

# Radiative MHD Flow over a Non-Isothermal Stretching Sheet in a Porous Medium

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## Abstract

In this communication, we present a numerical study for the steady two-dimensional radiative MHD boundary-layer flow of an incompressible, viscous, electrically conducting fluid caused by a non-isothermal linearly stretching sheet placed at the bottom of fluid saturated porous medium. The governing system of partial differential equations are converted to ordinary differential equations by using the similarity transformations, which are then solved numerically. The dimensionless temperature distribution and temperature gradient are computed for different thermophysical parameters viz the radiation parameter, the permeability parameter, magnetic parameter, wall temperature parameter and the Prandtl number.

**Keywords:** Stretching sheet, porous medium, thermal radiation, MHD, convective heat transfer, numerical study.

## 1. Introduction

The radiative heat transfer in porous medium has not been much investigated though it is interesting to note that the porous medium absorbs or emits radiation that is transferred to or from a fluid. The fluid can be regarded to be transparent to radiation, because the dimensions for the radiative transfer among the solid elements of porous structure are usually much less than the radiative mean free

path for scattering or absorption in the fluid (Howell(2000)). Simultaneous convection and radiation heat transfer finds numerous applications in many technological and industrial processes involving high temperatures, such as, space technology, furnace designs, nuclear reactors, heating and cooling chambers, fossil fuel combustion, energy processes, evaporation from large open water reservoirs etc. It is pertinent to record that contrary to conduction and convection heat transfer, thermal radiation is a complex phenomena to account for because of its spectral and directional dependence in addition to the difficulty of determining accurate physical property values of the medium. Furthermore, the inclusion of radiation term in the energy equation makes the equation highly non-linear. However, some reasonable simplifications are used to make system solvable. To be specific, one of these simplifications is the assumption of optically dense medium in which radiation travels only a short distance before being absorbed or scattered. This assumption physically means that the local radiation intensity at a point is assumed to emerge only within the neighbourhood of that point. A comprehensive literature of the radiation heat transfer has been given in the well presented texts by Sparrow and Cess (1970), Özisik (1973), and Siegel and Howell (1992). Though, the radiation heat transfer is significant in many flow regimes but unfortunately very little is known about the effects of radiation on the boundary layer. The effects of radiation on heat transfer in porous medium have been studied by Whitaker (1980), Plumb et al. (1981), Tong and Tien (1983), Bakier et al. (1996), Mansour (1997), Raptis (1998), Bakier (2001), Chamkha et al. (2001), Raptis et al. (2004).

In the present work we aim at analyzing the combined convection radiation heat transfer numerically in the boundary layer arising from a horizontal linearly stretching sheet placed at the bottom of porous medium. The radiation heat flux is approximated with the Rosseland approximation. Due to simple geometry and closed form exponential solution of this well known stretching surface problem (Crane(1970)) many investigators have attempted the problem with different assumptions [Gupta and Gupta (1977), Arunachalam and Rajappa (1978), Chiam (1982), Grubka and Bobba (1985), Vajravelu and Hadjinicolaan (1993), Chiam (1995), Chauhan and Vyas (1995), Anderson and Valnes (1998)]. To the best of the knowledge of the authors the numerical study presented here has not been reported so far.

## **2. Formulation of the problem**

Consider the steady two-dimensional forced convection boundary-layer flow of viscous, incompressible, electrically conducting fluid in a fluid saturated horizontal porous medium caused by linearly stretching sheet placed at the bottom

of the porous medium. A Cartesian co-ordinate system is used. The  $x$ -axis is along the sheet and  $y$ -axis is normal to the  $x$ -axis (see figure 1). Two equal and opposite forces are applied along the sheet so that the wall is stretched, keeping the position of the origin unaltered. The stretching velocity varies linearly with the distance from the origin. A uniform magnetic field of strength  $B_0$  is applied normal to sheet. We assume that the wall temperature  $T_w > T_\infty$ , where  $T_\infty$  is the uniform temperature of the ambient fluid.

