Adv. Theor. Appl. Mech., Vol. 1, 2008, no. 3, 121 - 130

Static Analysis of the Cantilever Subjected to Subtangential Follower Forces

B. S. Shvartsman

International University Audentes,
Tondi 55, 11316 Tallinn, Estonia
bshwar@hot.ee

Abstract

The numerical method for static analysis of the non-uniform cantilever column under distributed subtangential forces is suggested. The boundaries of divergence for tapered cantilever columns subjected to a uniformly distributed subtangential load are examined. It is found that in case of uniform cantilever for various distributions of the subtangential forces the transition value of the parameter of nonconservativeness is equal to 0.5 and buckling load in this point is exactly four times larger than buckling load of the corresponding conservative problem.

Keywords: Nonconservative systems; follower forces; buckling

1. Introduction

Stability of structures subjected to compressive follower forces has been treated by many researchers [1-7]. It is known that the type of instability (flutter or divergence) of these nonconservative systems depends on the amount of boundary conditions, laws of distribution of loading and stiffness, and other parameters. For certain regions of the parameters, these structures are of the divergence type, whereas outside these regions they are of the flutter type.

The rectilinear uniform cantilever column subjected to subtangential end force P is a classical problem in the field of nonconservative stability problems (Figure 1). Here α is the tip angle of the column, $\gamma\alpha$ is the angle between the force and the vertical direction, the parameter $\gamma \in [0,1]$ is known as parameter of nonconservativeness. The case $\gamma = 0$ corresponds to Euler's column (unidirectional vertical force) and $\gamma = 1$ represents Beck's column (tangential follower force). If $\gamma = 0$ the system is conservative (self-adjoint) and hence the loss of stability occurs by divergence (static instability). Under increase of γ from zero the transition from divergence to flutter (dynamic instability) take place when γ is larger than specific value $\gamma_{tr} = 0.5$ called transition value. The transition point is a double critical point at which the first and second buckling load coincides [1-7].

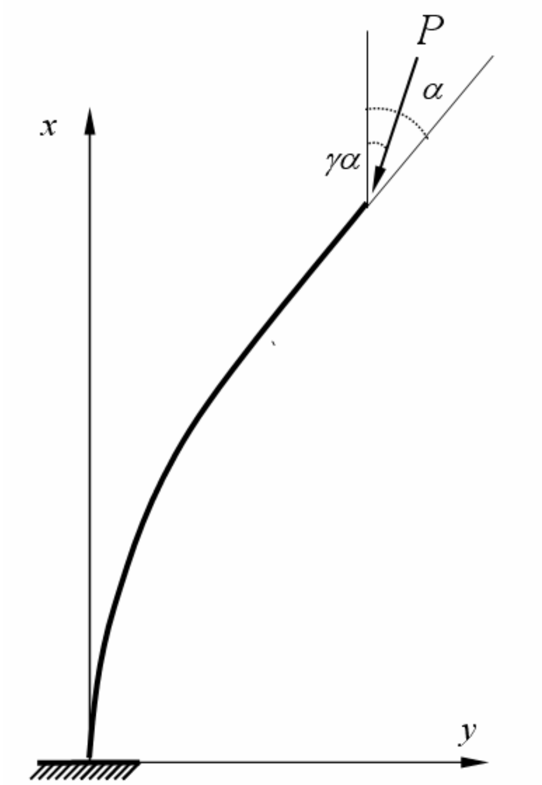

Figure 1. A cantilever column subjected to a tip concentrated subtangential follower load.

The similar behavior takes place for non-uniform cantilevers subjected to tip-concentrated or distributed subtangential load, however value γ_{cr} depends on laws of distribution of loading and stiffness of a column [5-7]. For $\gamma \in [0, \gamma_{cr}]$, divergence appears and critical load can be detected by purely static considerations. For the $\gamma > \gamma_{tr}$ there are no critical loads in the Euler sense (divergence) and stability analysis calls for dynamic approach [1-7].

It was shown [8,9] that the non-uniform cantilever column under distributed or tip concentrated tangential load ($\gamma = 1$) can lose stability only by flutter, that is $\gamma_{tr} < 1$.

