetric and asymmetric distributions have been considered for samples of sizes $n = 7,12,14,15,18.$ Using 100000 replications, estimates of the means, variances and mean square errors were computed. If the underlying distribution is symmetric, the efficiency of SMRSS relative to SRS, SSRS and SRSS, respectively are given by

$$eff\left(\overline{X}_{SMRSS}, \overline{X}_{SRS}\right) = \frac{Var\left(\overline{X}_{SRS}\right)}{Var\left(\overline{X}_{SMRSS}\right)}, \tag{5}$$

$$eff\left(\overline{X}_{SMRSS}, \overline{X}_{SSRS}\right) = \frac{Var\left(\overline{X}_{SSRS}\right)}{Var\left(\overline{X}_{SMRSS}\right)}, \tag{6}$$

and

$$eff\left(\overline{X}_{SMRSS}, \overline{X}_{SRSS}\right) = \frac{Var\left(\overline{X}_{SRSS}\right)}{Var\left(\overline{X}_{SMRSS}\right)}. \tag{7}$$

But if the distribution is asymmetric, the variances in the denominator above are replaced by MSE. Simulation results are summarized in Tables 1-6.

Table 1: The efficiency of SMRSS relative to SRSS, SSRS and SRS for $n = 14$ and samples sizes $n_1 = 8$ and $n_2 = 6$

	$eff\left(\overline{X}_{SMRSS}, \overline{X}_{SRSS}\right)$	$eff\left(\overline{X}_{SMRSS}, \overline{X}_{SSRS}\right)$	$eff\left(\overline{X}_{SMRSS}, \overline{X}_{SRS}\right)$
Uniform (0,1)	3.7143	3.9048	3.8571
Normal (0,1)	8.6176	7.1275	6.9902
Student T (3)	8.3784	9.9820	9.4414
Geometric (0.5)	4.0569	4.5358	4.4548
Exponential (1)	4.2400	6.0165	5.9008
Gamma (1,2)	3.4134	6.0186	5.8804
Beta (1,2)	3.5385	3.0769	3.0769
Beta (5,2)	4.2584	4.4049	4.1205
LogNormal (0,1)	6.0034	6.4732	6.9564
Weibull (1,2)	5.4074	5.7778	5.6667

Table 2: The efficiency of SMRSS relative to SRSS, SSRS and SRS for $n = 7$ and samples sizes $n_1 = 4$ and $n_2 = 3$

Distribution	$eff\left(\overline{X}_{SMRSS}, \overline{X}_{SRSS}\right)$	$eff\left(\overline{X}_{SMRSS}, \overline{X}_{SSRS}\right)$	$eff\left(\overline{X}_{SMRSS}, \overline{X}_{SRS}\right)$
Uniform (0,1)	2.1954	2.3908	2.3678
Normal (0,1)	3.2764	3.7571	3.6925
Student T (3)	4.7615	6.0976	5.8424
Geometric (0.5)	3.1237	3.0990	3.0437
Exponential (1)	2.7028	3.4202	3.3404
Gamma (1,2)	3.5801	3.1729	3.0573
Beta (1,2)	2.7273	2.8409	2.8182
Beta (5,2)	2.6154	2.8462	2.8462
LogNormal (0,1)	2.2697	3.0162	3.9329
Weibull (1,2)	1.8614	1.8855	1.8494

Table 3: The efficiency of SMRSS relative to SRSS, SSRS and SRS for $n = 12$ and samples sizes $n_1 = 5$ and $n_2 = 7$

Distribution	$eff\left(\overline{X}_{SMRSS}, \overline{X}_{SRSS}\right)$	$eff\left(\overline{X}_{SMRSS}, \overline{X}_{SSRS}\right)$	$eff\left(\overline{X}_{SMRSS}, \overline{X}_{SRS}\right)$
Uniform (0,1)	6.3333	5.9167	5.7500
Normal (0,1)	4.9645	6.0638	5.9078
Student T (3)	6.8450	4.5477	4.4378
Geometric (0.5)	2.5728	3.3050	3.2143
Exponential (1)	4.7614	4.3503	4.2183
Gamma (1,2)	4.5723	3.0393	2.9813
Beta (1,2)	3.5714	3.3571	3.2857
Beta (5,2)	3.2843	3.8029	3.6300
Log Normal (0,1)	5.3050	5.5106	5.8582
Weibull (1,2)	4.1455	3.3455	3.2545

Table 4: The efficiency of SMRSS relative to SRSS, SSRS and SRS for $n=18$ and samples sizes $n_1=4$, $n_2=6$ and $n_3=8$.

