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Magnetohydrodynamic Flow in a Channel with Varying Viscosity under Transverse Magnetic Field

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Abstract

Flow of a viscous, incompressible, electrically conducting fluid with varying viscosity through a channel in the presence of a transverse magnetic field is studied in this paper. In the special case investigated here we take viscosity as a function of distance from the mid section of the channel. Exact solutions for velocity is obtained. Solutions in particular cases when viscosity is constant but magnetic field is not zero and when magnetic field is zero but viscosity is not constant are also obtained. Velocity is exhibited graphically for various different cases.

Keywords: Variable viscosity, electrically conducting fluid, viscous incompressible fluid, laminar flow, channel flow, magnetic permeability, shearing stress

1 Introduction

Viscosity of a fluid is generally taken to be a constant but it may depend on temperature and concentration; it may also change due to suspending solid particles in the form of ash or soot or as a result of corrosion. Considerable amount of work has been done on fluid flow problems with temperature dependent viscosity; to cite a few cases, we may mention the papers by Dai, Dong and Szeri [2], Saikrishnan, Roy [12], Hazema [5] and Eswara and Bommiah[1].

But very little literature is avilable on the concentration dependent viscosity. Fluid flow problems of stratified fluid viscosities have been taken up amongst others by Malik and Hooper [10], Gelu Pasa and Olivier Titaud [3], Meiburg et. al. [9] and Payr, Vanparthy and Meiburg [11]. Considered the flow problem with viscosity as exponential function of concentration and solved the problem for superimposed upper and lower layers of fluids of different viscosities. Since it may be seen that the concentration is spatially varying quantity it is reasonable to assume that viscosity may be taken as a function of space coordinates. Haber and Brenner [4] and J. B. Shukla [7] studied a fluid flow problem involving spatially varying viscosity.

In the present study we consider the flow of a viscous, incompressible and electrically conducting fluid in a channel when viscosity is a function of distance from its mid section. The general case is too complicated to handle analytically; it is being investigated and will be presented in a forthcoming paper. Therefore, here we are contented with two particular interesting cases of parabolic variation of viscosity; case I, when the viscosity is maximum at the mid section, and case II, when the viscosity is minimum at the midsection. Analytical solutions in both cases have been obtained. Computational results are presented through the direct numerical integration of the concerned differential equation in the first case. In the second case computational results follow from the obtained analytical solution.

2 Formulation and solution of problem

In this paper we consider the two-dimensional steady laminar flow of a viscous incompressible, electrically conducting fluid between two parallel straight plates. Let $\overline{x}$ be the direction of the flow, $\overline{y}$ the direction normal to the flow and the width of the plates parallel to the $\overline{z}$ direction be large compared with the distance '$2\overline{h}$' between the plates. A transverse magnetic field $\overline{\mathbf{H}}_0$ is impressed across the flow. We consider variable viscosity ($\overline{\mu}$), a function of $\overline{y}$. Flow is governed by magnetohydrodynamic equations with Lorentz force as the external force, subject to relevant boundary conditions which will be specified in the sequel for the concerned problem. Magnetohydrodynamic equations for the present problem is given below

$$\nabla \times \overline{\mathbf{H}} = 4\pi \overline{\mathbf{j}} \tag{1}$$

$$\nabla \times \overline{\mathbf{E}} = -\overline{\mu}_m \frac{\partial \overline{\mathbf{H}}}{\partial \overline{t}} = 0 \tag{2}$$

$$\nabla \cdot \overline{\mathbf{H}} = 0 \tag{3}$$

$$\overline{\mathbf{j}} = \overline{\sigma}\left(\overline{\mathbf{E}} + \overline{\mu}_m \overline{\mathbf{V}} \times \overline{\mathbf{H}}\right) \tag{4}$$

$$\nabla \cdot \overline{\mathbf{V}} = 0 \tag{5}$$

$$\begin{aligned} \overline{\rho}\,(\overline{\mathbf{V}} \cdot \nabla)\overline{\mathbf{V}} &= -\nabla\overline{p} + \overline{\mu}\nabla^2\overline{\mathbf{V}} + 2(\nabla\overline{\mu} \cdot \nabla)\overline{\mathbf{V}} \\ &\quad +(\nabla\overline{\mu}) \times (\nabla \times \overline{\mathbf{V}}) + \overline{\mu}_m \overline{\mathbf{j}} \times \overline{\mathbf{H}} \end{aligned} \tag{6}$$

Where $\overline{\mu}$ is the viscosity of fluid, $\overline{\mu}_m$ the magnetic permeability, $\overline{\mathbf{V}}$ the fluid velocity, $\overline{\mathbf{H}}$ the magnetic field, $\overline{\mathbf{E}}$ the electric field, $\overline{\mathbf{j}}$ the current density vector, $\overline{\rho}$ the density, $\overline{p}$ the pressure. It may be noted that quantities having bar on the top are dimensional quantities and equation (6) is written for varying viscosity. To get the coupled equation for $\overline{\mathbf{V}}$ and $\overline{\mathbf{H}}$ we eliminate $\overline{\mathbf{j}}$ and $\overline{\mathbf{E}}$ amongst equations (1), (2) and (4) and $\overline{\mathbf{j}}$ between equations (1) and (6). Making use of equations (3) and (5), we obtained the resulting equations in the form.