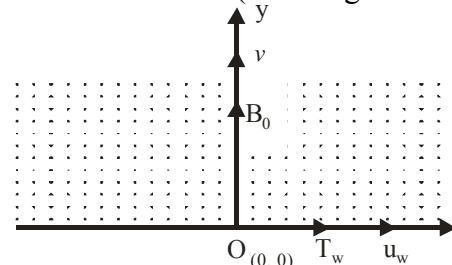


Figure 1: Physical model

We also assume that the fluid is optically dense, Newtonian and without phase change. Further it is assumed that both the fluid and the porous medium are in local thermal equilibrium. We also consider that both the surroundings and the fluid are maintained at a constant temperature  $T_\infty$  far away from the sheet. The Rosseland approximation is followed to describe the heat flux in the energy equation. On neglecting the induced magnetic field, the external electric field, the electric field due to polarization of charges, ohmic and viscous dissipations, the governing boundary layer equations can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\nu}{k} u - \frac{\sigma}{\rho} B_0^2 u \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} \tag{3}$$

and the boundary conditions are:

$$y = 0 : u = cx, v = 0, T = T_w(x) = T_\infty + Dx^\alpha \tag{4}$$

$$y \rightarrow \infty : u = 0, T = T_\infty \tag{5}$$

where  $c > 0$  and  $D$  are constants,  $\nu$  is the kinematic viscosity of the fluid,  $\sigma$  is the electrical conductivity,  $\rho$  is the density,  $T$  is the temperature,  $\kappa$  is the thermal conductivity,  $c_p$  is the specific heat at constant pressure and  $q_r$  is the radiation heat flux. Using Rosseland approximation for radiation [Brewster (1972)] we can write  $q_r = (-4\sigma^*/3k^*) (\partial T^4 / \partial y)$  where  $\sigma^*$ ,  $k^*$  are Stephan-Boltzmann constant and mean absorption coefficients respectively. Temperature difference within the flow is assumed to be sufficiently small so that  $T^4 \cong 4T_\infty^3 T - 3T_\infty^4$ , i.e.  $T^4$  may be

expressed as a linear function of temperature  $T$ , using a truncated Taylor series about the free stream temperature  $T_\infty$ .

### 3. Analysis

We introduce the similarity variables

$$\psi = \sqrt{c\nu} x f(\eta), \quad \eta = \sqrt{c/\nu} y \quad \text{and} \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad (6)$$

where  $\Psi$  is the stream function defined as

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \quad (7)$$

Using eqs.(6) and (7) the solution of eq.(1) can be written as

$$u = c x f'(\eta) \quad \text{and} \quad v = -\sqrt{\nu c} f(\eta) \quad (8)$$

In view of eqs.(6) and (8); eq.(2) reduces to

$$f'''' + f f'' - f'^2 - (\lambda + M) f' = 0 \quad (9)$$

$$\text{with boundary conditions} \quad f(0) = 0, \quad f'(0) = 1, \quad f'(\infty) = 0 \quad (10)$$

Here  $\lambda = ck/\nu$  is the permeability parameter and  $M = \sigma B_0^2 / c\rho$  is the magnetic field parameter. Here prime denotes differentiation with respect to  $\eta$ .

The boundary conditions given by (10) suggest a solution of the form :

$$f(\eta) = A + B e^{-s\eta} \quad (11)$$

where the constants  $A$ ,  $B$  and  $s$  are given by

$$A = 1/s, \quad B = -1/s, \quad s = \sqrt{1 + (\lambda + M)} \quad (12)$$

Thus the exact solution of eq.(9) reads

$$f(\eta) = \frac{1}{s} \left( 1 - e^{-s\eta} \right) \quad (13)$$

The skin friction  $\tau^*$  at the wall in the non-dimensional form is given as:

$$\tau^* = \frac{\tau}{\mu c x \sqrt{\frac{c}{\nu}}} = f''(0) = -s \quad (14)$$

To solve the energy eq.(3) the temperature distribution is assumed in the form of a similar solution as

$$T = T_\infty + Dx^\alpha \theta(\eta) \quad (15)$$

Using eqs.(6), (8) and (15) in eq.(3) one obtains

$$\theta'' - \left( \frac{3NPr}{3N+4} \right) (\alpha \theta f' - f \theta') = 0 \quad (16)$$

$$\text{subject to the boundary conditions } \theta(0) = 1, \quad \theta(\infty) = 0 \quad (17)$$

where  $Pr = \rho \nu c_p / \kappa$  is Prandtl number and  $N = \kappa k^2 / 4\sigma T_\infty^3$  is radiation parameter.