Distribution	$eff\left(\overline{X}_{SMRSS}, \overline{X}_{SRSS}\right)$	$eff\left(\overline{X}_{SMRSS}, \overline{X}_{SSRS}\right)$	$eff\left(\overline{X}_{SMRSS}, \overline{X}_{SRS}\right)$
Uniform (0,1)	2.3656	4.9000	4.6000
Normal (0,1)	5.2581	9.1875	8.6719
Student T (3)	3.0210	3.7574	3.5362
Geometric (0.5)	2.7917	3.6260	3.6797
Exponential (1)	4.7183	4.3333	4.7937
Gamma (1,2)	4.0745	4.5963	4.8199
Beta (1,2)	3.4244	3.6380	3.5826
Beta (5,2)	3.0967	3.3340	3.8507
LogNormal (0,1)	4.0728	5.5402	4.9272
Weibull (1,2)	3.4396	3.6316	3.2632

Table 5: The efficiency of SMRSS relative to SRSS, SSRS and SRS for $n=18$ and samples sizes $n_1=10$ and $n_2=8$.

Distribution	$eff\left(\overline{X}_{SMRSS}, \overline{X}_{SRSS}\right)$	$eff\left(\overline{X}_{SMRSS}, \overline{X}_{SSRS}\right)$	$eff\left(\overline{X}_{SMRSS}, \overline{X}_{SRS}\right)$
Uniform (0,1)	1.9202	4.6785	4.5577
Normal (0,1)	4.6017	6.1648	6.1099
Student T (3)	3.0127	3.2016	3.5168
Geometric (0.5)	2.7899	3.3097	3.2714
Exponential (1)	4.7160	4.2196	4.4216
Gamma (1,2)	3.8344	4.1800	2.2955
Beta (1,2)	3.2984	3.5011	3.3524
Beta (5,2)	3.0441	3.2961	3.2984
LogNormal (0,1)	3.9748	5.4730	4.5031
Weibull (1,2)	3.0000	3.5588	3.2000

Tables 1-6, in general, indicate that greater efficiency is attained using SMRSS method as opposed to the other contending methods that have been discussed when estimating the population mean of the variable of interest. To be more specific, based on the simulation results we can conclude that:

1. SMRSS is more efficient than SRSS based on the same number of measured units. For example, when $n=14$, the efficiency of SMRSS with respect to SRSS is 3.4134 for estimating the mean of the gamma distribution.

Table 6: The efficiency of SMRSS relative to SRSS, SSRS and SRS for $n = 15$ and samples sizes $n_1 = 3$, $n_2 = 5$ and $n_3 = 7$.

Distribution	$eff\left(\overline{X}_{SMRSS}, \overline{X}_{SRSS}\right)$	$eff\left(\overline{X}_{SMRSS}, \overline{X}_{SSRS}\right)$	$eff\left(\overline{X}_{SMRSS}, \overline{X}_{SRS}\right)$
Uniform (0,1)	4.4673	6.2034	5.7898
Normal (0,1)	3.1356	6.0763	5.6441
Student T (3)	5.6212	8.3276	7.3448
Geometric (0.5)	5.8190	6.5113	6.0452
Exponential (1)	4.5667	5.9011	5.2967
Gamma (1,2)	3.3541	3.7608	3.7656
Gamma (2,3)	4.5078	4.1630	4.9407
Beta (1,2)	3.1195	3.9500	3.5788
Beta (5,2)	3.9805	4.3135	4.9072
Log Normal (0,1)	4.3056	6.2432	5.7108
Weibull (1,2)	5.5543	5.8444	5.6412

2. When the performance of SMRSS are compared to either SRSS, SSRS or SRS, it is found that SMRSS is more efficient, as shown by all the values of relative efficiency which are greater than 1.
3. The efficiency of the suggested estimators is found to be more superior when the underlying distributions are symmetric as compared to asymmetric.
4. The relative efficiency of SMRSS estimators to those estimators based on SRS, SSRS and SRSS are increasing as the sample size increases. For example, if the underlying distribution is normal with mean 0 and variance 1, the efficiency of SMRSS estimators relative to SSRS for $n = 7$ and 12, are 3.7571 and 6.0638, respectively.
5. The efficiency is increasing as the number of strata is increasing. For example, when $n = 18$, the efficiency values of SMRSS w.r.t. SRSS are 5.2581 and 4.6017 with three and two strata, respectively for estimating the population mean of normal distribution.

In Tables 7-12 we summarized the bias values for different sampling methods considered in this study. Note that the estimators are unbiased when the distribution is symmetric.