In the present paper, numerical method for static analysis of stability non-uniform cantilever columns subjected to distributed subtangential follower forces is suggested. The shooting method is used to solve the linear boundary-value problem for the first two buckling loads and the algorithm to find transition value from divergence to flutter is suggested. The results for three different types of the tapered cantilever subjected to a uniformly distributed subtangential load are examined. It is found that in case of uniform cantilever for various distributions of subtangential forces the transition value parameter of nonconservativeness is equal to 0.5 and critical load in this point is exactly four times larger than buckling load of the corresponding conservative problem.

2. Formulation of the problem

Consider a rectilinear non-uniform cantilever column subjected to a distributed subtangential follower load $q(s)$, which forms the angle $\gamma\varphi(s)$ with the vertical direction at any point of the column axis (Figure 2). The arc length coordinate s measured from the tip of the column, the angle between the vertical axis and the tangent to the deformed curve of the column is $\varphi(s)$ and parameter of nonconservativeness is γ .

We shall be further limited by case of divergence-type instability. Then based on the Euler-Bernoulli beam bending theory, the static linear governing equation can be written in the non-dimensional form:

$$(f(s)\varphi')'' + p(Q(s)\varphi)' + \gamma\, p\, q(s)\varphi = 0 \qquad (1)$$

and the associated boundary conditions are

$$\varphi'(0)=0\,, \quad (f(0)\varphi'(0))' = 0\,, \quad \varphi(1)=0\,, \quad s \subset [0;1] \qquad (2)$$

where p - non-dimensional parameter of load and

$$Q(s) = \int_0^s q(s)ds \qquad (3)$$

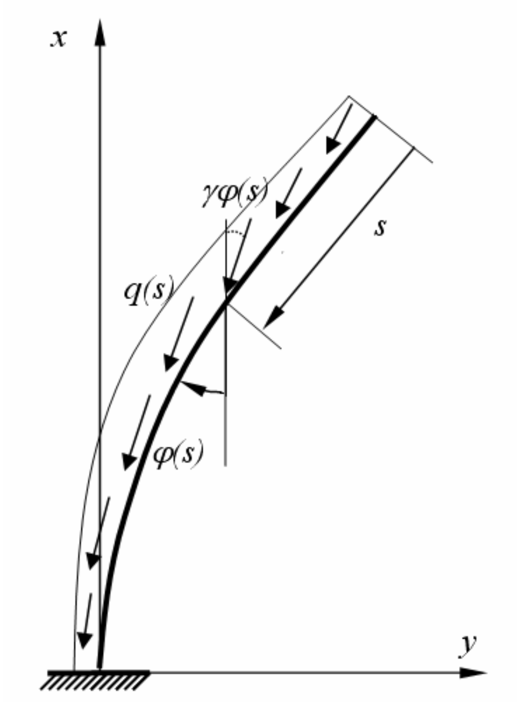

Figure 2. A cantilever column subjected to a distributed subtangential follower load.

The moment of inertia for the three tapered columns considered in this paper is represented by [7]:

$$I(s) = I_R \ f(s) = I_R \ (1 - \varepsilon + \varepsilon \ s)^n \ , \tag{4}$$

where $I_R = B_R D^3 / 12$, $\varepsilon = 1 - B_T / B_R$, $n = 1$ for a column of rectangular cross-section with linear breadth (B) taper (depth being constant); $I_R = B \, D_R^{\ 3} / 12$, $\varepsilon = 1 - D_T / D_R$, $n = 3$ for a column of rectangular cross-section with linear depth (D) taper (breadth being constant); and $I_R = \pi D_R^{\ 4} / 64$, $\varepsilon = 1 - D_T / D_R$, $n = 4$ for a column of circular cross-section with linear diameter (D) taper; the subscripts R and T indicate the quantities at the root and at the tip of the column. The taper parameter value $\varepsilon = 0$ in the present analysis corresponds to the uniform column.