Equation (16) subject to boundary conditions (17) has been solved numerically using the finite difference scheme with  $\lambda$ ,  $M$ ,  $N$ ,  $\alpha$ ,  $Pr$  as prescribed parameters. The non linear differential eq.(16) is transformed into difference equation at each grid point in the solution space resulting into tridiagonal system of equations which is then solved by Gauss-elimination method. The finite difference methods enjoy an upper hand over other numerical methods such as shooting methods, Runge Kutta methods in terms of accuracy and flexibility in setting the limiting conditions far from the surface. The values of  $\eta_\infty$  for which  $\theta(\eta)$  decays exponentially to zero for different set of values for the parameters  $\lambda$ ,  $M$ ,  $N$ ,  $\alpha$ ,  $Pr$  was chosen after some preliminary investigations. A grid independent study was carried out to examine the effect of the step size  $\Delta\eta$  and the edge of the boundary layer  $\eta_\infty$  on the solution in the quest for their optimization. The  $\eta_{\max}$  i.e.  $\eta$  at  $\infty$  was so chosen that the solution shows little further changes for  $\eta$  larger than  $\eta_{\max}$ . A step size of  $\Delta\eta=0.0001$  was found to be satisfactory for a convergence criteria of  $10^{-6}$  in all cases and the value of  $\eta_\infty=50$  was found to be sufficiently large for the velocity to reach the relevant stream velocity. Further  $\theta'(0)$  has also been computed numerically by employing Newton forward difference interpolation formula for differentiation.

#### 4. Results and Discussion

In order to get an insight of the phenomena under study we illustrate the results obtained from numerical computations by plotting the numerical values in figure 2–figure 9.

It is clear from figure 2 that an increase in permeability parameter  $\lambda$  produces a decrease in temperature while with an increase in magnetic field parameter  $M$  the temperature increases. The temperature is found to decrease with an increase in wall temperature parameter  $\alpha$  and Prandtl number  $Pr$  (fig 3) and also an increase with radiation parameter  $N$  (fig 4).

Figure 5 shows that though the temperature gradient increases with an increase in permeability parameter  $\lambda$  it decreases with an increase in magnetic field parameter  $M$ . The temperature gradient increases with an increase in wall temperature parameter  $\alpha$  as well as with an increase in the radiation parameter  $N$  (fig 6). Figure 7 shows that though the temperature gradient increases with an increase in permeability parameter  $\lambda$  it decreases with an increase in magnetic field parameter  $M$ . The temperature gradient increases with an increase in wall temperature parameter  $\alpha$  as well as with an increase in the Prandtl no  $Pr$  (fig 8).

Fig-9 depicts the values for skin friction and indicates that it decreases with an increase in magnetic field parameter  $M$ .

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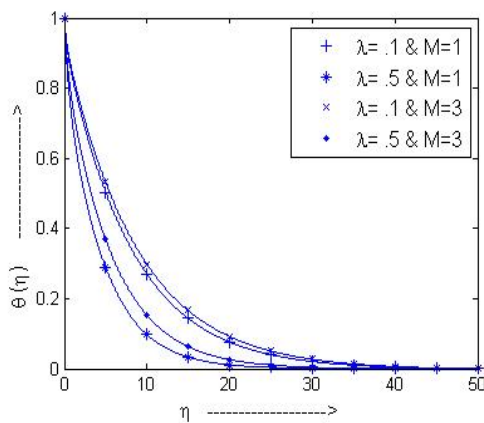


Figure 2: Temperature distribution when  $\lambda$ ,  $M$  vary while  $\alpha = 2$ ,  $Pr = 1$  and  $N = 1$

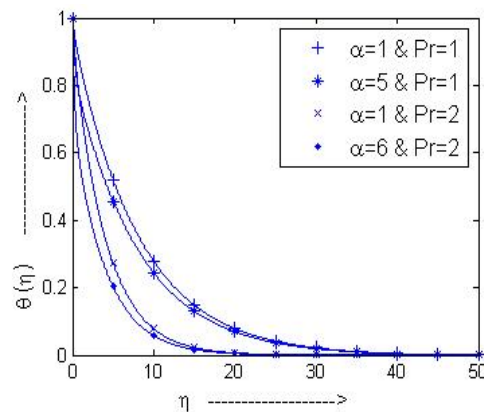


Figure 3: Temperature distribution when  $\alpha$ ,  $Pr$  vary while  $\lambda = 0.1$ ,  $M = 1$  and  $N = 1$