Table 7: The bias values of SMRSS, SRSS, SSRS and SRS for $n = 14$ and samples sizes $n_1 = 8$ and $n_2 = 6$

Distribution	**SMRSS**	**SRSS**	**SSRS**	**SRS**
Geometric (0.5)	0.0127	0.0640	0.0727	0.0716
Exponential (1)	0.0356	0.0391	0.0726	0.0713
Gamma (1,2)	0.1437	0.2171	0.2910	0.2856
Beta (1,2)	0.0048	0.0062	0.0081	0.0079
Beta (5,2)	0.0336	0.0360	0.0371	0.0365
Log Normal (0,1)	0.0528	0.0992	0.1979	0.1949
Weibull (1,2)	0.0449	0.0475	0.0572	0.0562

Table 8: The bias values of SMRSS, SRSS, SSRS and SRS for $n = 7$ and samples sizes $n_1 = 4$ and $n_2 = 3$

Distribution	**SMRSS**	**SRSS**	**SSRS**	**SRS**
Geometric (0.5)	0.0041	0.0171	0.1453	0.1422
Exponential (1)	0.0607	0.0825	0.1455	0.1428
Gamma (1,2)	0.2202	0.2710	0.5798	0.5705
Beta (1,2)	0.0111	0.0112	0.0160	0.0159
Beta (5,2)	0.0723	0.0772	0.0743	0.0728
Log Normal (0,1)	0.1403	0.1957	0.3918	0.3885
Weibull (1,2)	0.0946	0.0970	0.1142	0.1120

Table 9: The bias values of SMRSS, SRSS, SSRS and SRS for $n = 12$ and samples sizes $n_1 = 5$ and $n_2 = 7$

Distribution	**SMRSS**	**SRSS**	**SSRS**	**SRS**
Geometric (0.5)	0.0144	0.0421	0.0852	0.0831
Exponential (1)	0.0285	0.0443	0.0852	0.0832
Gamma (1,2)	0.1253	0.2792	0.3403	0.3340
Beta (1,2)	0.0079	0.0085	0.0094	0.0093
Beta (5,2)	0.0401	0.0410	0.0436	0.0426
Log Normal (0,1)	0.0744	0.1467	0.2313	0.2259
Weibull (1,2)	0.0570	0.0594	0.0671	0.0654

Table 10: The bias values of SMRSS, SRSS, SSRS and SRS for $n = 18$ and samples sizes $n_1 = 4$, $n_2 = 6$ and $n_3 = 8$.

Distribution	**SMRSS**	**SRSS**	**SSRS**	**SRS**
Geometric (0.5)	0.0030	0.0067	0.0590	0.0555
Exponential (1)	0.0063	0.0079	0.0589	0.0553
Gamma (1,2)	0.0410	0.0855	0.2330	0.2223
Beta (1,2)	0.0016	0.0016	0.0068	0.0062
Beta (5,2)	0.0098	0.0102	0.0300	0.0284
Log Normal (0,1)	0.0221	0.0297	0.1601	0.1511
Weibull (1,2)	0.0146	0.0152	0.0462	0.0436

Table 11: The bias values of SMRSS, SRSS, SSRS and SRS for $n = 15$ and samples sizes $n_1 = 3$, $n_2 = 5$ and $n_3 = 7$.

Distribution	**SMRSS**	**SRSS**	**SSRS**	**SRS**
Geometric (0.5)	0.0054	0.0109	0.0720	0.0668
Exponential (1)	0.0137	0.0166	0.0718	0.0665
Gamma (1,2)	0.0714	0.0908	0.2868	0.2662
Beta (1,2)	0.0012	0.0014	0.0080	0.0074
Beta (5,2)	0.0110	0.0110	0.0367	0.0340
Log Normal (0,1)	0.0127	0.0209	0.1956	0.1806
Weibull (1,2)	0.0119	0.0160	0.0565	0.0524

Table 12: The values of bias of SMRSS, SRSS, SSRS and SRS for $n = 18$ and samples sizes $n_1 = 10$ and $n_2 = 8$

Distribution	**SMRSS**	**SRSS**	**SSRS**	**SRS**
Geometric (0.5)	0.0045	0.0201	0.0560	0.0555
Exponential (1)	0.0298	0.0522	0.0562	0.0553
Gamma (1,2)	0.0873	0.1271	0.2253	0.2223
Beta (1,2)	0.0042	0.0059	0.0063	0.0062
Beta (5,2)	0.0261	0.0294	0.0287	0.0262
Log Normal (0,1)	0.0569	0.0808	0.1522	0.1518
Weibull (1,2)	0.0369	0.0434	0.0441	0.0437

It seems that the values of bias are smaller for SMRSS as compared to other methods for all the distributions considered.

5. Conclusion

In this paper, we have suggested a new estimator of the population mean using SMRSS. The performance of the estimator based on SMRSS is compared with SRSS, SSRS and SRS. It is found that SMRSS estimator is unbiased of the population mean of symmetric distributions and it is more efficient than SRSS, SSRS and SRS based on the same number of measured units. Also, the efficiency increases as the number of strata increases.

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Received: January, 2010