$$\lambda\nabla^2\overline{\mathbf{H}} = -(\overline{\mathbf{H}} \cdot \nabla)\overline{\mathbf{V}} + (\overline{\mathbf{V}} \cdot \nabla)\,\overline{\mathbf{H}} \tag{7}$$

$$\begin{aligned} \overline{\rho}(\overline{\mathbf{V}} \cdot \nabla)\overline{\mathbf{V}} &= -\nabla\left(\overline{p} + \frac{\overline{\mu}_m}{8\pi}\left|\overline{\mathbf{H}}\right|^2\right) + \overline{\mu}\nabla^2\overline{\mathbf{V}} + 2\,(\nabla\overline{\mu} \cdot \nabla)\overline{\mathbf{V}} \\ &\quad + (\nabla\overline{\mu}) \times (\nabla \times \overline{\mathbf{V}}) + \frac{\overline{\mu}_m}{4\pi}(\overline{\mathbf{H}} \cdot \nabla)\overline{\mathbf{H}} \end{aligned} \tag{8}$$

Where, $\lambda = \frac{1}{4\pi\overline{\mu}_m\overline{\sigma}}$, is the magnetic viscosity.

We now express equation(7) and (8) in cartisian coordinate system and use the following conditions for our purpose.

(1) Let $\overline{u}, \overline{v}, \overline{w}$ be components of velocity in $\overline{x}, \overline{y}$ and $\overline{z}$ directions. Since the flow is laminar and steady we have $\overline{v} = 0, \overline{w} = 0$ and $\frac{\partial}{\partial\overline{x}} = 0$.

(2) $\overline{H}_{\overline{z}} = 0, \overline{H}_{\overline{y}} = \overline{H}_0, \frac{\partial\overline{H}_{\overline{x}}}{\partial\overline{x}} = 0$ (By the same argument as shown by S. Globe[13]).

When these conditions are introduced the equations corrosponding to (7) and (8) reduces to the following three equations

$$\frac{\partial\overline{\mu}}{\partial\overline{y}}\frac{\partial\overline{u}}{\partial\overline{y}} + \overline{\mu}\frac{\partial^2\overline{u}}{\partial\overline{y}^2} + \frac{\overline{\mu}_m}{4\pi}\overline{H}_0\frac{\partial\overline{H}_{\overline{x}}}{\partial\overline{y}} = \frac{\partial\overline{p}}{\partial\overline{x}}, \tag{9}$$

$$\frac{\overline{\mu}_m}{4\pi}\overline{H}_x\frac{\partial\overline{H}_{\overline{x}}}{\partial\overline{y}} = -\frac{\partial\overline{p}}{\partial\overline{y}}, \tag{10}$$

$$\lambda \frac{\partial^2 \overline{H}_{\overline{x}}}{\partial \overline{y}^2} + \overline{H}_0 \frac{\partial \overline{u}}{\partial \overline{y}} = 0. \tag{11}$$

Here, $\overline{u}$is a function of $\overline{y}$. Since flow is fully developed $\frac{\partial \overline{H}_{\overline{x}}}{\partial \overline{z}} = 0$ and $\frac{\partial \overline{H}_{\overline{x}}}{\partial \overline{x}} = 0$, we conclude that $\overline{H}_{\overline{x}}$ is a function of $\overline{y}$ only, and then it follows from equation (9) that $\frac{\partial \overline{p}}{\partial \overline{x}}$ must be independent of $\overline{x}$. By differentiating equation (10) with respect to $\overline{x}$, it can be seen that $\frac{\partial \overline{p}}{\partial \overline{x}}$ is also independent of $\overline{y}$. Therefore $\frac{\partial \overline{p}}{\partial \overline{x}}$ is constant, say $-\overline{P}$. Once $\overline{H}_{\overline{x}}$ is determined, the variation of $\overline{p}$ across the channel may be found by using equation (10). Rewriting equations (9) and (11) with ordinary derivatives, we obtained

$$\frac{d\overline{\mu}}{d\overline{y}} \frac{d\overline{u}}{d\overline{y}} + \overline{\mu} \frac{d^2 \overline{u}}{d\overline{y}^2} + \frac{\overline{\mu}_m}{4\pi} \overline{H}_0 \frac{d\overline{H}_{\overline{x}}}{d\overline{y}} = -\overline{P}, \tag{12}$$

$$\lambda \frac{d^2 \overline{H}_{\overline{x}}}{d\overline{y}^2} + \overline{H}_0 \frac{d\overline{u}}{d\overline{y}} = 0 \tag{13}$$

Integrating equation(13) and using the condition $\left(\frac{d\overline{H}_{\overline{x}}}{d\overline{y}}\right)_{\overline{y}=\overline{h}} = 0$,we get

$$\lambda \frac{d\overline{H}_{\overline{x}}}{d\overline{y}} = -\overline{H}_0 \, \overline{u} \tag{14}$$

Justification of condition $\left(\frac{d\overline{H}_{\overline{x}}}{d\overline{y}}\right)_{\overline{y}=\overline{h}} = 0$ is given below.