It is necessary to find values p and function $\varphi(s)$ for fixed γ. Once the slope $\varphi(s)$ has been found, the Cartesian coordinates of the buckling mode shape of the column is readily determined from the relations

$$x(s) = 1 - s, \quad y(s) = \int_s^1 \varphi \, ds \tag{5}$$

3. Method of solution

Let us introduce the following notation in order to solve the linear boundary value problem (1),(2):

$$\varphi_1 = \varphi\ ,\quad \varphi_2 = f(s)\varphi'\ ,\quad \varphi_3 = (f(s)\varphi')'\ . \qquad (6)$$

As a result, the equations (1),(2) is reduced to the normal system of linear differential equations

$$\varphi_1' = \varphi_2 / f(s),\ \ \varphi_2' = \varphi_3,\ \ \varphi_3' = -p(Q\varphi_1)' + \gamma\, p\, q(s)\varphi_1 \ (7)$$

with initial conditions

$$\varphi_1(0) = 1\ ,\quad \varphi_2(0) = 0\ ,\quad \varphi_3(0) = 0 \qquad (8)$$

and supplementary condition

$$\varphi_1(1) = 0\,. \qquad (9)$$

According to shooting method, the solution of the problem is reduced to a set of initial-value problems (7),(8) which can be integrated over a given interval $s \in [0;1]$ by a standard numerical method. The values $p(\gamma)$ are iteratively found for fixed values γ from condition (9). Alternatively, the values $\gamma(p)$ can be found for fixed values of parameter p .

Because of the fact that the transition point is a double critical point at which the first and second buckling load coincide [1-7], the value of γ_{tr} can be found numerically from condition

$$\gamma'(p) = 0 \qquad (10)$$

4. Analysis of the tapered cantilever column

Using the method of solution outlined above the first two buckling loads and transition value for the three different types of the tapered cantilever subjected to a uniformly distributed subtangential load $q(s) = 1$ is examined (Figures 3-5). Differential equations (7),(8) were integrated numerically by the fourth-order Runge-Kutta method with a fixed step size equal to 0.01.The numerical results are presented in Tables 1 were compared with the solutions obtained for step size equal to 0.02. The discrepancy between these solutions was found to be within 0.01%.

Table 1. The results for tapered cantilever under uniformly distributed subtangential load

n	ε	$p_1(0)$	$p_2(0)$	γ_{tr}	$p(\gamma_{tr})$	$p(\gamma_{tr})/p_1(0)$
1,3,4	0	7.8373	55.9770	0.5000	31.3494	4.000
1	0.2	7.4976	51.8109	0.4900	29.2511	3.901
1	0.4	7.1352	47.3872	0.4778	27.0218	3.787
1	0.6	6.7421	42.5951	0.4624	24.6068	3.650
3	0.2	6.8300	44.1721	0.4695	25.3486	3.711
3	0.4	5.7789	33.1251	0.4316	19.6041	3.392
3	0.6	4.6568	22.8190	0.3815	14.0825	3.024
4	0.2	6.5027	40.6872	0.4592	23.5409	3.620
4	0.4	5.1309	27.3469	0.4082	16.4867	3.213
4	0.6	3.6956	16.0401	0.3416	10.2155	2.764

It is found that for all these cases the transition value γ_{tr} and the buckling load $p(\gamma_{tr})$ decrease as the taper parameter ε increases. It is seen from Table 1 that in case of the tapered cantilever $(\varepsilon > 0)$ the following inequalities are valid: $\gamma_{tr} < 0.5, \quad p(\gamma_{tr})/p_1(0) < 4$. The same results was found for tapered cantilever under various distributions of the subtangential load $q(s)$. For the $\gamma > \gamma_{tr}$ there are no critical divergence loads and stability analysis calls for dynamic approach [1-7]