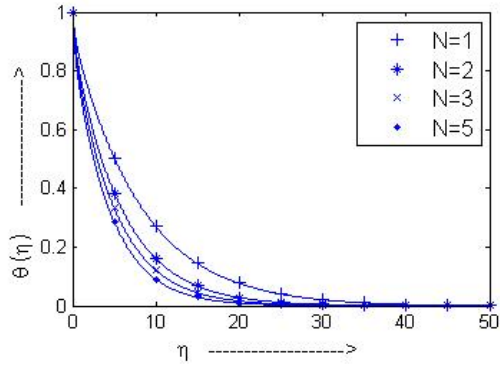


Figure 4: Temperature distribution when  $N$  vary while  $\lambda = 0.1$ ,  $M = 1$ ,  $\alpha = 2$  and  $Pr = 1$

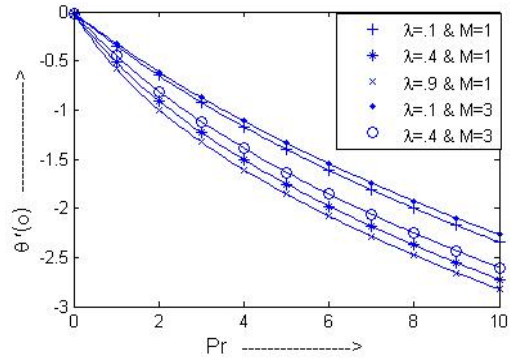


Figure 5: Temperature gradient when  $\lambda$  and  $M$  vary while  $\alpha = 2$  and  $N = 1$

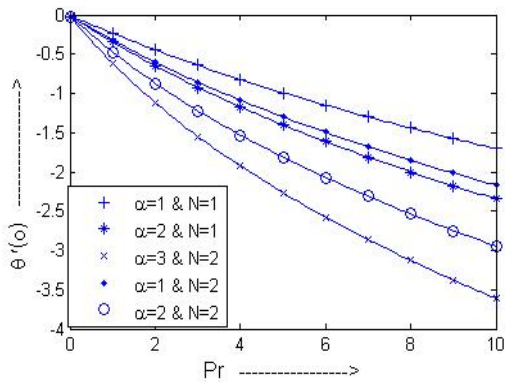


Figure 6: Temperature gradient when  $\alpha$  and  $N$  vary while  $\lambda = 0.1$  and  $M = 1$

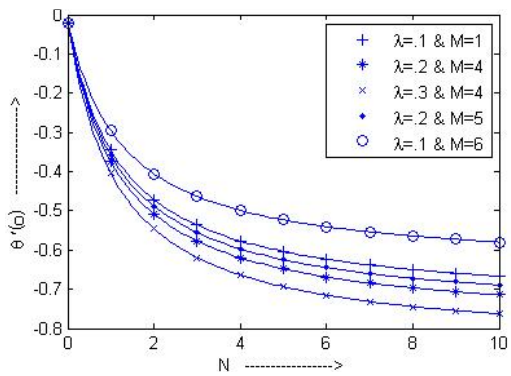


Figure 7: Temperature gradient when  $\lambda$  and  $M$  vary while  $\alpha = 2$  and  $Pr = 1$

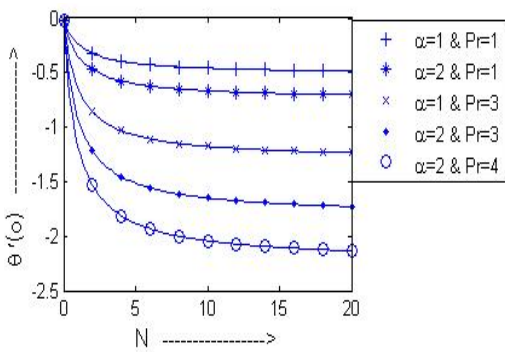


Figure 8: Temperature gradient when  $\alpha$  and  $Pr$  vary while  $\lambda = 0.1$  and  $M = 1$

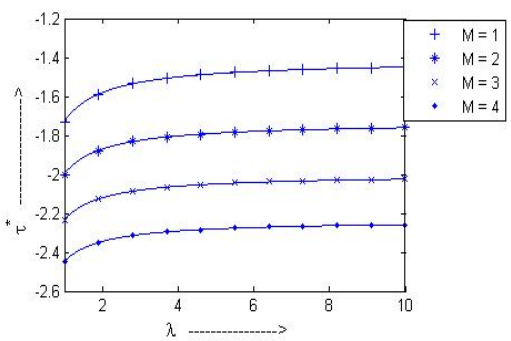


Figure 9: Skin friction for the case of variable  $M$

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