From (1) we have $4\pi \overline{j}_{\overline{z}} = -\frac{d\overline{H}_{\overline{x}}}{d\overline{y}}$

But $\overline{j}_{\overline{z}} = \overline{\sigma} \, \overline{\mu}_m \left(\overline{\mathbf{V}} \times \overline{\mathbf{H}}\right)_{\overline{z}}$ (with $\overline{E}_{\overline{z}} = 0$ as in Globe[13])

Since by no slip condition $\overline{\mathbf{V}} = 0$ when $\overline{y} = \pm \overline{h}$. Hence $\overline{j}_{\overline{z}}$ vanishes there and hence $\frac{d\overline{H}_{\overline{x}}}{d\overline{y}}$ also vanishes there.

Substituting $\frac{d\overline{H}_{\overline{x}}}{d\overline{y}}$ from equation(14) in equation(12)we have

$$\overline{\mu} \frac{d^2 \overline{u}}{d\overline{y}^2} + \frac{d\overline{\mu}}{d\overline{y}} \frac{d\overline{u}}{d\overline{y}} - \frac{\overline{\mu}_m \overline{H}_0^2}{4\pi\lambda} \, \overline{u} = -\overline{P} \tag{15}$$

Now it will be convenient to express the equation(15) in non-dimensional form by introducing the following transformations of variables

$\overline{y} = \overline{h} \, y, \overline{u} = \overline{u_0} \, u, \; \overline{\mu} = \overline{\mu}_0 \, \mu(r), \; \overline{P} = \frac{\overline{\mu}_0 \overline{u}_0}{\overline{h}^2} P.$

Where, $\overline{u}_0$ and $\overline{\mu}_0$ are characteristic velocity and characteristic viscosity corresponding to the classical case of constant viscosity and non-zero magnetic field.

Equation (15) in non-dimensional variables y, u, μ and P can be written as

$$\mu \frac{d^2 u}{dy^2} + \frac{d\mu}{dy} \frac{du}{dy} - m^2 u = -P \tag{16}$$

Where, $m^2 = \frac{\overline{\mu}_m^2 \overline{H}_0^2 \ \overline{h}^2 \overline{\sigma}}{\overline{\mu}_0}$ is a dimensionless parameter.

Equation(16) is a differential equation of order two.Two boundary conditions are required for the solution. These are

$$u\,(1) = 0 \ and \ u\,(-1) = 0 \tag{17}$$

The analytic solution of the equation (16) for the general variation of viscosity is difficult to deal, hence here we consider two special cases.

2.1 Case-I

When $\mu = (1 - \epsilon \ y^2)$, where $0 < \epsilon < 1$ is a nondimensional viscosity variation parameter. In this case governing equation of motion (16) becomes

$$\left(1 - \epsilon y^2\right) \frac{d^2u}{dy^2} - 2\epsilon y \frac{du}{dy} - m^2 u = -P \tag{18}$$

It should be noted that $\epsilon = 0$ corresponds to the constant viscosity case. Solution of above equation(18) subject to the boundary conditions (17) is

$$\begin{aligned} u\,(y) \ &= \ \frac{P}{m^2} + \ C_1 L_P \left[-\frac{1}{2} + i \frac{\sqrt{4m^2 - \epsilon}}{2\sqrt{\epsilon}}, y\sqrt{\epsilon} \right] \\ &\quad + C_2 \ L_Q \left[-\frac{1}{2} + i \frac{\sqrt{4m^2 - \epsilon}}{2\sqrt{\epsilon}}, y\sqrt{\epsilon} \right] \end{aligned} \tag{19}$$

Here, $L_P[-\frac{1}{2} + i\frac{\sqrt{4m^2-\epsilon}}{2\sqrt{\epsilon}}, y\sqrt{\epsilon}\ ]$ and $L_Q[-\frac{1}{2} + i\frac{\sqrt{4m^2-\epsilon}}{2\sqrt{\epsilon}}, y\sqrt{\epsilon}\ ]$ are Legendre's function of the first and second kind of the degree $(-\frac{1}{2} + i\frac{\sqrt{4m^2-\epsilon}}{2\sqrt{\epsilon}} \ )$ and constants C_1 and C_2 are,