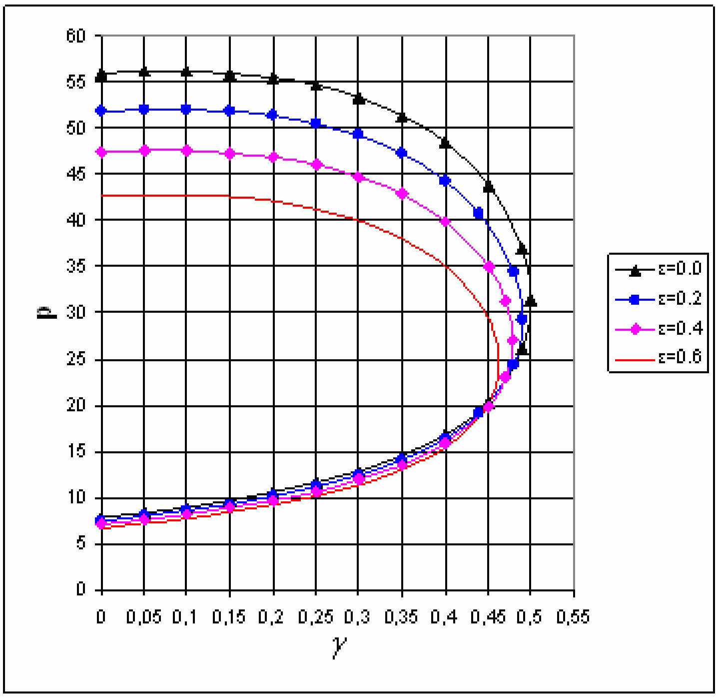

Figure 3. The first and second buckling loads for column of rectangular cross-section with linear breadth taper (n = 1).

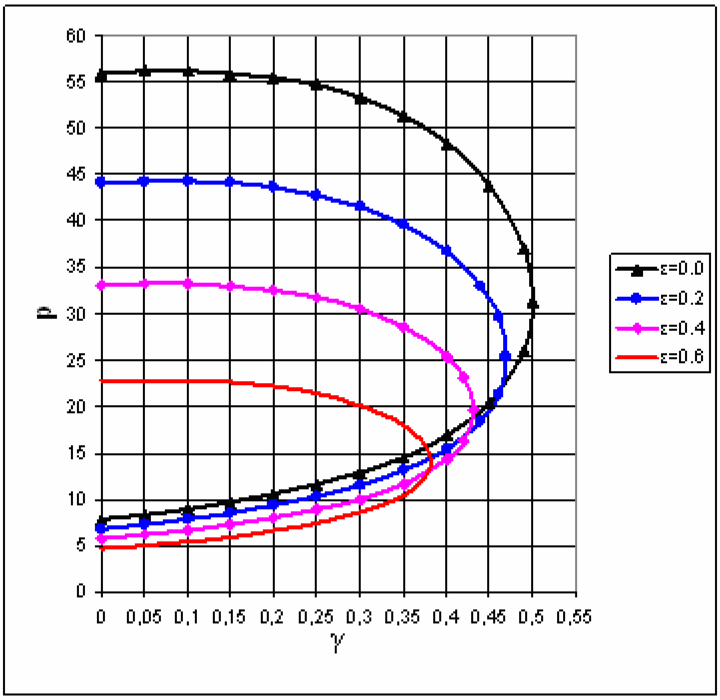

Figure 4. The first and second buckling loads for column of rectangular cross-section with linear depth taper (n = 3)

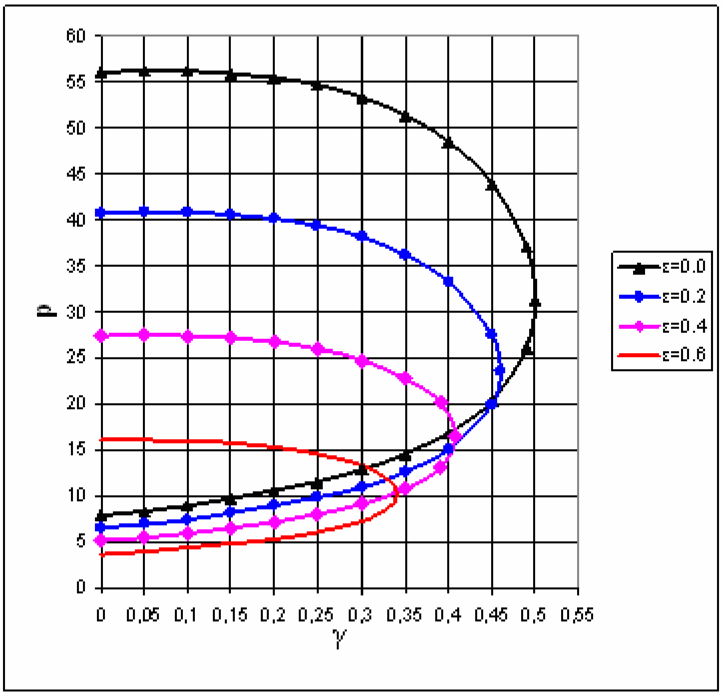

Figure 5. The first and second buckling loads for column of circular cross-section with linear diameter taper (n = 4)