$$C_1 = \frac{P(L_Q[-\frac{1}{2} + \frac{i\sqrt{4m^2-\epsilon}}{2\sqrt{\epsilon}}, -\sqrt{\epsilon}\ ] - L_Q[-\frac{1}{2} + \frac{i\sqrt{4m^2-\epsilon}}{2\sqrt{\epsilon}}, \sqrt{\epsilon}\ ])}{\begin{array}{c} m^2(L_P[-\frac{1}{2} + \frac{i\sqrt{4m^2-\epsilon}}{2\sqrt{\epsilon}}, -\sqrt{\epsilon}\ ] \ L_Q[-\frac{1}{2} + \frac{i\sqrt{4m^2-\epsilon}}{2\sqrt{\epsilon}}, \sqrt{\epsilon}\ ] \\ -L_P[-\frac{1}{2} + \frac{i\sqrt{4m^2-\epsilon}}{2\sqrt{\epsilon}}, \sqrt{\epsilon}\ ] \ L_Q[-\frac{1}{2} + \frac{i\sqrt{4m^2-\epsilon}}{2\sqrt{\epsilon}}, -\sqrt{\epsilon}\ ]) \end{array}}, \tag{20}$$

$$C_2 = \frac{P(L_P[-\frac{1}{2} + \frac{i\sqrt{4m^2-\epsilon}}{2\sqrt{\epsilon}}, -\sqrt{\epsilon}\ ] - L_P[-\frac{1}{2} + \frac{i\sqrt{4m^2-\epsilon}}{2\sqrt{\epsilon}}, \sqrt{\epsilon}\ ])}{\begin{array}{c} m^2(L_P[-\frac{1}{2} + \frac{i\sqrt{4m^2-\epsilon}}{2\sqrt{\epsilon}}, \sqrt{\epsilon}\ ] \ L_Q[-\frac{1}{2} + \frac{i\sqrt{4m^2-\epsilon}}{2\sqrt{\epsilon}}, -\sqrt{\epsilon}\ ] \\ -L_P[-\frac{1}{2} + \frac{i\sqrt{4m^2-\epsilon}}{2\sqrt{\epsilon}}, -\sqrt{\epsilon}\ ] \ L_Q[-\frac{1}{2} + \frac{i\sqrt{4m^2-\epsilon}}{2\sqrt{\epsilon}}, \sqrt{\epsilon}\ ]) \end{array}} \tag{21}$$

Solution, when $\epsilon = 0$ but $m \neq 0$ and when $m = 0$ but $\epsilon \neq 0$ can be obtain by solving the reduced equation of motion (18) for $\epsilon = 0$ and $m = 0$ separately. That are

$$u = \frac{P}{m^2}\left[1 - \frac{\cosh(my)}{\cosh(m)}\right], \tag{22}$$

$$u = P\left[\frac{Log\,(1-\epsilon\, y^2) - Log\,(1-\epsilon)}{2\epsilon}\right] \tag{23}$$

Velocity profiles of the flow in a channel under transverse magnetic field with the viscosity variation $\mu = (1-\epsilon y^2)$ for different cases are shown in figures [1], [2], [3] and [4].

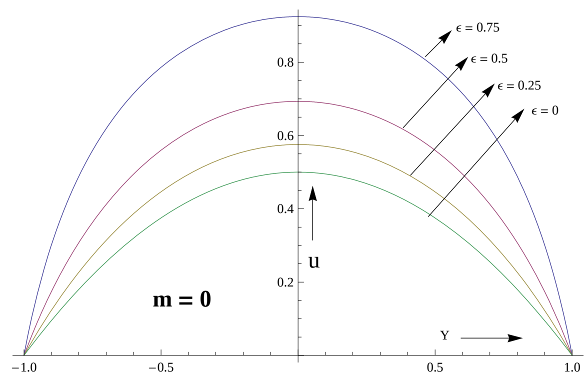

Figure 1: Velocity profile of flow in the channel with viscosity variation is $\mu = (1-\epsilon y^2)$ when $m = 0$ for $\epsilon = 0, 0.25, 0.5, 0.75$

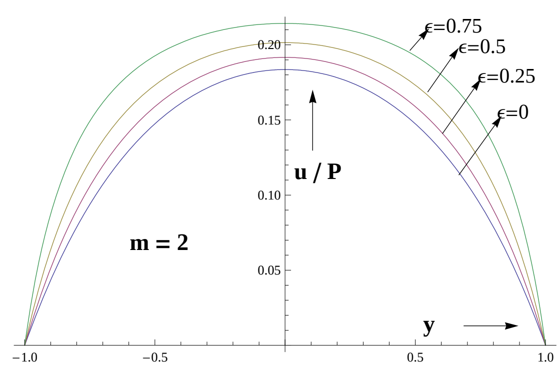

Figure 2: Velocity profile of flow in the channel with viscosity variation is $\mu = (1-\epsilon y^2)$ when $m = 2$ for $\epsilon = 0, 0.25, 0.5, 0.75$

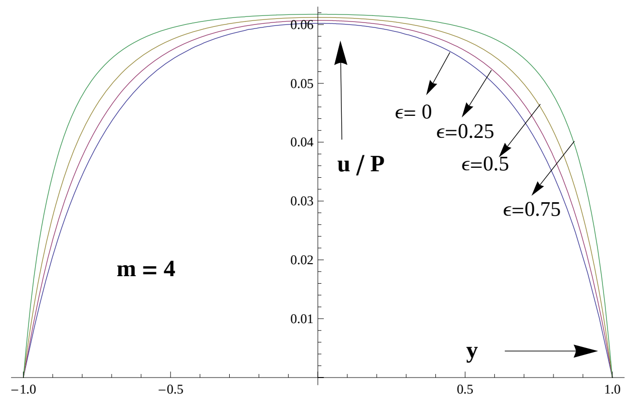

Figure 3: Velocity profile of flow in the channel with viscosity variation is $\mu = (1 - \epsilon y^2)$ when $m = 4$ for $\epsilon = 0, 0.25, 0.5, 0.75$

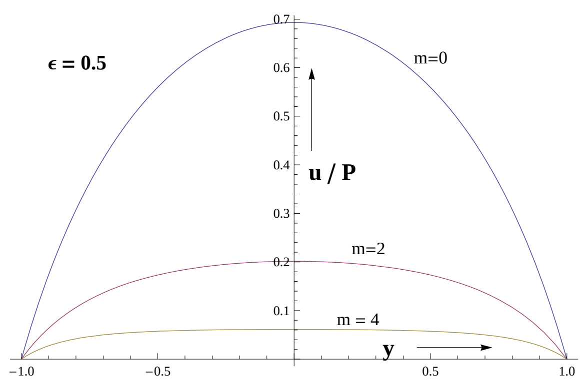

Figure 4: Velocity profile of flow in the channel with viscosity variation $\mu = (1 - 0.5y^2)$ for $m = 0, 2, 4$

3 Shearing stress at the wall

Dimensionless shear stress at the wall for the flow when viscosity variation is $\mu = (1 - \epsilon\ y^2)$ can be determined as follows:

$(\tau_{yx})_{y=1} = -\mu(\frac{du}{dy})_{y=1}$

Substituting u from equation (19) in the above equation and substituting derivatives of $L_P\left[-\frac{1}{2} + ib, y\sqrt{\epsilon}\right]$ and $L_Q\left[-\frac{1}{2} + ib, y\sqrt{\epsilon}\right]$ [Ref.6]. we get

$$\begin{aligned}(\tau_{yx})_{y=1} &= \sqrt{\epsilon}\ (\frac{1}{2} + ib)[C_1\{-y\sqrt{\epsilon}L_P[-\frac{1}{2} + ib, y\sqrt{\epsilon}\ ] \\ &\quad + L_P[\frac{1}{2} + ib, y\sqrt{\epsilon}\ ]\} + C_2\{-y\sqrt{\epsilon}L_Q[-\frac{1}{2} + ib, y\sqrt{\epsilon}\ ] \\ &\quad + L_Q[\frac{1}{2} + ib, y\sqrt{\epsilon}\ ]\}]_{y=1}\end{aligned}$$

$$\begin{aligned}(\tau_{yx})_{y=1} &= \sqrt{\epsilon}\ (\frac{1}{2} + ib)[C_1\{-\sqrt{\epsilon}L_P[-\frac{1}{2} + ib, \sqrt{\epsilon}\ ] \\ &\quad + L_P[\frac{1}{2} + ib, \sqrt{\epsilon}\ ]\} + C_2\{-\sqrt{\epsilon}L_Q[-\frac{1}{2} + ib, \sqrt{\epsilon}\ ] \\ &\quad + L_Q[\frac{1}{2} + ib, \sqrt{\epsilon}\ ]\}]\end{aligned} \tag{24}$$

Where, $b = \frac{\sqrt{4m^2-\epsilon}}{2\sqrt{\epsilon}}$ and C_1, C_2 are given by equation (20) and (21).

Dimensionless shearing stress at any point for $m = 0$ and $\epsilon \neq 0$ is

$$\tau_{yx} = -\left(1 - \epsilon y^2\right) \frac{d}{dy}\left[\frac{PLog(1 - \epsilon\ y^2) - PLog(1 - \epsilon)}{2\epsilon}\right] \tag{25}$$

$$\tau_{yx} = Py \tag{26}$$

Thus shearing stress is independent of ϵ when $m = 0$.

Dimensionless shearing stress at the wall for $m = 0, m = 2, m = 4, m = 6$ when $\epsilon = 0.5$ is,

$(\tau_{yx})_{y=1} = P$, when $m = 0$.
$(\tau_{yx})_{y=1} = 0.39234\ P$, when $m = 2$.
$(\tau_{yx})_{y=1} = 0.19045\ P$, when $m = 4$.
$(\tau_{yx})_{y=1} = 0.1240\ P$, when $m = 6$.

Variation of shearing stress with respect to y for $m = 2, 4$ and 6 when viscosity variation is $\mu = (1 - 0.5\ y^2)$ is shown in figure(5).