5. Analysis of the uniform cantilever

Table 2 demonstrates the results of the solutions for uniform cantilever under various distributions of the subtangential follower load $q(s)$. Differential equations (7),(8) were integrated numerically by the fourth-order Runge-Kutta method with a fixed step size equal to 0.005. It is important to notice that in any case of distribution of the subtangential follower load $q(s)$ the transition value $\gamma_{tr} = 0.5$ and critical load in this point is exactly four times larger than buckling load of the corresponding conservative problem (the last column of the Table 2).

It is found numerically that first buckling shape do not have any inflection points, while the second buckling shape has one inflection point. The transition shape form ($\gamma = \gamma_{tr}$) has an inflection point at the root of column. In other words, for uniform cantilever the condition $\varphi'(1) = 0$ corresponds to transition point at which the first and second buckling loads coincide and can be used instead of (10).

It is easy to explain the similar results for uniform cantilever under a tip-concentrated subtangential follower force [1,3,5,6]. In this particular case the solution at the transition point $\gamma_{tr} = 0.5$ has a simple geometric interpretation. It corresponds to buckling of the beam with two hinges [3]. As a result, the inflection point is located at the root of column and $p(\gamma_{tr}) = \pi^2 = 4p_1(0)$.

Table 2. The first two buckling loads for and for uniform cantilever under various distributions of the subtangential load

$q(s)$	$p_1(0)$	$p_2(0)$	$p_1(0.5) = p_2(0.5)$	$p_1(0.5)/p_1(0)$
1	7.837347	55.978	31.349390	4.00000
s	32.201907	209.967	128.807630	4.00000
s^2	81.770715	507.676	327.082877	4.00000
s^3	165.219232	994.814	660.876994	4.00000
$1-s$	10.243339	79.322	40.973355	4.00000
$1-2s$	14.582210	141.545	58.328842	4.00000
$4s(1-s)$	13.064346	94.006	52.257386	4.00000

5. Conclusions

The static stability of the rectilinear non-uniform cantilever column subjected to distributed subtangential forces is considered. The results for three different types of the tapered cantilever subjected to a uniformly distributed subtangential load are examined. It is shown numerically that in the case of uniform cantilever for various distributions of the subtangential follower forces the transition value of the parameter of nonconservativeness is equal to 0.5 and critical load in this point is exactly four times larger than buckling load of the corresponding conservative problem.

References

[1] V.V. Bolotin, *Nonconservative problems of the theory of elastic stability*, Pergamon Press, Oxford. UK, 1963.

[2] I. Elishakoff, Controversy associated with the so-called ‘‘follower forces’’: critical overview, *Applied Mechanics Reviews*, 58 (2005), 117–142.

[3] Z. Kordas, M. Zyczkowski, On the loss of stability of a rod under a super-tangential force, *Archiwum Mechaniki Stosowanej*, 15 (1963), 7-31.

[4] A.N. Kounadis, The existence of regions of divergence instability for nonconservative systems under follower forces, *International Journal of Solids and Structures*, 19 (1983), 725-733.

[5] M.A. Langthjem, Y. Sugiyama, Dynamic stability of columns subjected to follower loads: a survey, *Journal of Sound and Vibration*, 8 (2000), 809–851.

[6] H. Leipholz, *Stability of elastic systems*. Sijthoff and Noordhoff, The Netherlands, 1980.

[7] B.N. Rao, G.V. Rao, Applicability of static or dynamic criterion for the stability of a non-uniform cantilever column subjected to a tip-concentrated subtangential follower force, *Journal of Sound and Vibration*, 122 (1988), 188-191.

[8] B.S. Shvartsman, Direct method for analysis of flexible beam under a follower load, *Proceedings of Computational of Mechanics for the Next Millennium*, Vol. 1, Singapore, 1999, 155-160

[9] B.S. Shvartsman, Large deflections of a cantilever beam subjected to a follower force. *Journal of Sound and Vibration*, 304 (2007), 969-973.

Received: May 1, 2008