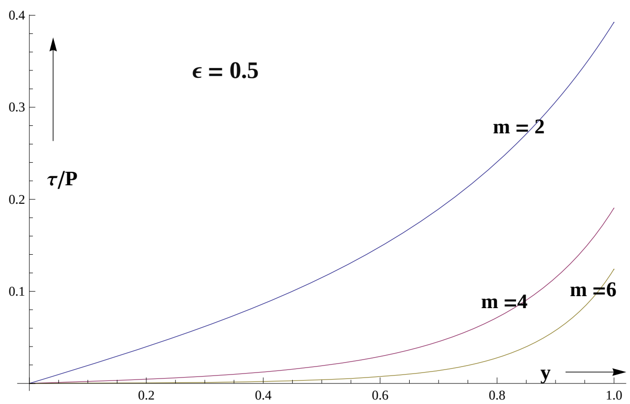

Figure 5: Shearing stress at any point for $m = 2, 4$ and 6 when $\mu = (1 - 0.5y^2)$.

3.1 Case-II

When $\mu = (1 + \epsilon\, y^2)$,where $0 < \epsilon < 1$ is a nondimensional viscosity variation parameter.In this case governing equation of motion (16) becomes

$$\left(1 + \epsilon y^2\right) \frac{d^2u}{dy^2} + 2\epsilon y \frac{du}{dy} - m^2 u = -P \tag{27}$$

Solution of above equation(18) subject to the boundary conditions (17) is

$$\begin{aligned} u(y) \;\; &= \;\; \frac{P}{m^2} + \; C_1 \; L_P \left[\frac{-\sqrt{\epsilon} + \sqrt{4m^2+\epsilon}}{2\sqrt{\epsilon}}, iy\sqrt{\epsilon}\right] \\ &\quad + C_2 L_Q \left[\frac{-\sqrt{\epsilon} + \sqrt{4m^2+\epsilon}}{2\sqrt{\epsilon}}, iy\sqrt{\epsilon}\right] \end{aligned} \tag{28}$$

Here C_1 and C_2 are

$$C_1 = \frac{P(L_Q[\frac{-\sqrt{\epsilon}+\sqrt{4m^2+\epsilon}}{2\sqrt{\epsilon}}, -i\sqrt{\epsilon}\,] - L_Q[\frac{-\sqrt{\epsilon}+\sqrt{4m^2+\epsilon}}{2\sqrt{\epsilon}}, i\sqrt{\epsilon}\,])}{\begin{array}{c} m^2(L_P[\frac{-\sqrt{\epsilon}+\sqrt{4m^2+\epsilon}}{2\sqrt{\epsilon}}, -i\sqrt{\epsilon}\,]\; L_Q[\frac{-\sqrt{\epsilon}+\sqrt{4m^2+\epsilon}}{2\sqrt{\epsilon}}, i\sqrt{\epsilon}\,] \\ -L_P[\frac{-\sqrt{\epsilon}+\sqrt{4m^2+\epsilon}}{2\sqrt{\epsilon}}, i\sqrt{\epsilon}\,]\; L_Q[\frac{-\sqrt{\epsilon}+\sqrt{4m^2+\epsilon}}{2\sqrt{\epsilon}}, -i\sqrt{\epsilon}\,]) \end{array}}, \tag{29}$$

$$C_2 = \frac{P(L_P[\frac{-\sqrt{\epsilon}+\sqrt{4m^2+\epsilon}}{2\sqrt{\epsilon}}, -i\sqrt{\epsilon}\,] - L_P[\frac{-\sqrt{\epsilon}+\sqrt{4m^2+\epsilon}}{2\sqrt{\epsilon}}, i\sqrt{\epsilon}\,])}{\begin{array}{c} m^2(L_P[\frac{-\sqrt{\epsilon}+\sqrt{4m^2+\epsilon}}{2\sqrt{\epsilon}}, i\sqrt{\epsilon}\,]\; L_Q[\frac{-\sqrt{\epsilon}+\sqrt{4m^2+\epsilon}}{2\sqrt{\epsilon}}, -i\sqrt{\epsilon}\,] \\ -L_P[\frac{-\sqrt{\epsilon}+\sqrt{4m^2+\epsilon}}{2\sqrt{\epsilon}}, -i\sqrt{\epsilon}\,] L_Q[\frac{-\sqrt{\epsilon}+\sqrt{4m^2+\epsilon}}{2\sqrt{\epsilon}}, i\sqrt{\epsilon}\,]) \end{array}} \tag{30}$$

Solution of equation (27) when $\epsilon = 0$ but $m \neq 0$ is same as in case(I) and is given by equation(22). When $m = 0$ but $\epsilon \neq 0$ solution can be obtain by solving the reduced equation of motion (18) for $m = 0$ separately .That is

$$u(y) = P\left[\frac{Log(1+\epsilon) - Log(1+\epsilon y^2)}{2\epsilon}\right] \tag{31}$$

Velocity profiles of the flow with the viscosity variation $\mu = (1 + \epsilon\ y^2)$ for different cases are shown below in figures [6],[7],[8]and[9].

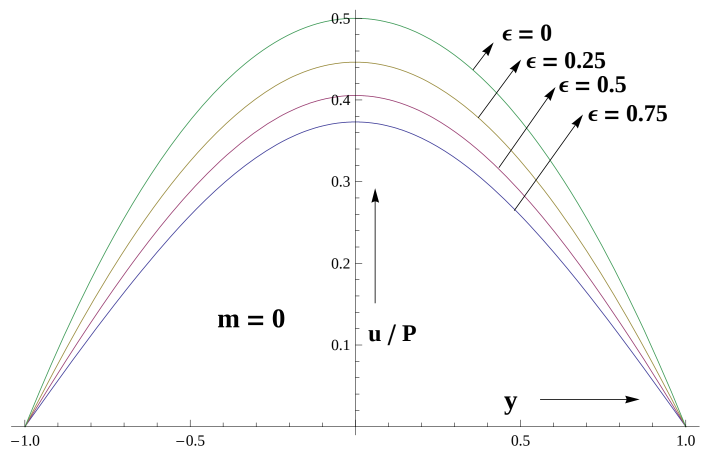

Figure 6: Velocity profile of flow in the channel with viscosity variation $\mu = (1 + \epsilon y^2)$ when $m = 0$ for $\epsilon = 0, 0.25, 0.5, 0.75$.

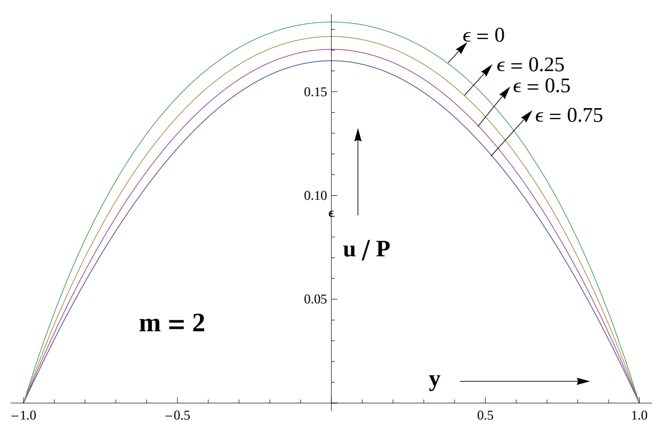

Figure 7: Velocity profile of flow in the channel with viscosity variation $\mu = (1 + \epsilon y^2)$ when $m = 2$ for $\epsilon = 0, 0.25, 0.5, 0.75$.

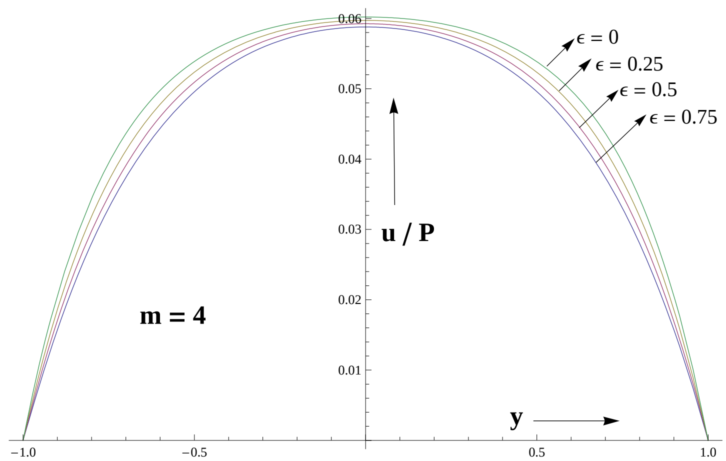

Figure 8: Velocity profile of flow in the channel with viscosity variation $\mu = (1 + \epsilon y^2)$ when $m = 4$ for $\epsilon = 0, 0.25, 0.5, 0.75$.

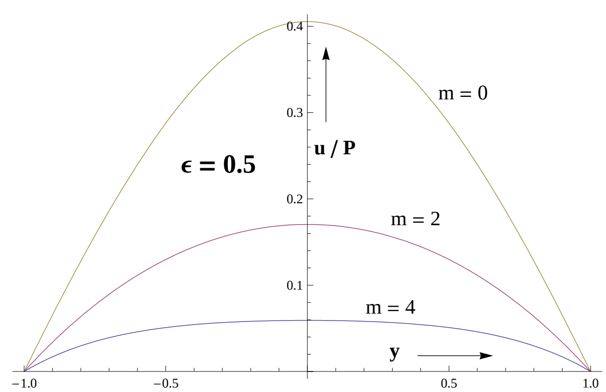

Figure 9: Velocity profile of flow in the channel when viscosity variation is $\mu = (1 + 0.5y^2)$ for $m = 0, 2, 4$

4 Shearing stress at the wall

Shear stress at the wall for the flow when viscosity variation is $\mu = (1 + \epsilon\ y^2)$ can be determined as follows:

$$(\tau_{yx})_{y=1} = -\mu(\frac{du}{dy})_{y=1} \tag{32}$$

Substituting u from equation (28) in the above equation and substituting derivatives of $L_P\left[-\frac{1}{2}+\frac{\sqrt{4m^2+\epsilon}}{2\sqrt{\epsilon}},\ iy\sqrt{\epsilon}\right]$ and $L_Q\left[-\frac{1}{2}+\frac{\sqrt{4m^2+\epsilon}}{2\sqrt{\epsilon}},\ iy\sqrt{\epsilon}\right]$. we get

$$\begin{aligned}(\tau_{yx})_{y=1} \quad = \quad & i\sqrt{\epsilon}\ (\frac{1}{2}+a)[C_1\ \{ - iy\sqrt{\epsilon}L_P[-\frac{1}{2}+a, iy\sqrt{\epsilon}\ ]+ \\ & L_P[\frac{1}{2}+a, iy\sqrt{\epsilon}\ ]\} + C_2\ \{ - iy\sqrt{\epsilon}L_Q[-\frac{1}{2}+a, iy\sqrt{\epsilon}\ ] \\ & +L_Q[\frac{1}{2}+a, iy\sqrt{\epsilon}\ ]\}]_{y=1}\end{aligned}$$

$$\begin{aligned}(\tau_{yx})_{y=1} \quad = \quad & i\sqrt{\epsilon}(\frac{1}{2}+a)[C_1\{ - i\sqrt{\epsilon}L_P[-\frac{1}{2}+a, i\sqrt{\epsilon}\ ] \\ & +L_P[\frac{1}{2}+a, i\sqrt{\epsilon}\ ]\ \} + C_2\{ - i\sqrt{\epsilon}L_Q[-\frac{1}{2}+a, i\sqrt{\epsilon}\ ] \\ & +L_Q[\frac{1}{2}+a, i\sqrt{\epsilon}\ ]\ \}]\end{aligned} \tag{33}$$

Where, $a = \frac{\sqrt{4m^2+\epsilon}}{2\sqrt{\epsilon}}$ and C_1, C_2 are given by Equation (29) and (30).
When $m = 0, \epsilon \neq 0$ dimensionless shearing stress at any point is

$$\tau_{yx} = -\left(1+\epsilon y^2\right)\frac{d}{dy}\left[\frac{PLog(1+\epsilon) - PLog(1+\epsilon\ y^2)}{2\epsilon}\right], \tag{34}$$

$$\tau_{yx} = Py \tag{35}$$

Thus as in case-small I shearing stress text is independent of ϵ when $m = 0$.
Dimensionless shearing stress at the wall for $m = 0, m = 2, m = 4, m = 6$ when $\epsilon = 0.5$ is,

$(\tau_{yx})_{y=1} = P$, when $m = 0$.
$(\tau_{yx})_{y=1} = 0.54002P$, when $m = 2$.
$(\tau_{yx})_{y=1} = 0.29227P$, when $m = 4$.
$(\tau_{yx})_{y=1} = 0.19776P$, when $m = 6$.

Variation of shearing stress with respect to y for $m = 2, 4$ and 6 when viscosity variation is $\mu = (1 + 0.5y^2)$ is shown in figure(10).

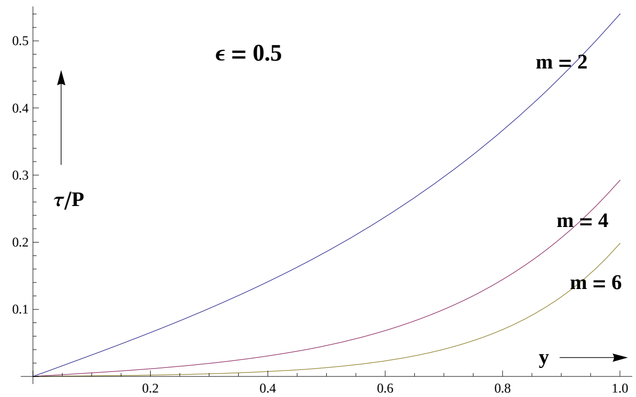

Figure 10: Shearing stress at any point for $m = 2, 4$ and 6 when $\mu = (1 + 0.5y^2)$.

5 Conclusion

Exact solutions have been obtained for the flow of viscous, incompressible, electrically conducting fluid through a cannel in the presence of transverse uniform magnetic field subject to the no slip condition on the non-conducting walls for two cases of viscosity variations. Solutions are exact thus valid for all values of the hydromagnetic parameter m. In the first case of viscosity variation $\mu = (1 - \epsilon\ y^2)$, we find from figures(1), (2) and (3) that for fixed magnetic field (i.e. for fixed m) as viscosity variation parameter ϵ increases, velocity increases because of the decrease in the average viscosity μ. In the second case of viscosity variation $\mu = (1 + \epsilon y^2)$, the finding is just opposite for the corresponding reason as depicted in figures (6), (7) and (8). On the other hand in both the cases for fixed ϵ the effect of the magnetic field is to decrease the velocity as shown in the figures (4) and (9). In general it is concluded that the effect of magnetic field is to flatten the velocity profile so that the core gets formed. This is because increase in the magnetic field leads to an increase in the Lorentz force opposing the flow. From figures (5) and (10) it is seen that shearing stress at the wall decreases as magnetic field increases for fixed ϵ in both the cases of viscosity variation. For $m = 0$, shearing stress at any point is independent of ϵ and depend only on the position of point.

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